Lesson 2 Part 2

Author: Atul Roy

In the lesson 2 part 1, we visited Examples of computation of probabilities based on counting and empirical evidence.

We are going to look at more examples, where we use the techniques of counting and tree diagrams.

Example :

1.

A small college has 1031 students. This semester, 126 of these are enrolled in accounting, 91 in Business Mathematics, and 35 in both. Let us use Venn Diagrams to determine

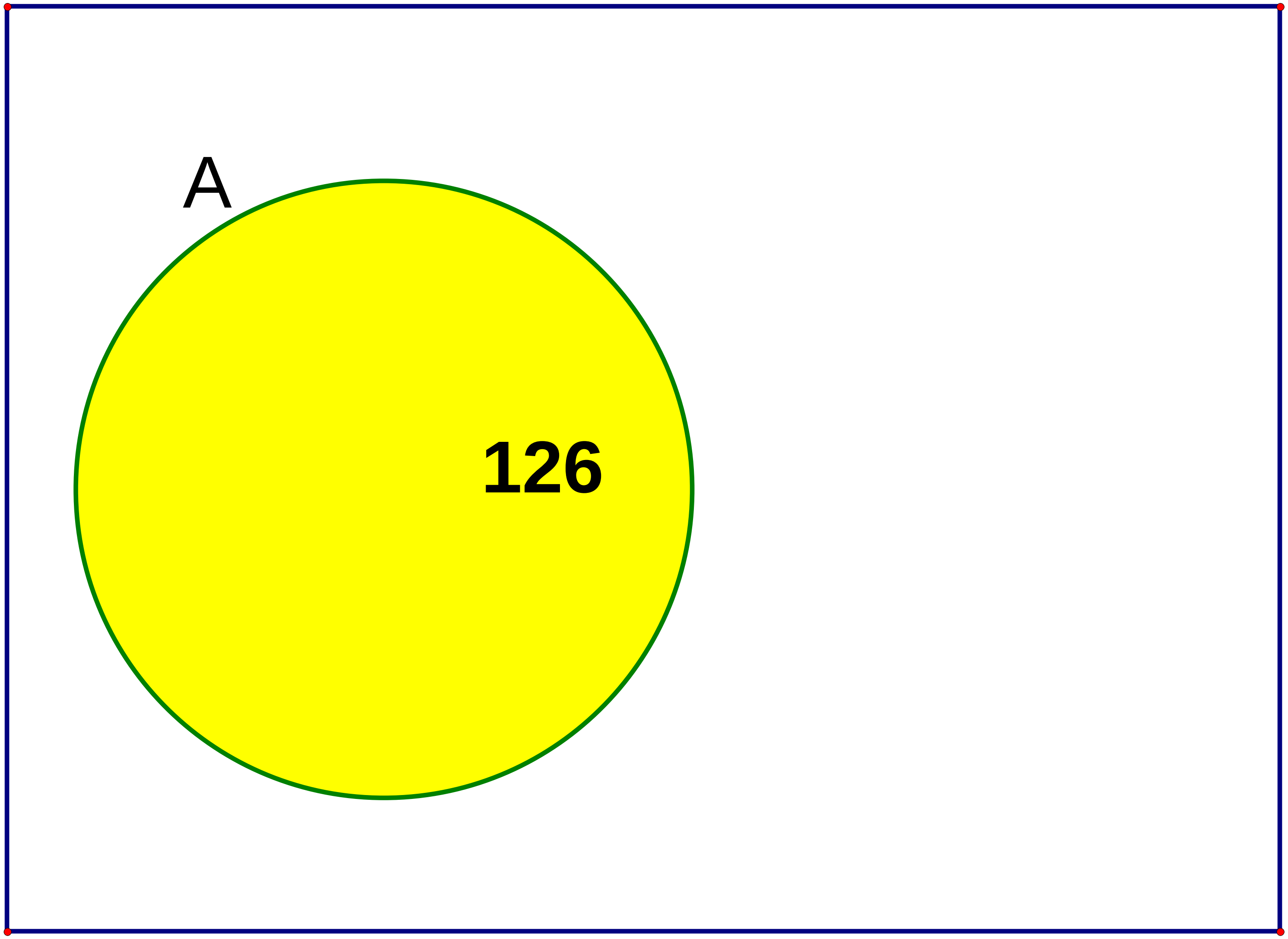
a) How many of the students are taking Accounting or Business Mathematics.

b) How many of the students are neither taking accounting nor business mathematics.

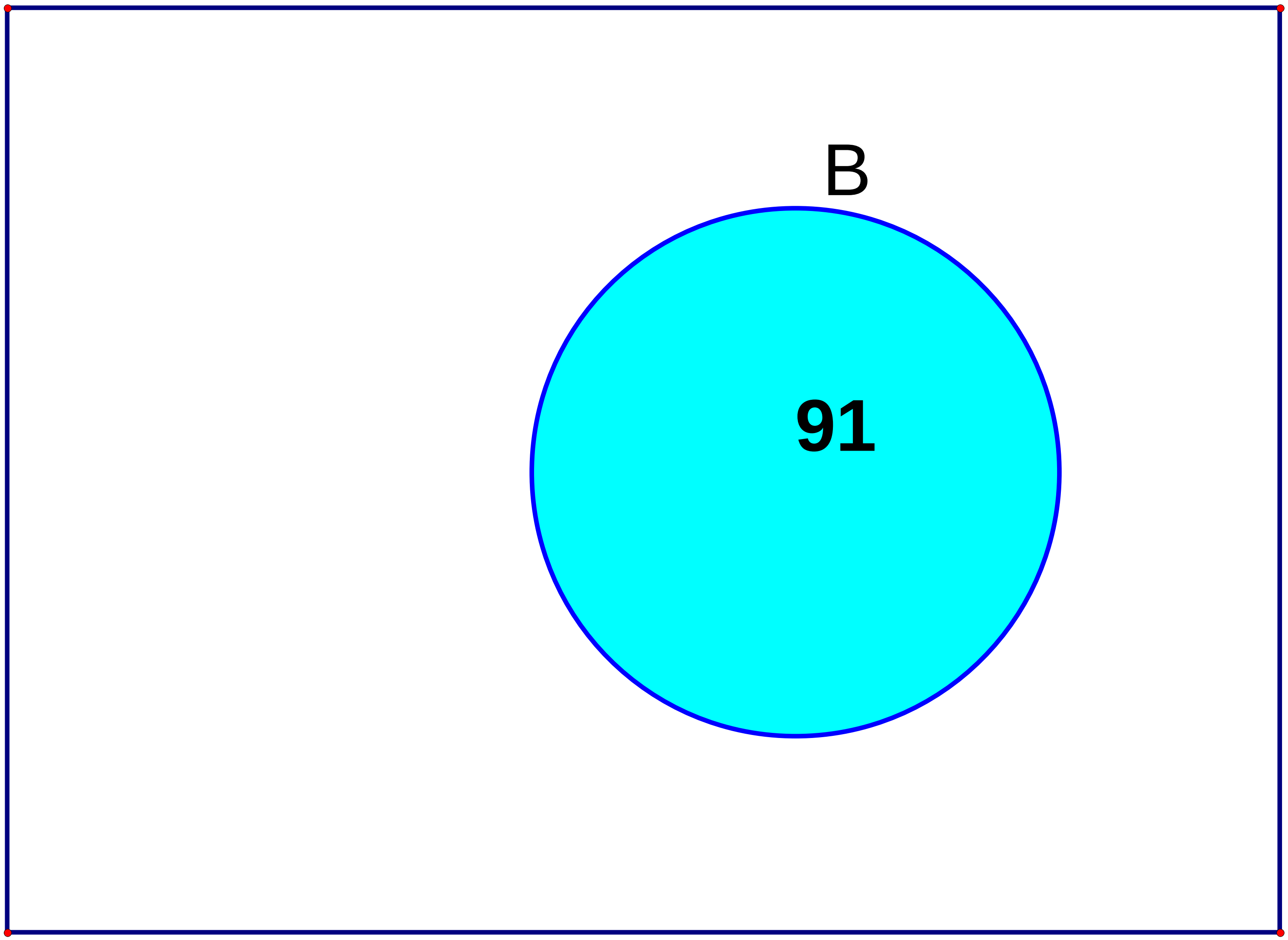
Let us use the symbols

A: The set of students taking Accounting
B: The set of students taking Business Mathematics

