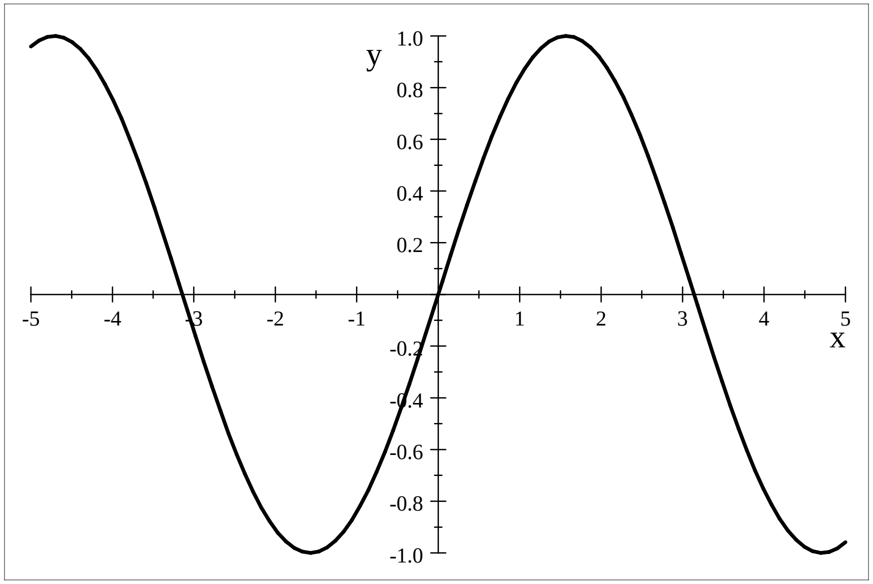
In this study, we are going to discuss the Inverse Trigonometric Functions

