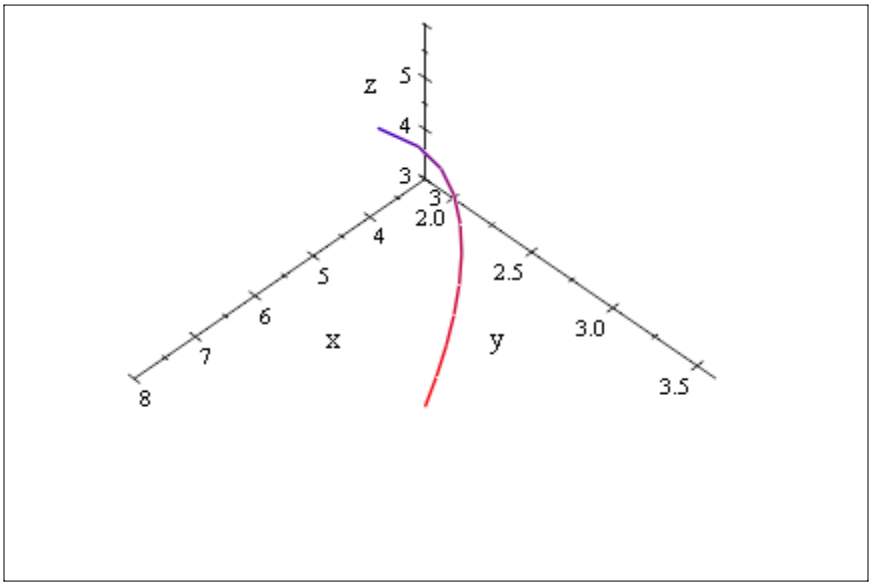
Exercise on the Page 880 of Calculus 5th Edition by James Stewart , 5th edition,

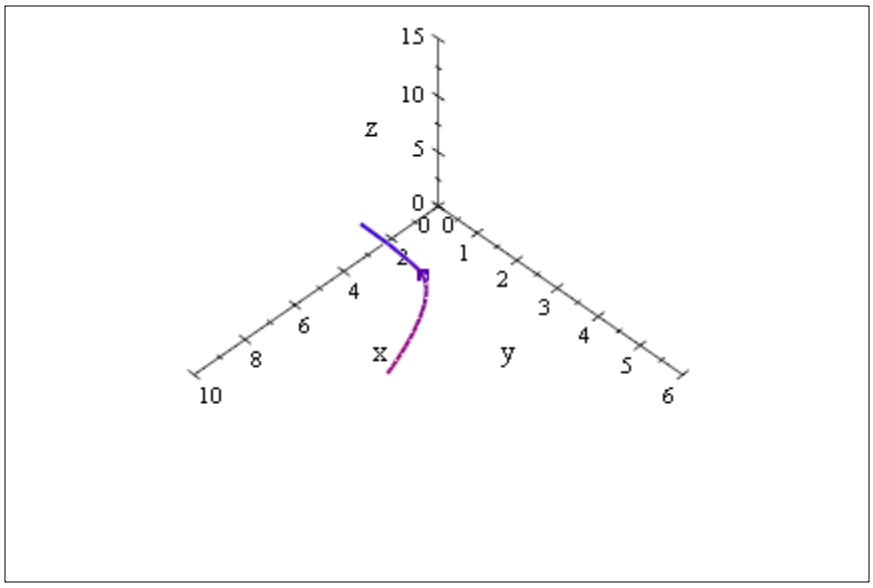
The position function of a spaceship is

$$\vec{r}(t) = (3+t)i + (2+\ln t)j + \left(7 - \frac{4}{t^2+1}\right)k$$

and the coordinates of a space station are $(6,4,9)$. The captain wants the spaceship to coast into the space station. When should the engines be turned off?