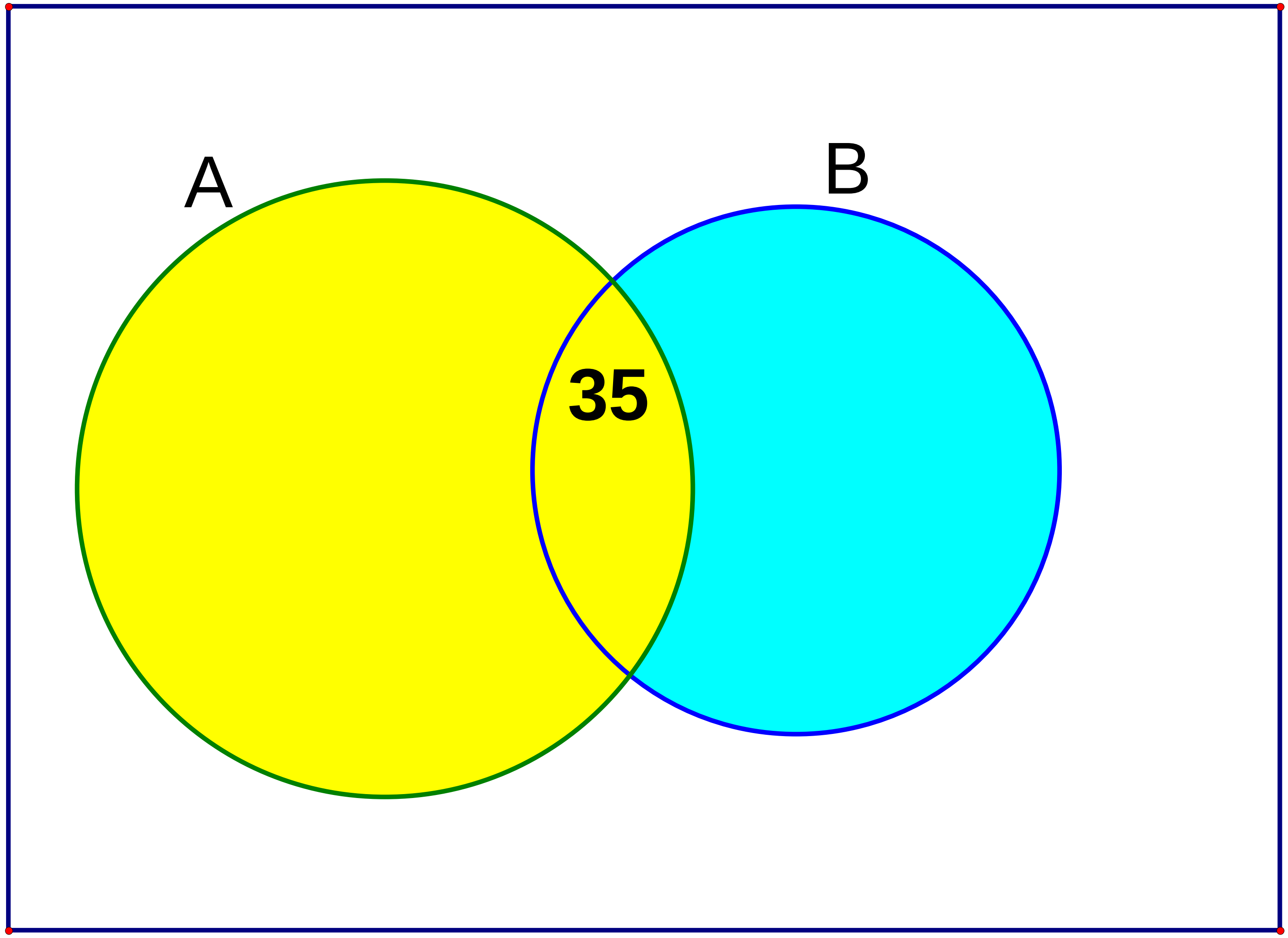
Note that the set A has 126 elements indicating that 126 students are taking accounting



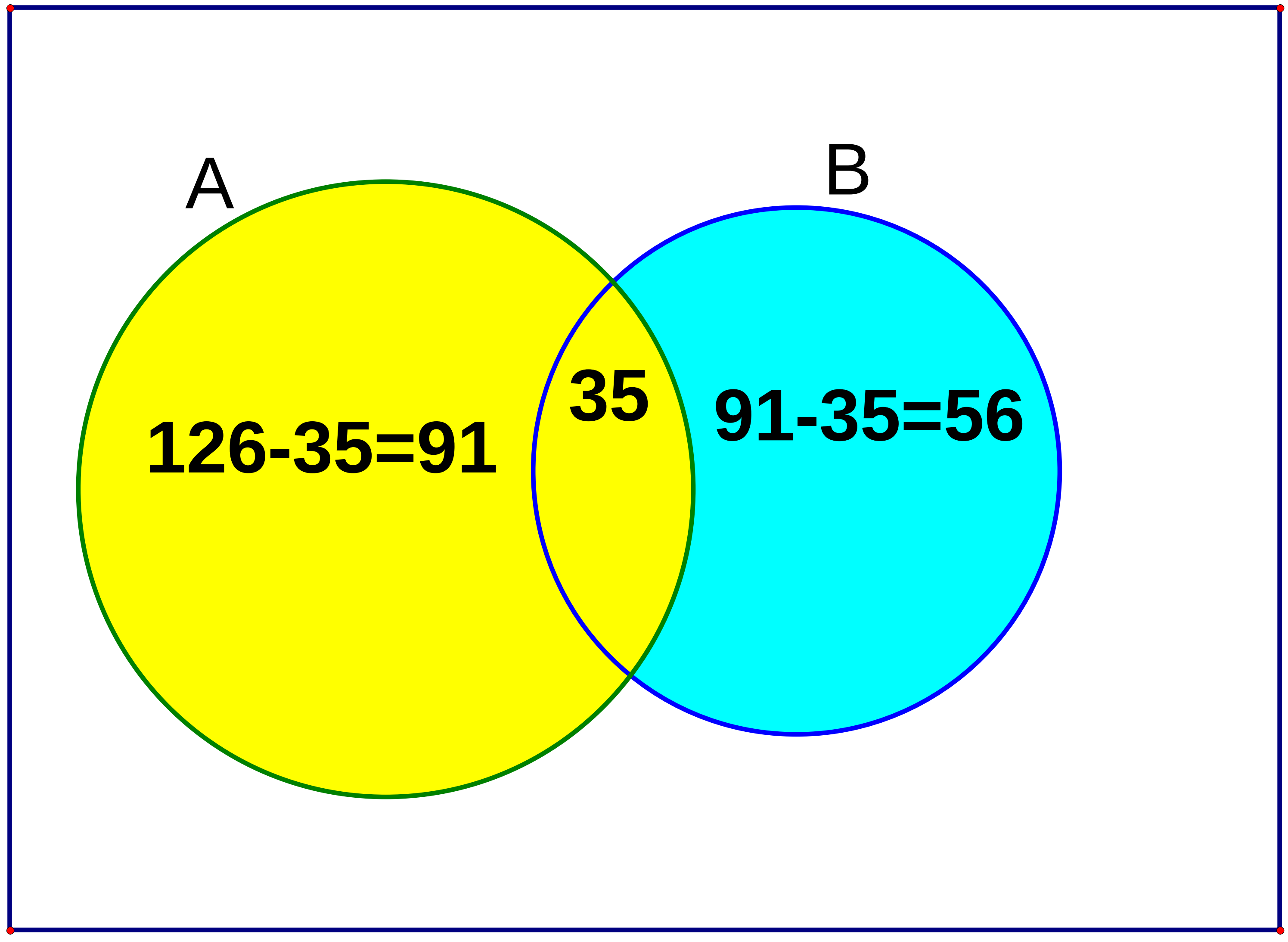
the set B has 91 elements indicating that 91 students are taking Business Mathematics