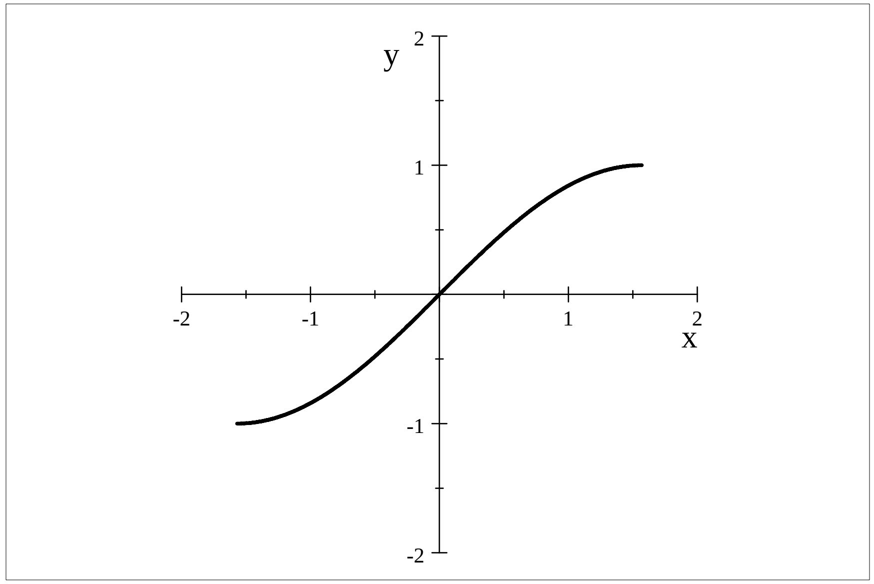
Note that graph of
$y = \sin x$



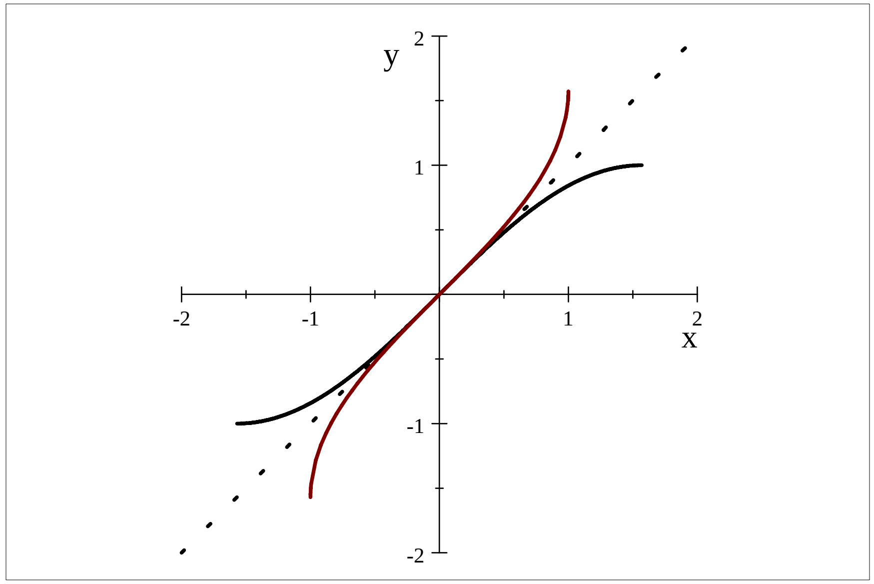
Fails the horizontal line test, therefore we are not able to write a function that is inverse of $f(x) = \sin x$