$$\left[3+t, 2+\ln t, 7-\frac{4}{t^2+1}\right]$$

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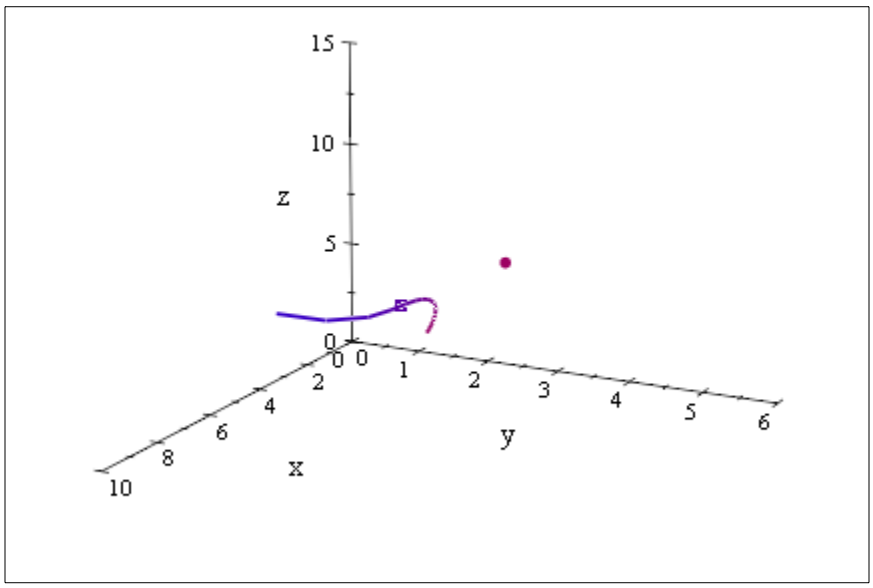
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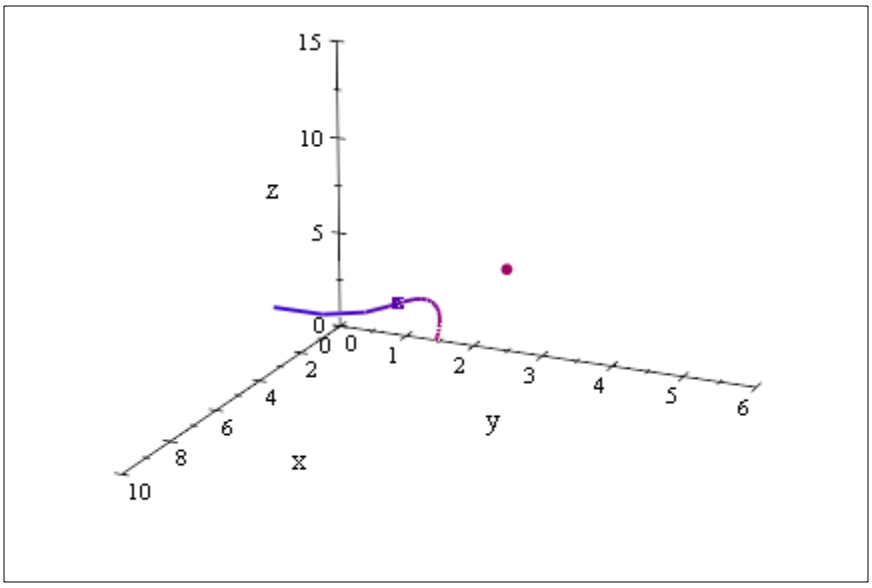
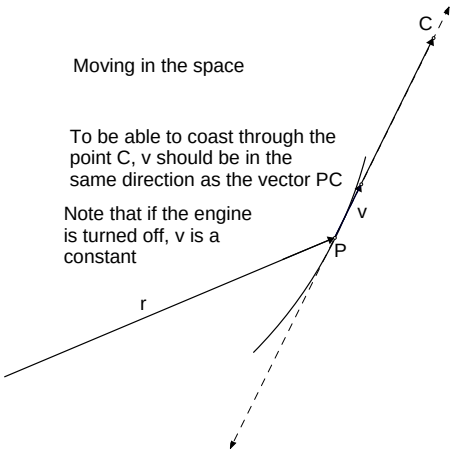


for P, the position $\vec{r} = \left\langle 3 + t, 2 + \ln t, 7 - \frac{4}{t^2 + 1} \right\rangle$