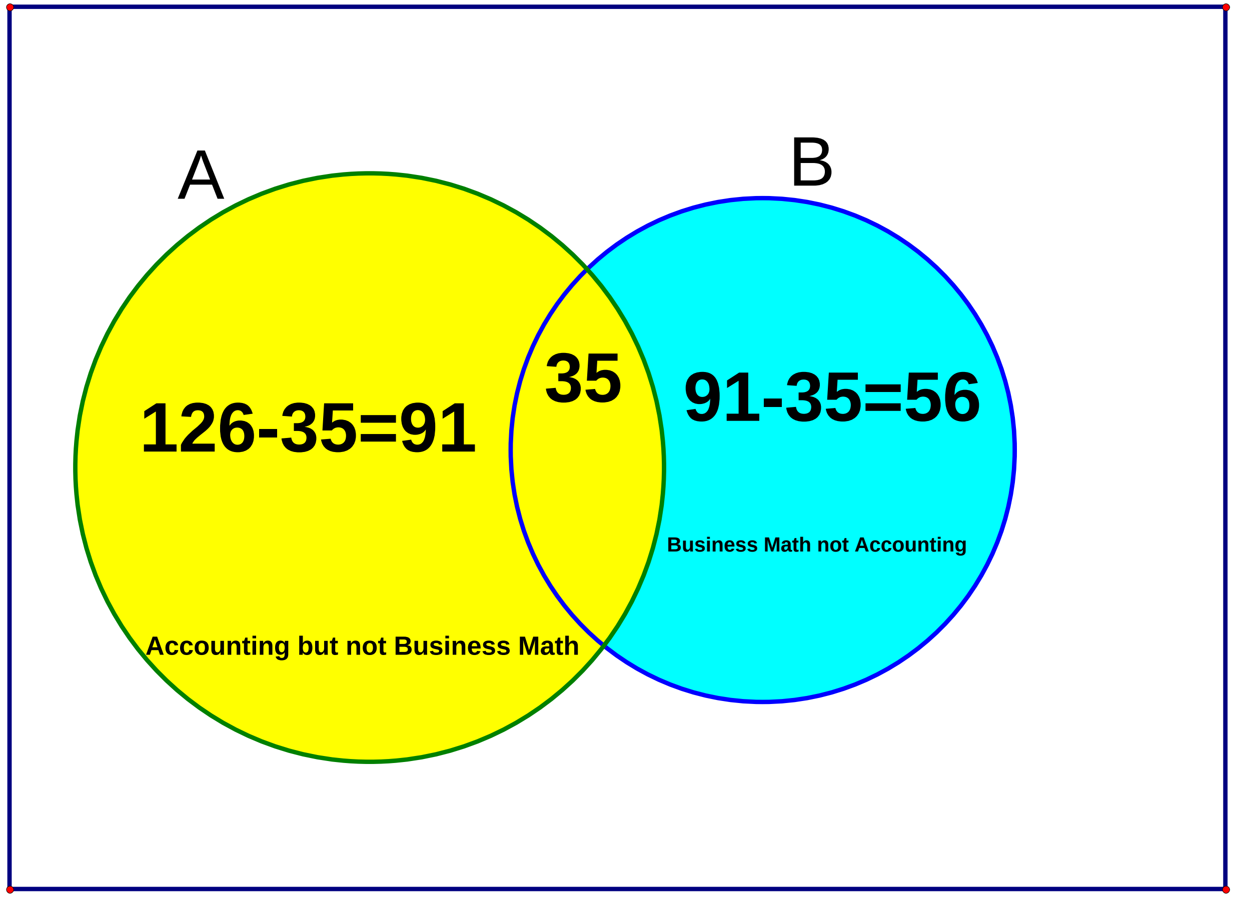


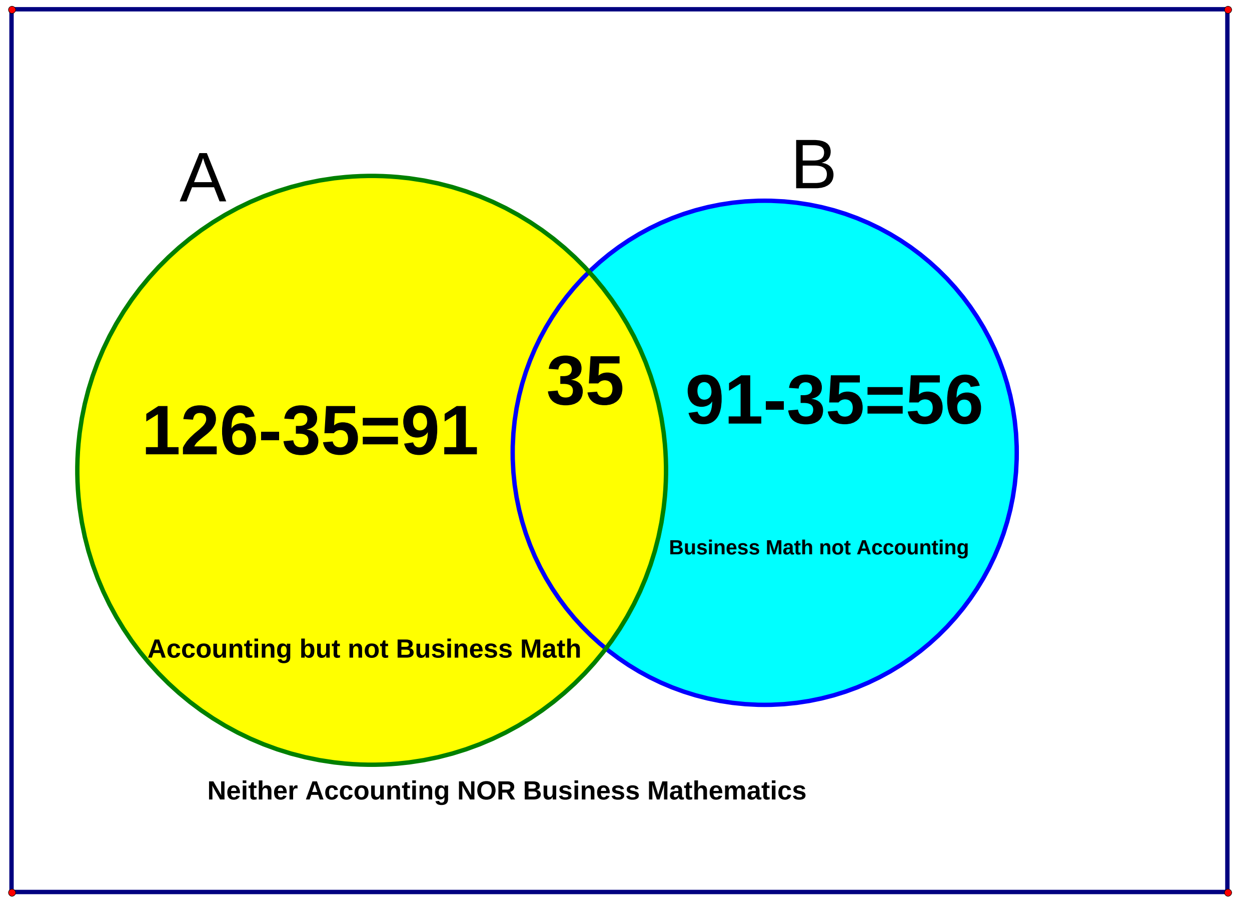
The set $A \cap B$, the set containing the common elements of A and B, that is the set of students taking both, Accounting and Business Mathematics has 35 elements in it

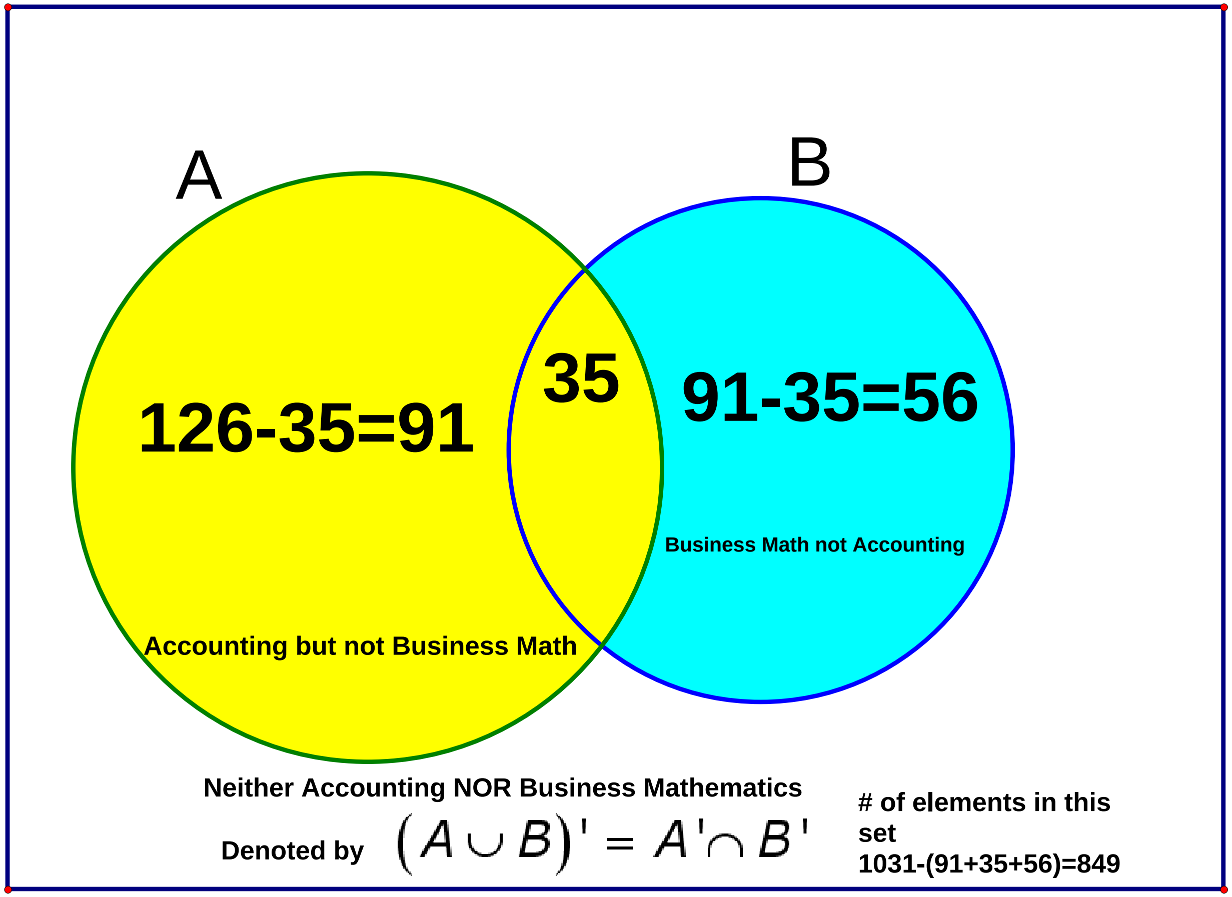


Now look at the following arithmetic that is given in the diagrams below


A
B
126-35=91
35
91-35=56


A
B
35
126-35=91
91-35=56
Business Math not Accounting
Accounting but not Business Math


A
B
35
126-35=91
91-35=56
Business Math not Accounting
Accounting but not Business Math
Neither Accounting NOR Business Mathematics



Now let us use the above diagram to answer the following questions:

If a student is randomly selected from the above college, to find the probability that the student is taking Accounting OR Business Mathematics

Symbolically, **we denote this set by the symbol** $A \cup B$ **that is the set** A **union** B
the set that contains the elements that belong to at least one of the wto sets.