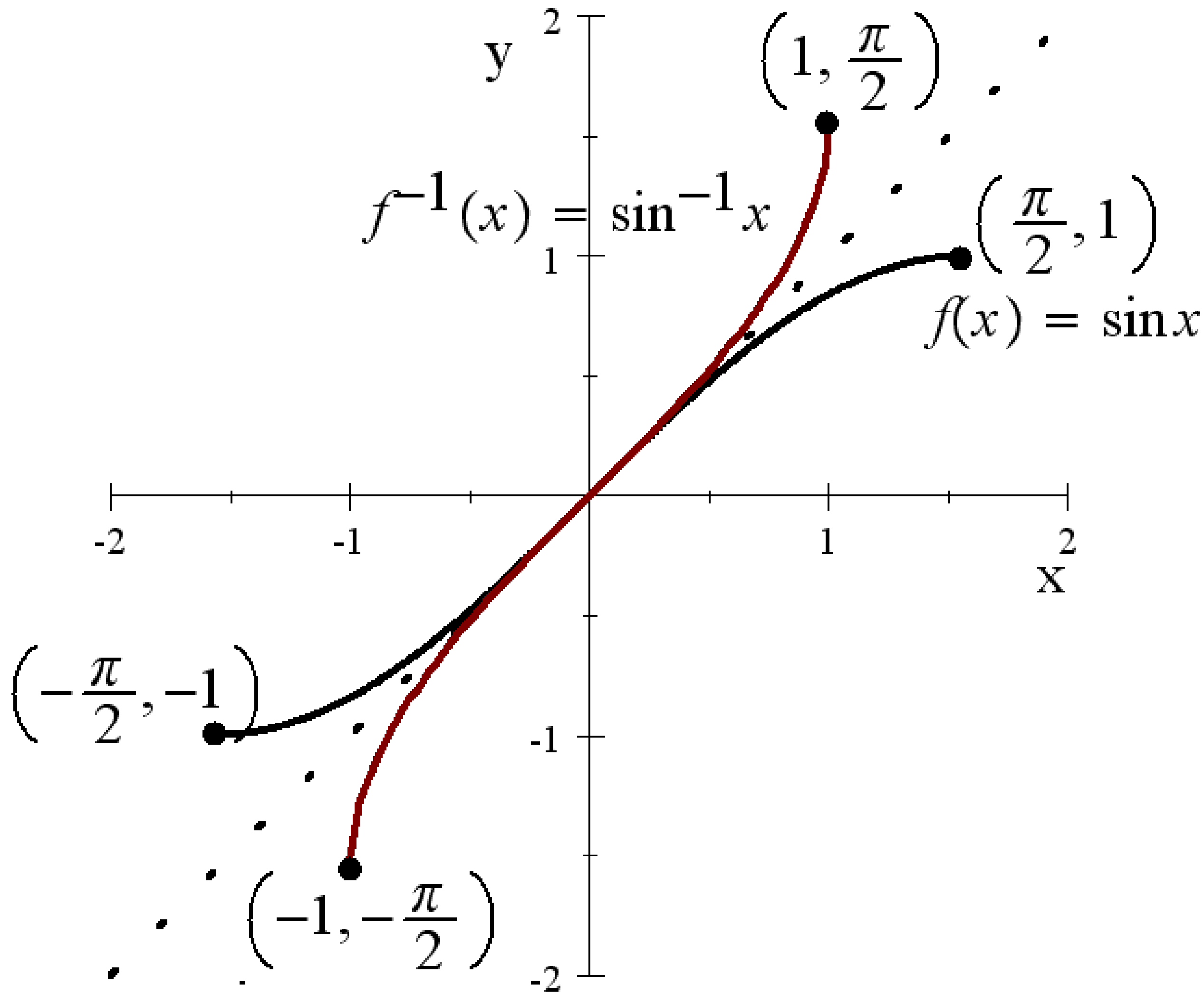
However, if we take

$f(x) = \sin x$ **with** $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$


y
2
1
-1
-2
-2
-1
1
2
x

We have a graph of an invertible function and we have


y
2
1
-1
-2
-2
-1
1
2
x



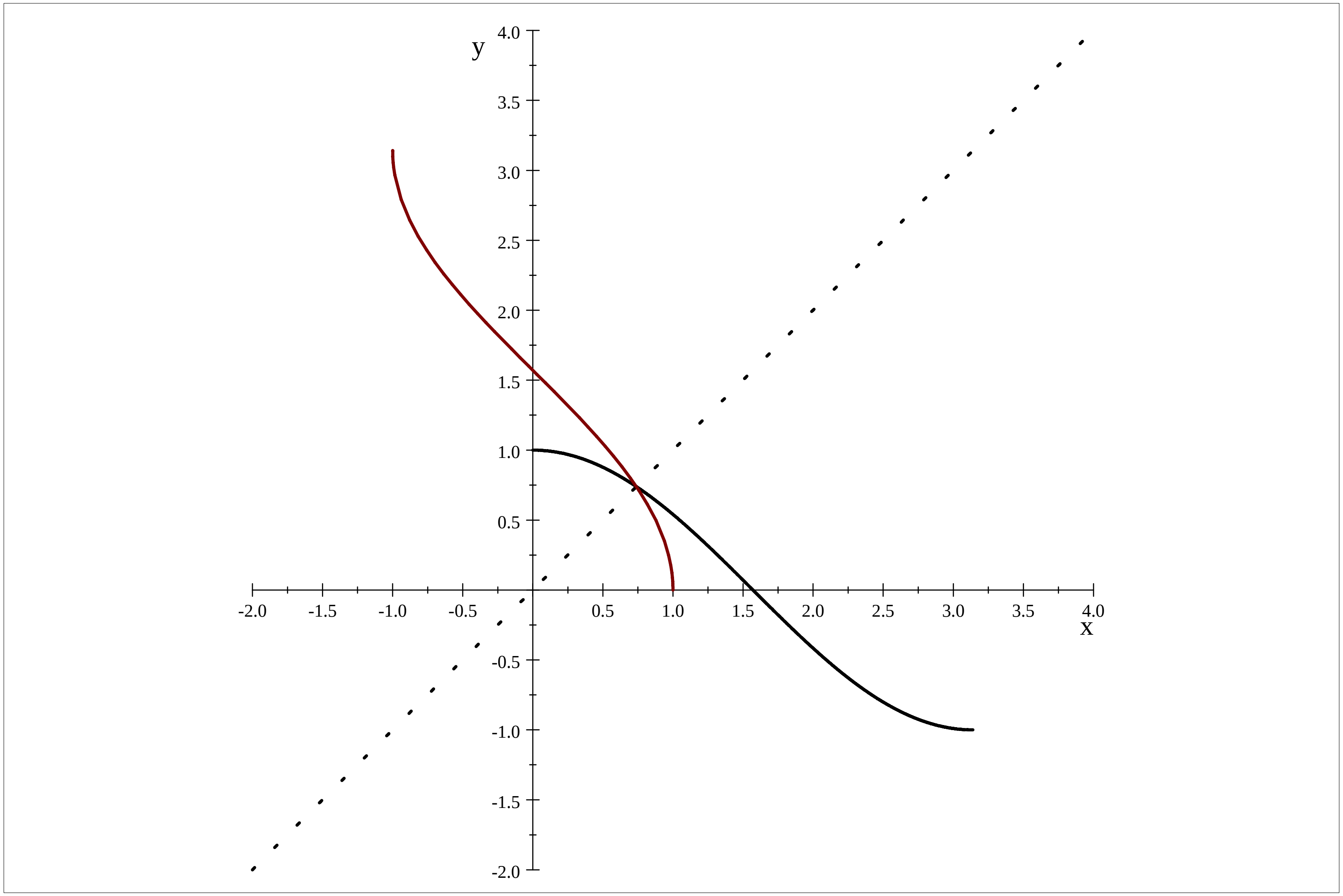
$\mathbf{f^{-1}}(x)=\sin^{-1}\mathbf{x}$ $\quad\quad$ $\mathbf{-1 \le x \le 1}$