$\vec{v} = \frac{d\vec{r}}{dt} = \left\langle 1, \frac{1}{t}, \frac{8t}{\left(t^2 + 1\right)^2} \right\rangle$

The coordinates of the space station C are $(6, 4, 9)$



We should be able to get a positive number μ so that

$\overrightarrow{PC} = \mu\vec{v}$

Remember that the components of $\overrightarrow{PC}$ are the differencs of the coordinates of C and P in order, that is

$$\overrightarrow{PC} = \left\langle 6-(3+t), 4-(2+\ln t), 9-\left(7-\frac{4}{1+t^2}\right)\right\rangle$$

For $\overrightarrow{PC} = \mu\vec{v}$, we must have

$$\left\langle 3-t, 2-\ln t, 2+\frac{4}{1+t^2}\right\rangle = \mu\left\langle 1, \frac{1}{t}, \frac{8t}{\left(t^2+1\right)^2}\right\rangle$$

or

$$\left\langle 3-t, 2-\ln t, 2+\frac{4}{1+t^2}\right\rangle = \left\langle \mu, \frac{\mu}{t}, \frac{8t\mu}{\left(t^2+1\right)^2}\right\rangle$$

$3-t=\mu$ ..... (1)

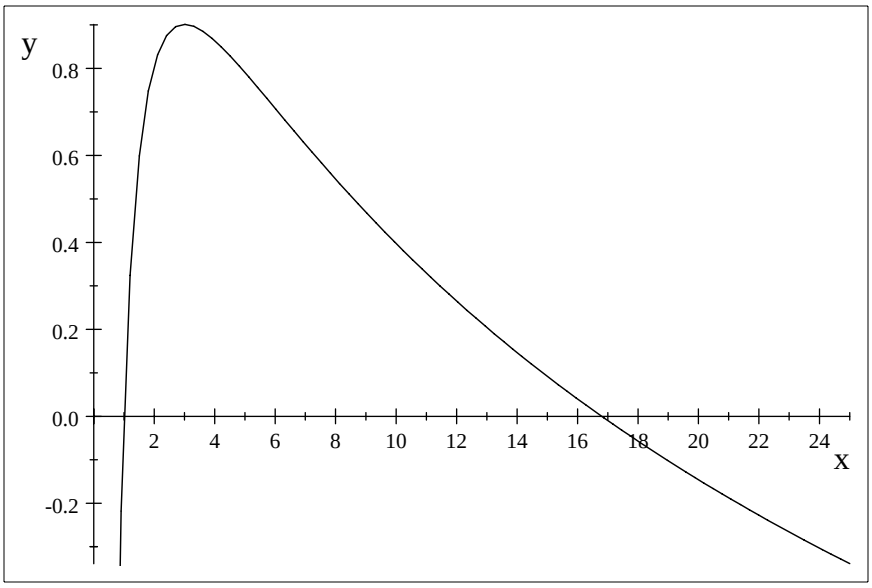
$2-\ln t=\frac{\mu}{t}$ ..... (2)

$2+\frac{4}{1+t^2}=\frac{8t\mu}{\left(t^2+1\right)^2}$ ..... (3)

Subsitute for $\mu = 3-t$ from the equation number (1) in the equation (2)

$2-\ln t=\frac{3-t}{t}$

$2-\ln t-\frac{3-t}{t}$



$t = 1$

$2 - \ln t = \frac{3-t}{t}$, Solution is: $\{[t = 16.801016190708342853]\}$

$$2 + \frac{4}{1+t^2} = \frac{8t\mu}{\left(t^2+1\right)^2}$$

$2 + \frac{4}{1+1^2} = \frac{8(1)(3-1)}{\left(1^2+1\right)^2}$ is true GREAT

$$2 + \frac{4}{1+16.801016190708342853^2} = \frac{8(16.801016190708342853)(3-16.801016190708342853)}{\left(16.801016190708342853^2+1\right)^2}$$

is false

$$2 + \frac{4}{1+16.801016190708342853^2} - \frac{8(16.801016190708342853)(3-16.801016190708342853)}{\left(16.801016190708342853^2+1\right)^2}$$

2.037237131355742153 9