The number of elements in $A \cup B$ **is** $91 + 35 + 56 = 182$

The number of students at the entire college is 1031 that is the universal set in this context has 1031 students

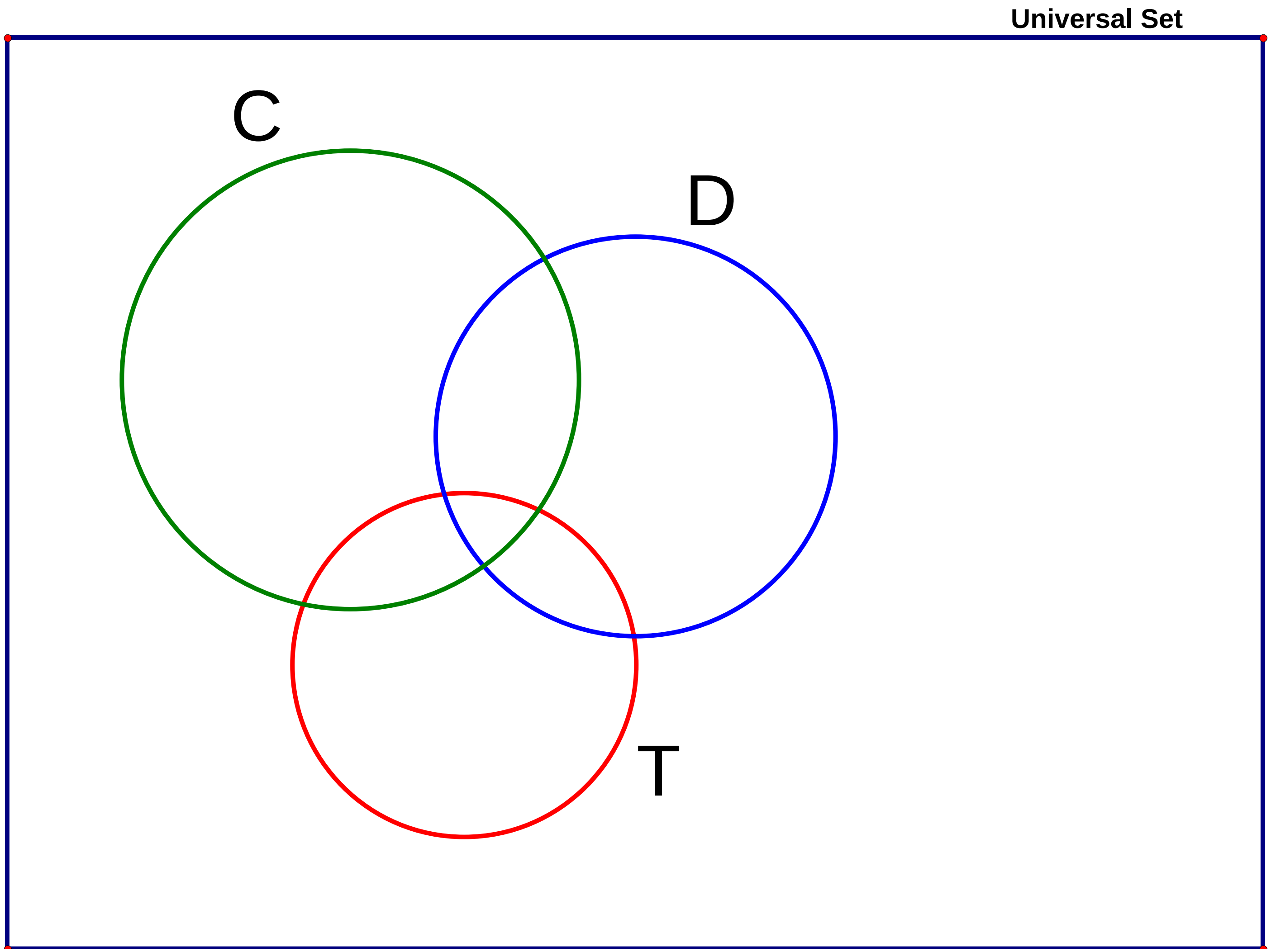
Therefore $P(A \cup B) = \frac{182}{1031}$

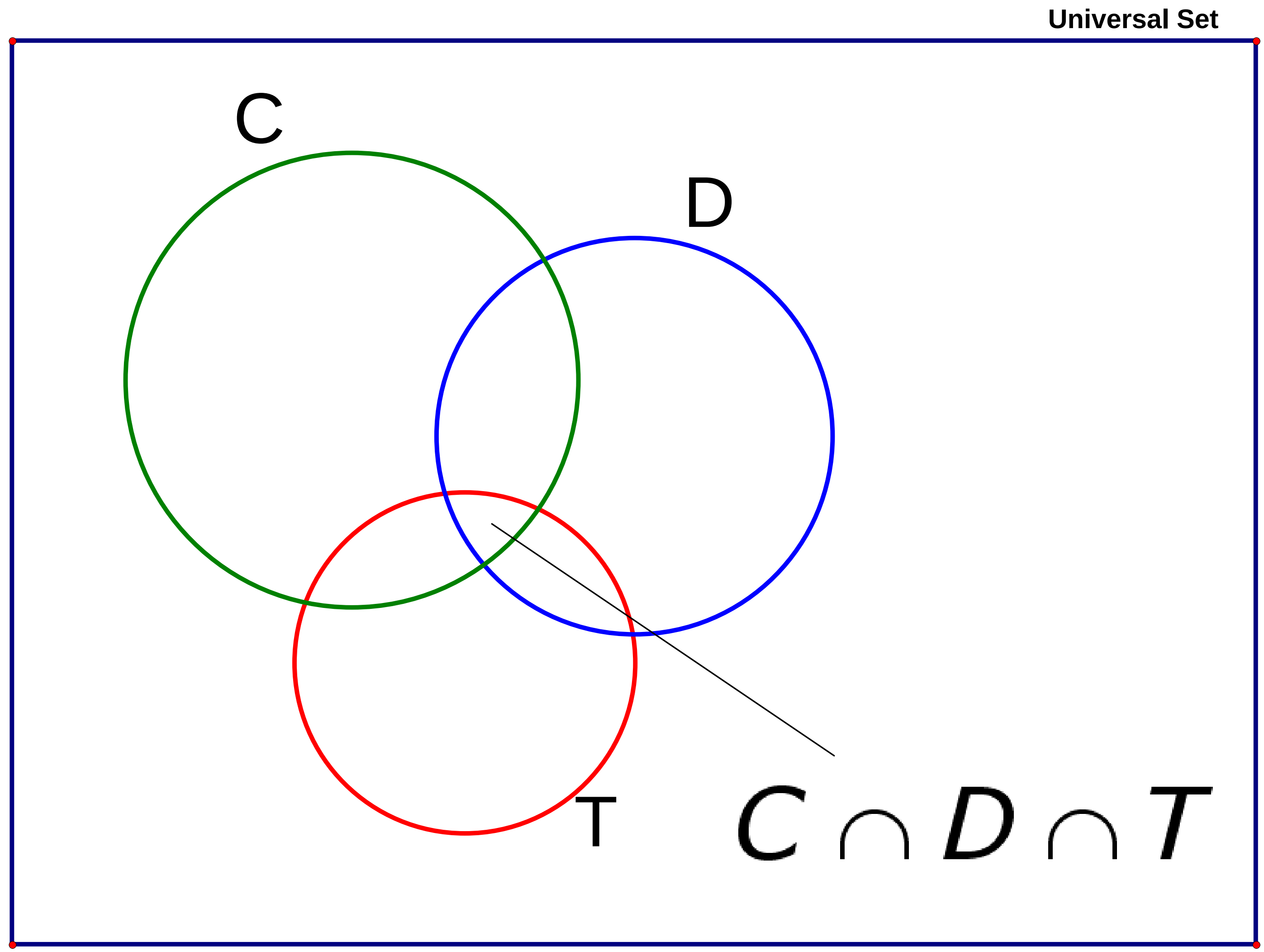
2.
In a certain county,
a) **31**% **of the residents have cable internet connection**.
b) **23**% **of the residents have dsl internet connection**.
c) **15**% **of the residents have a satelite internet connection**.
d) **7**% **have both cable and dsl**
e) **4**% **have both dsl and satelite**
f) **2**% **have both satelite and cable**
g) **1**% **have all three of these**.