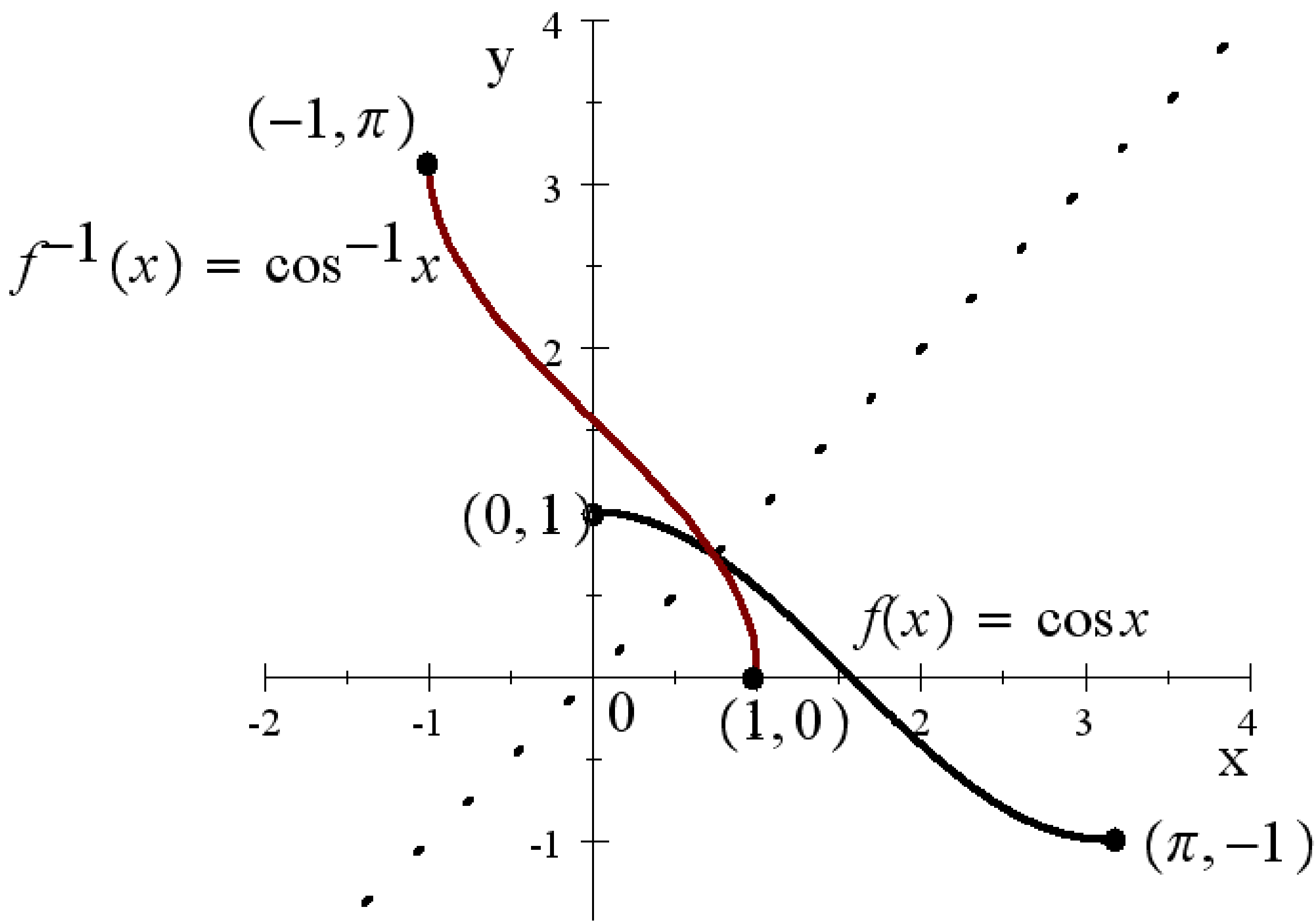
The domain of $f^{-1}(x) = \sin^{-1}x$ **is** $[-1, 1]$ **and the range is** $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\sin\boldsymbol{\theta} = \frac{1}{2} \Rightarrow \boldsymbol{\theta} = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$
$$\sin\theta = -\frac{1}{2} \Rightarrow \theta = \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$$