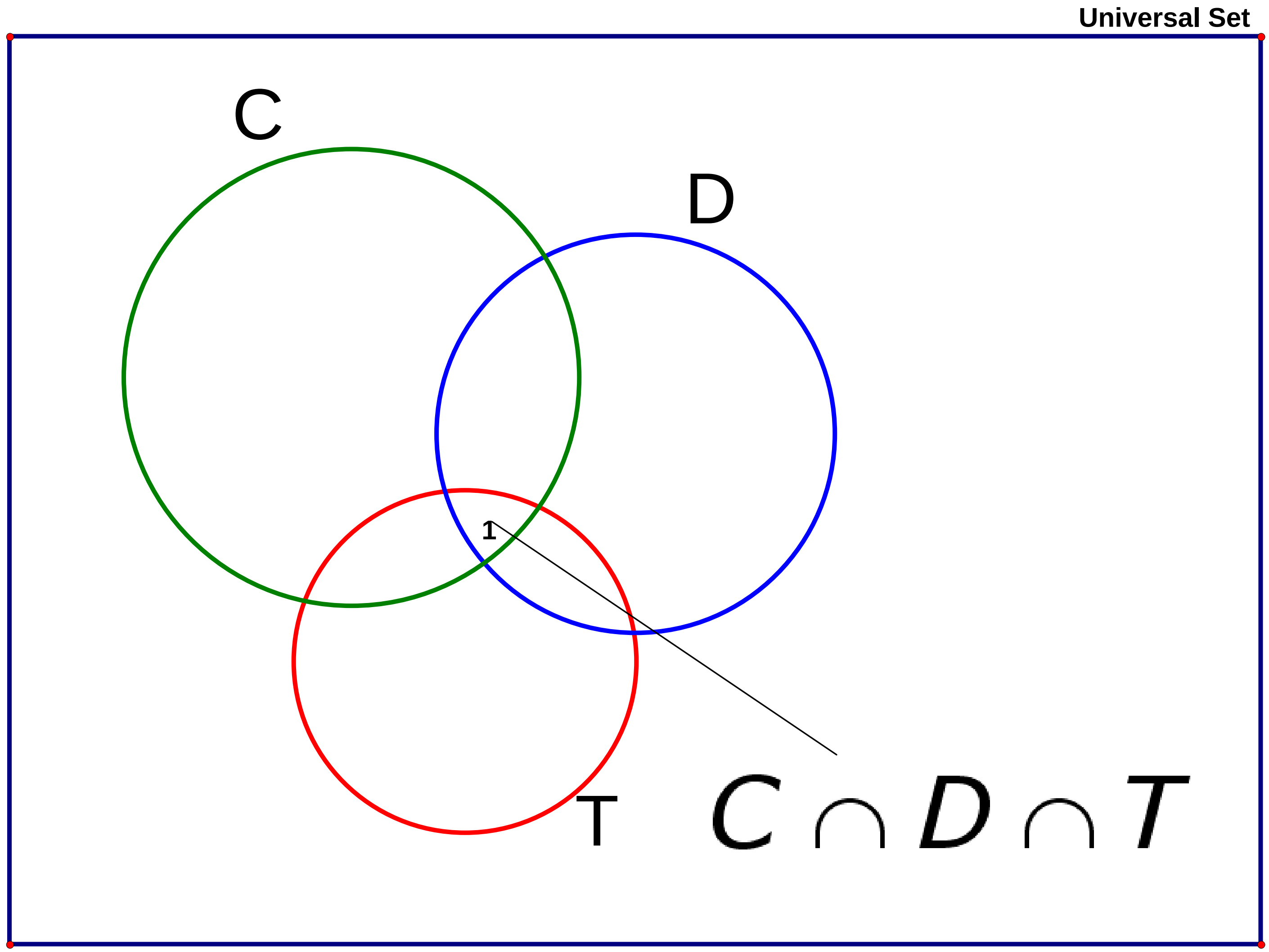
Assume that the above mentioned services are the only means of fast internet connectivity in the county, **answer the following questions**

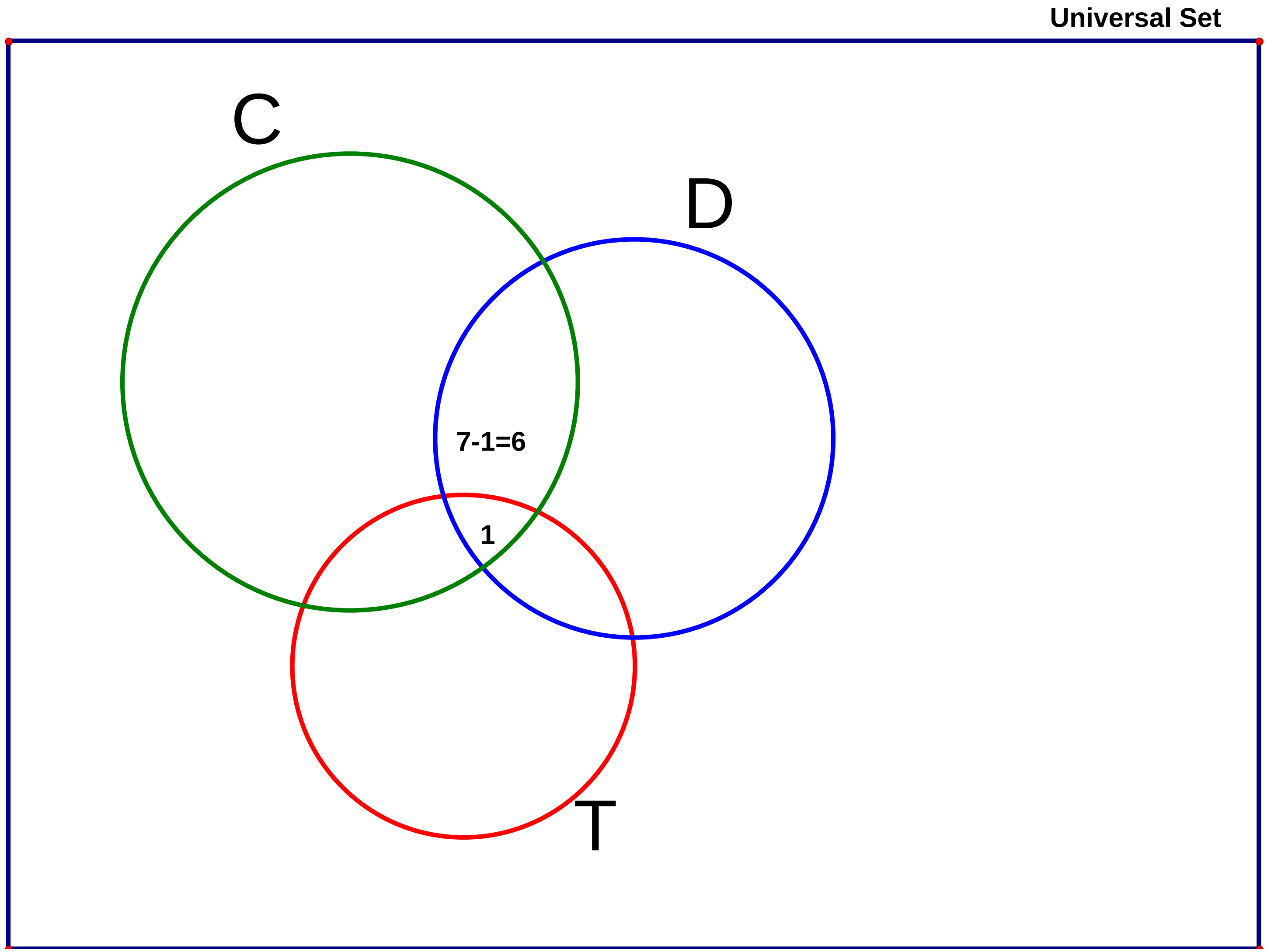
Let us treat the universal set as a set containing 100 people,

then we can fill up the diagram as



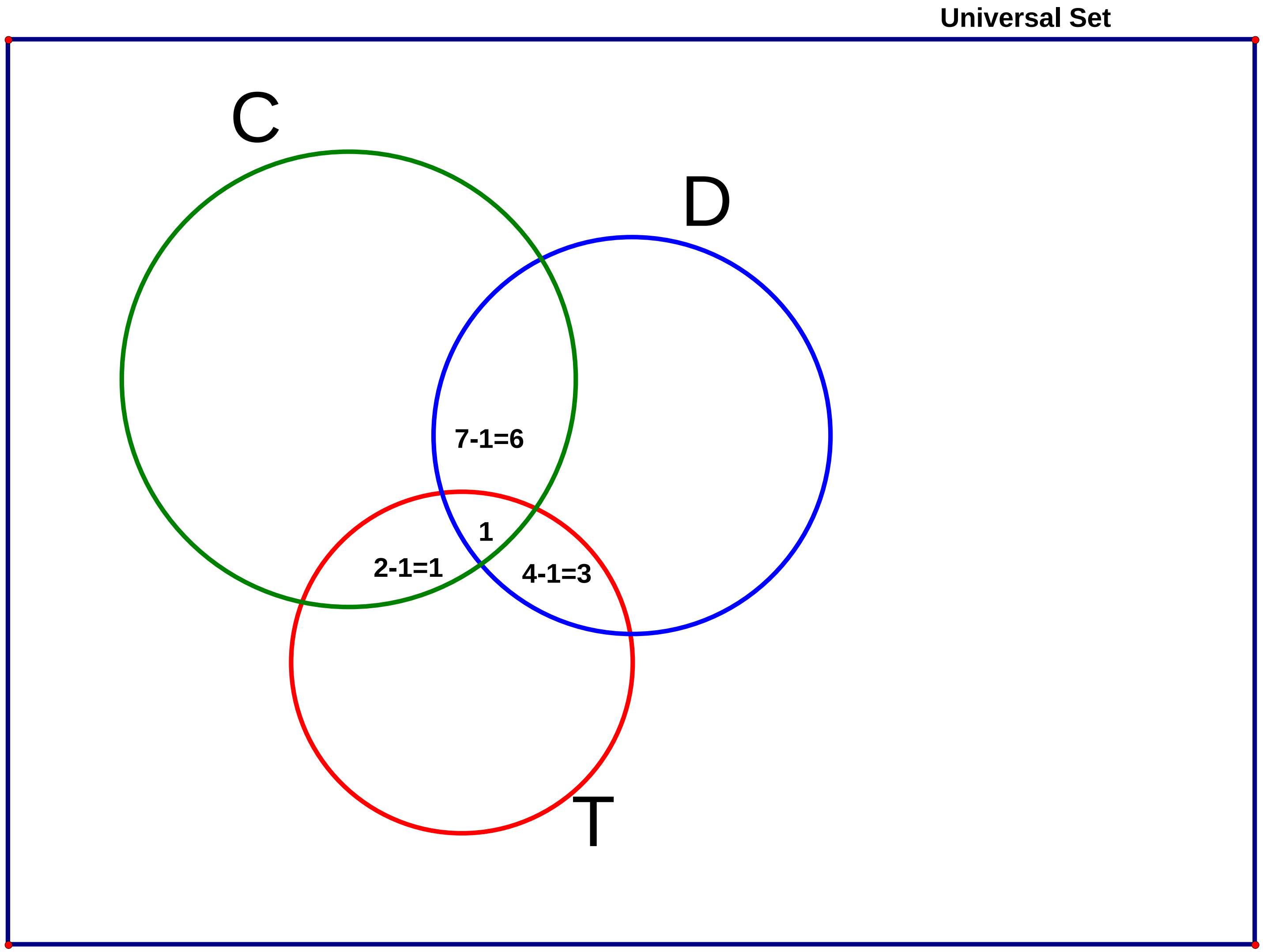

Universal Set
C
D
T
$C \cap D \cap T$


Universal Set
C
D
T
1
$C \cap D \cap T$



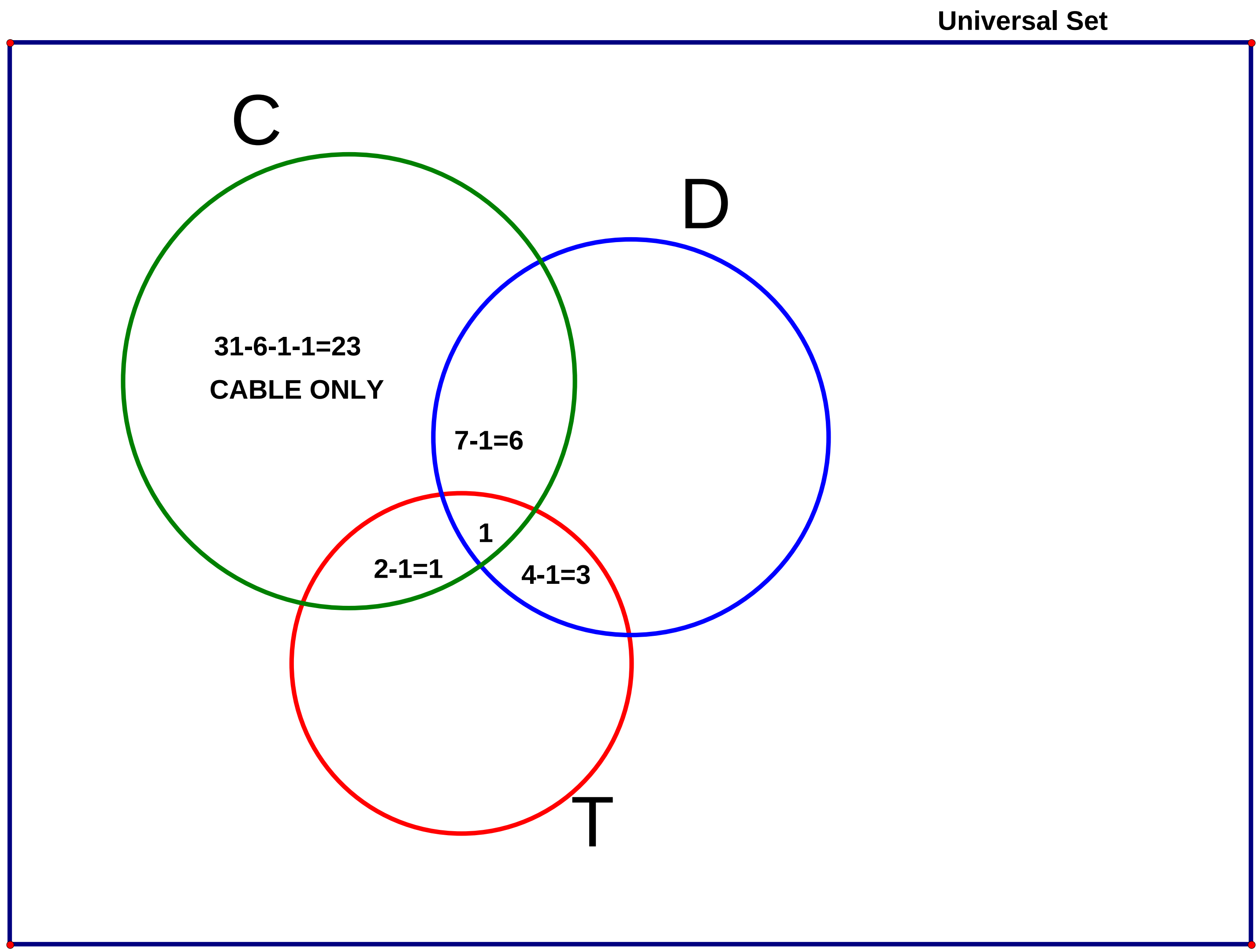
The above diagram shows the % of residents who have fast internet connection by cable and dsl but not by satelite