Like wise, if we took

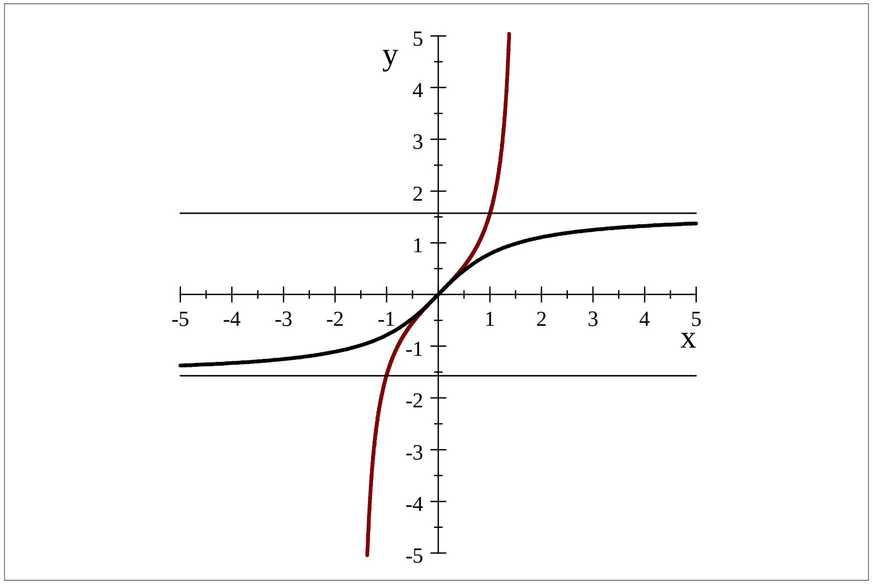
$f(x) = \cos x$ **with** $0 \le x \le \pi$
we can have $f^{-1}(x) = \cos^{-1}x$ **with** $-1 \le x \le 1$ **as a function as shown below**