as 6%

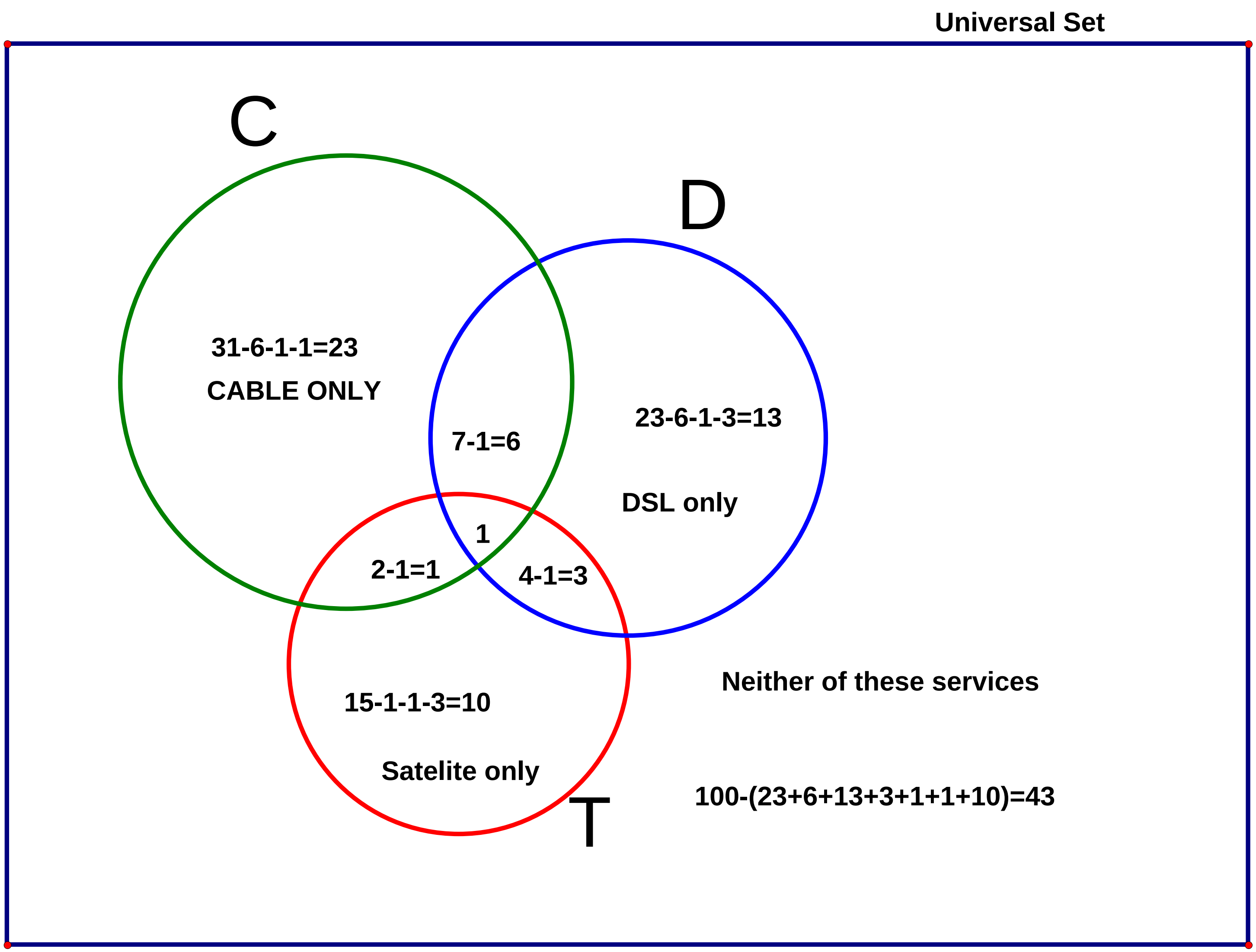


We can use the above diagram to conclude that
the percentage of residents who have fast internet connection by cable and dsl but not by satelite is 5%

the percentage of residents who have fast internet connection by cable and satelite but not dsl is 1%
the percentage of residents who have fast internet connection by dsl and satelite but not cable is 3%



We can conclude from the above diagram that
is the percentage of residents who have fast internet connection by cable only is 23



the percentage of residents who have fast internet connection by dsl only is 13

the percentage of residents who have fast internet connection by satelite only is 1

the percentage of residents who have no fast fast internet connection. is 43
Assume that the above three are the only means of fast internet connection in that county
If a resident is randomly selected:

i) Find the probability that the resident does have fast internet connection.
ii) Find the probability that the resident does not have fast internet connection.

The above calculations show that an answers to this question are
i) 0.57

ii) $1 - .57 = 0.43$

Example 3:

The following example is to illustrate conditional probability calculation.

The following shows a record from past 1271 days for a dangerous traffic intersection
B: Bad Weather **A: At least one accident**

	B		B'		Column Total
A	75	A and B	176	A and B'	251
A'	125	A' and B	895	A' and B'	1020
Row Total	200		1071		Grand Total 1271

Find the empirical probability

a) P(A) , the probability of at least one accident at that intersection

There were 251 accidents during 1271 days
Answer
P(A)$=\frac{251}{1271}$
b) P(A given that the weather is Bad)

Given B means we know that the weather is bad, that is we are restricted to the column

B	
75	**A and B**
125	**A' and B**
200	

During 200 days of bad weather an accident happened on 75 days
P(A given B)$=\frac{75}{200}$

A given B is also denoted by A|B

Definition:
Two events A and B are called statistically independent if
P(A)=P(A given B)

Note that in the example 3, A and B are not independent

Example 4:

Flip a fair quarter and a fair dime

{**HH**,**HT**,**TH**,**TT**} , **first place for quarter**, **second for dime**

A: **Head on the quarter**
B: **Head on the dime**

A$=${**HH**,**HT**}
B$=${**HH**,**TH**}
P(**A**)$=\frac{2}{4}$
P(**A given B**)$=\frac{1}{2}$
P(**B given A**)$=\frac{1}{2}$

P(**A**)$=$**P**(**A given B**)

Here A and B are independent.

....

Notations:

P(**A given B**) $=$ **P**(**A**|**B**)

Can easily see that
$$P(A|B)=\frac{P(A\cap B)}{P(B)},\ \text{provided that } P(B)\neq 0$$

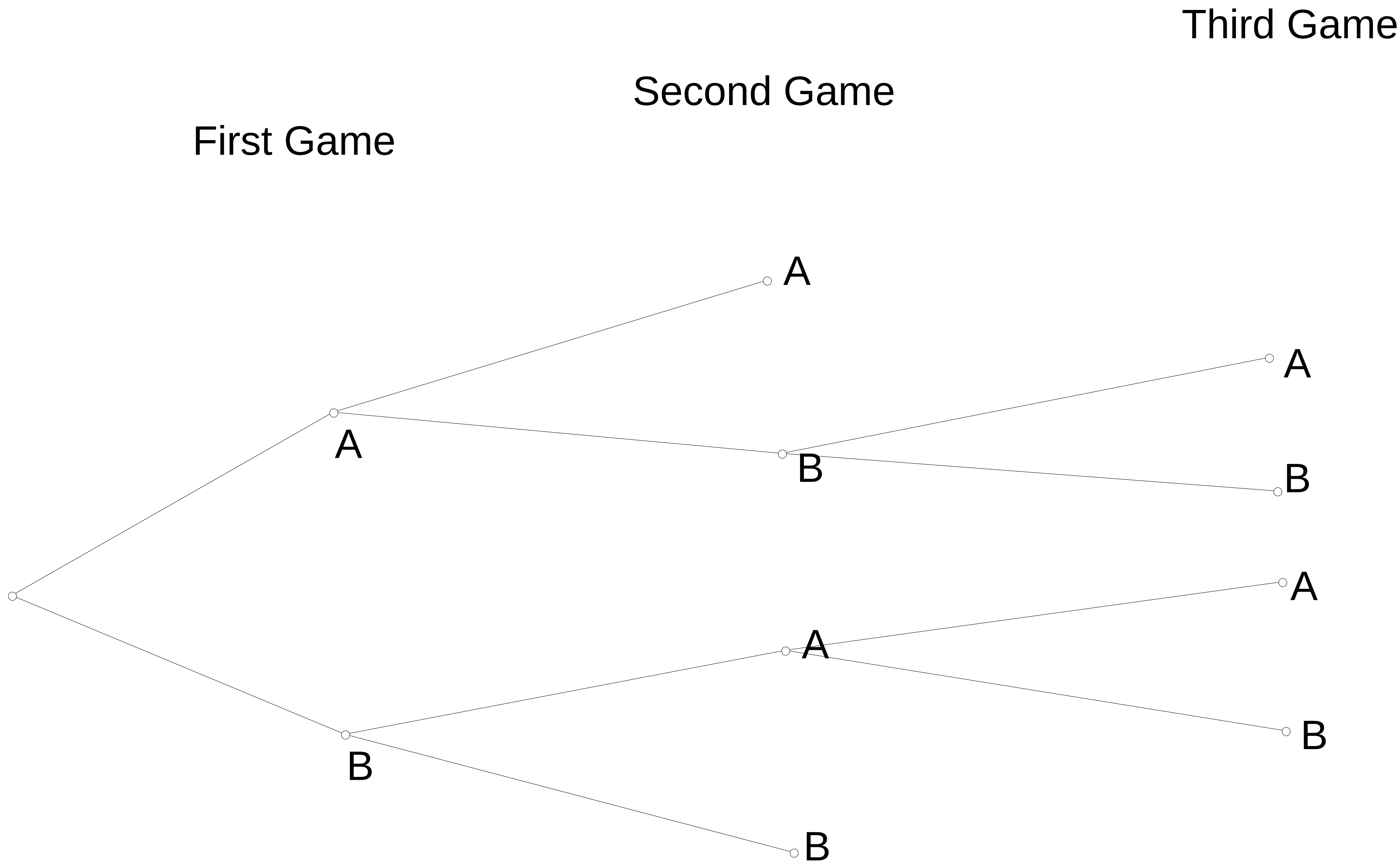
We have

$$\boxed{P(A\cap B)=P(B)P(A|B) \text{ as well as } P(B\cap A)=P(A)P(B|A) \text{ or } P(A\cap B)=P(A)P(B|A)}$$

Probability Calculations with the Use of Tree Diagrams:

Example 4:

Ann and Barbara are playing a tennis match which will be decided by three games.
How many winning possibilities are there?