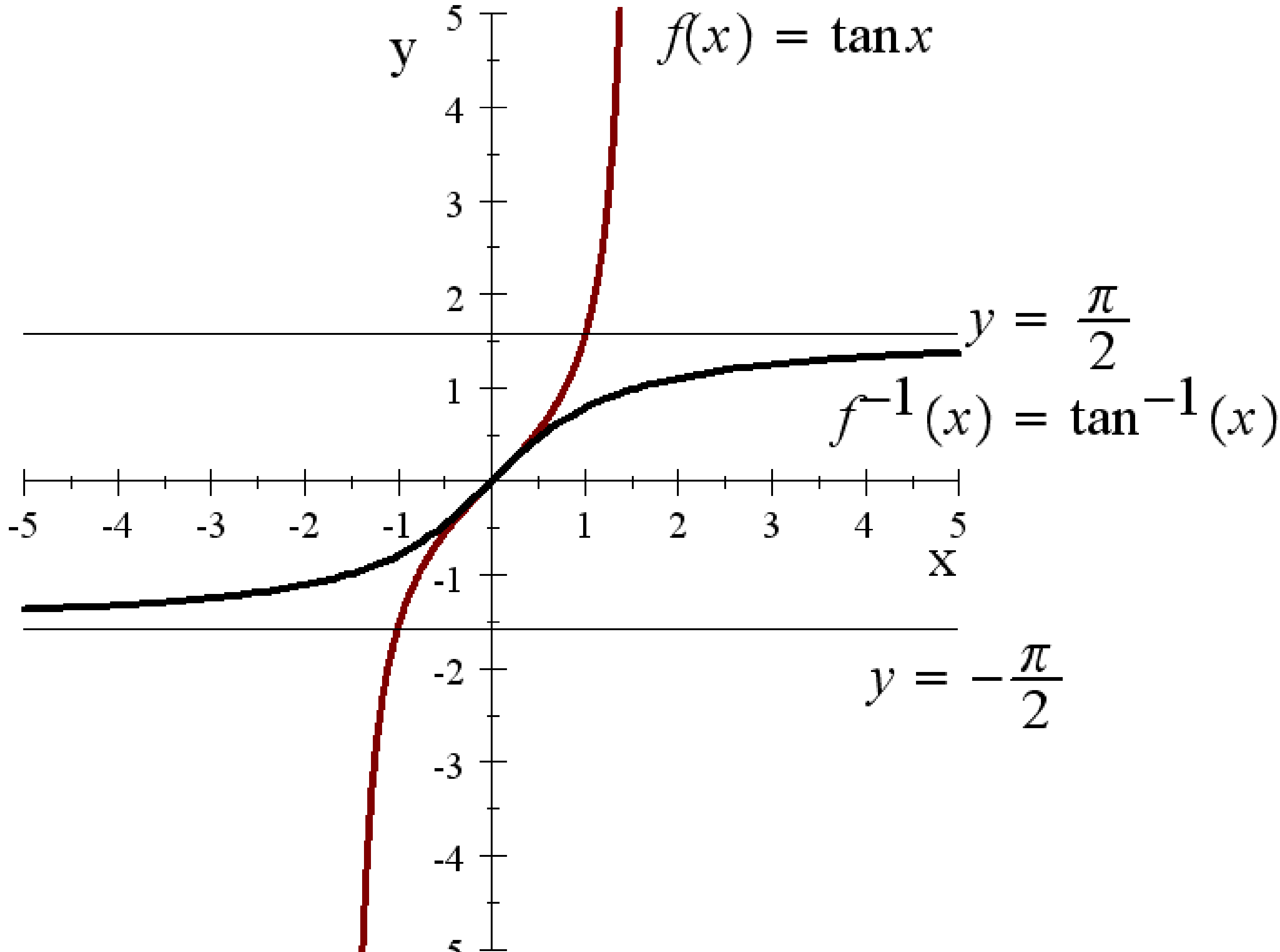

y
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
-0.5
-1.0
-1.5
-2.0
-2.0
-1.5
-1.0
-0.5
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
x



Similary, **for** $f(x) = \tan x$ **with** $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$f(x) = \tan x$ $\qquad f^{-1}(x) = \tan^{-1}(x)$ $\qquad y = -\frac{\pi}{2}$ $\qquad y = \frac{\pi}{2}$





Note that for

the function given by $y = \tan^{-1} x$

The domain is $(-\infty,\infty)$ **and the range is** $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Note that

$\tan^{-1}x \to \frac{\pi}{2}$ **as** $x \to \infty$
$\tan^{-1}x \to -\frac{\pi}{2}$ **as** $x \to -\infty$

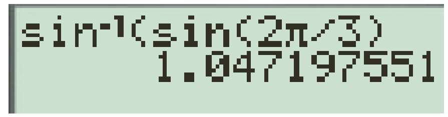
Worked out exercises from the Text

Page 521

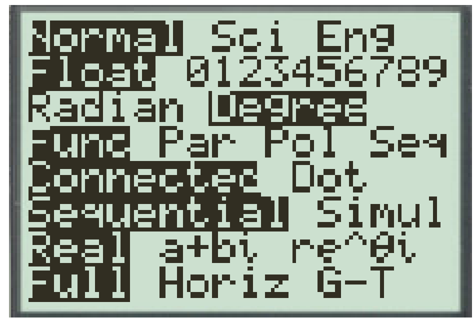
12. To find $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$
Caution: Note that according the definition above, we can not take $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right) = \frac{2\pi}{3}$ **because** $\frac{2\pi}{3}$ **is not in the range** $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ **of inverse sine.**

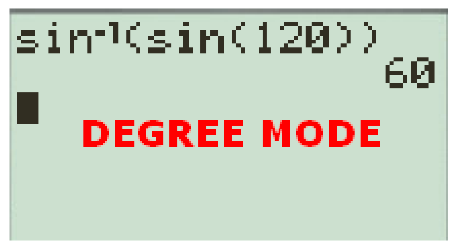
When evaluating a quantity of type $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$ **, it is always a good idea to first evaluate** $\sin\left(\frac{2\pi}{3}\right) = \frac{1}{2}\sqrt{3}$ **and then look at**
$\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right) = \sin^{-1}\left(\frac{1}{2}\sqrt{3}\right) = \frac{\pi}{3}$ **in** $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

When using your TI83 or TI84



OR in the degree mode





Keep the special triangles in mind

I am going to post more worked out exercises tomorrow