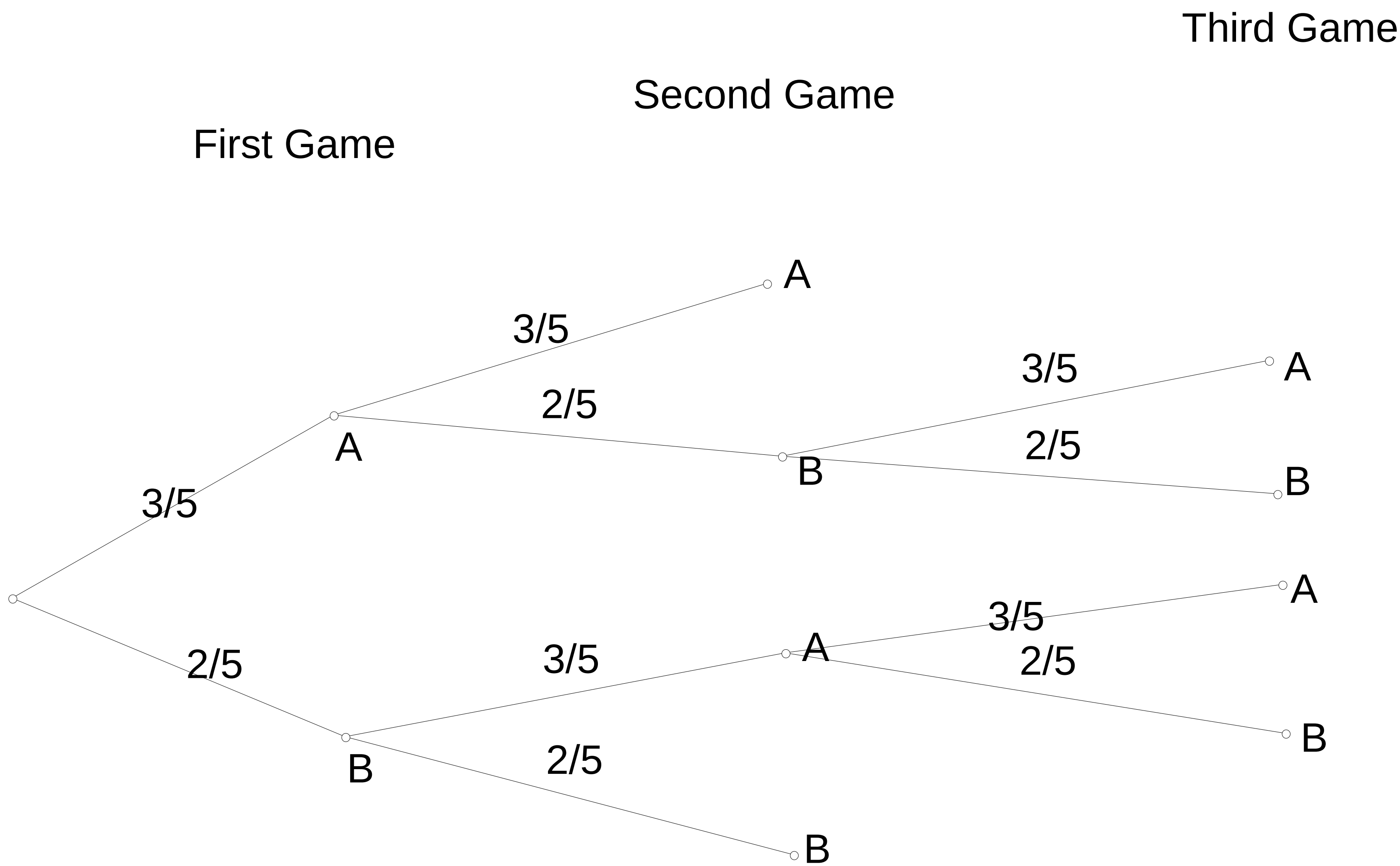


AA

ABA
ABB
BAA
BAB
BB

Example 4:

Ann and Barbara are playing a tennis match which will be decided by three games.
The probability of Ann winning any of the games is $\frac{3}{5}$
What is the probability that Ann will win the game?

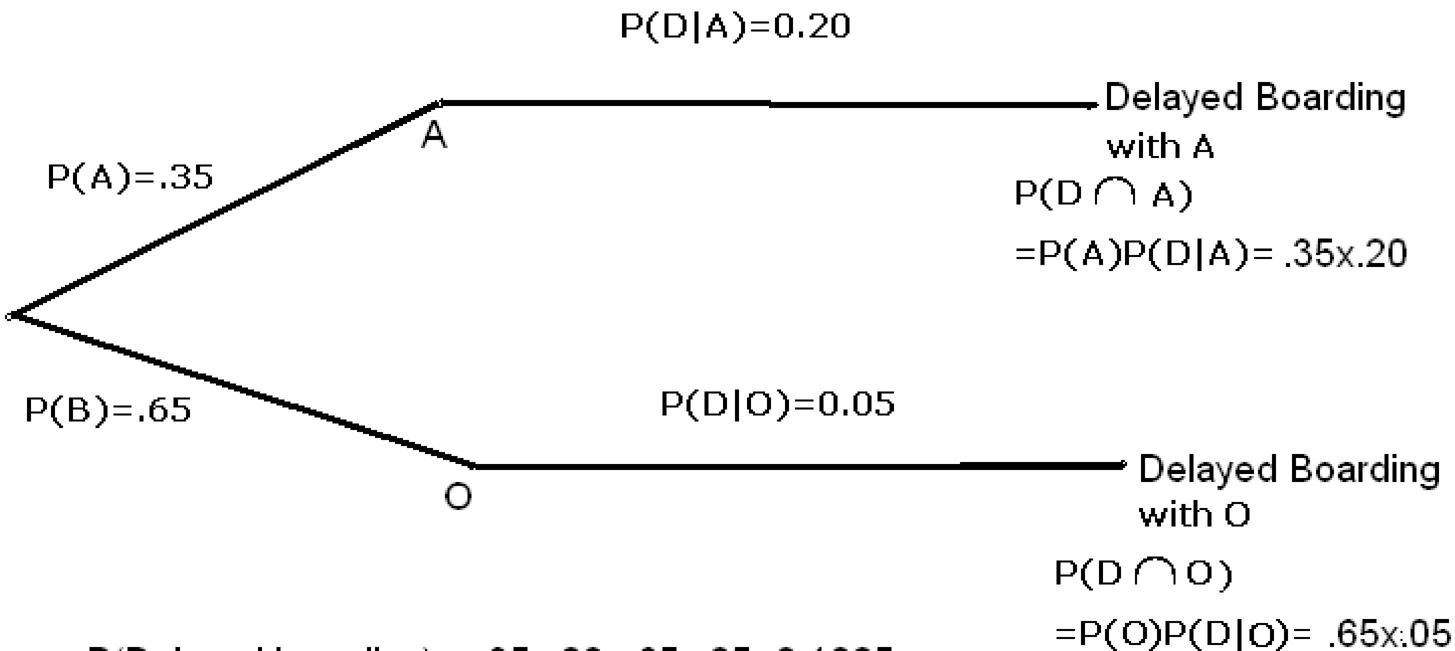


$$\frac{3}{5}\times\frac{3}{5}+\frac{3}{5}\times\frac{2}{5}\times\frac{3}{5}+\frac{2}{5}\times\frac{3}{5}\times\frac{3}{5}=\frac{81}{125}$$

Example 5: **35**% **of the flights at an airport are operated by the airline A and 65**% **by the others** ”**O**.”. **The airline A gets the boarding process organized on time 80**% **of the times and** ”**O**” **get the boarding process organized on time 95**% **of the times**.

Find the probability that a flight will have a delayed boarding.

A: Flight operated by A
O: Flight operated by O, the complement of A
D: Delayed Boarding on a flight

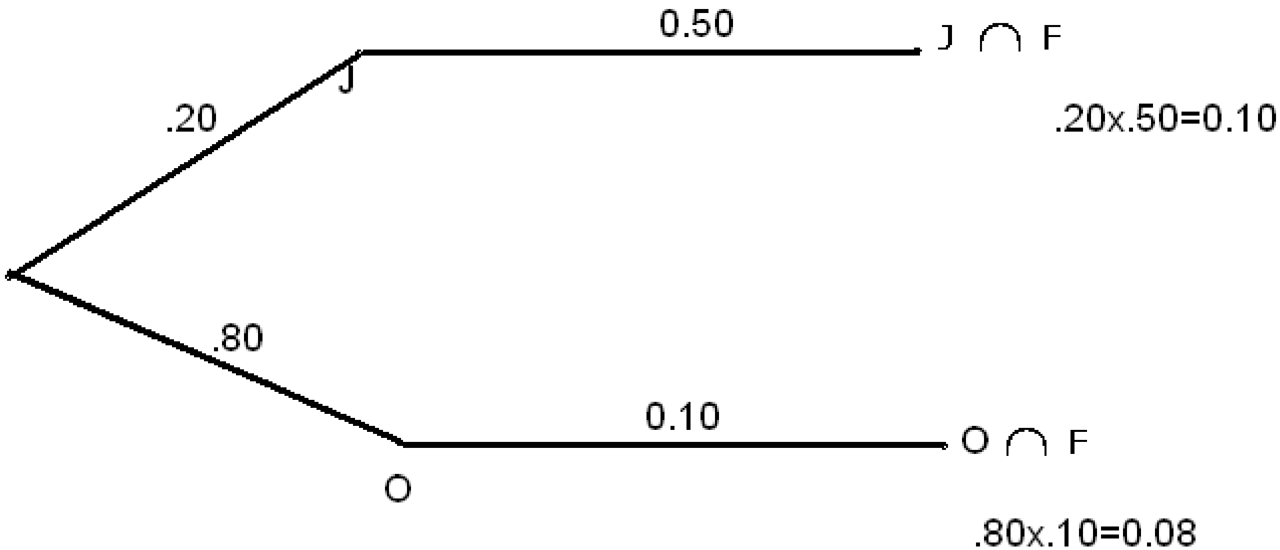


P(Delayed boarding) = .35x.20+.65x.05=0.1025

The following example should show you the use of Tree Diagram to compute the posterior probability using the Baye's Formula

Example 6: Jim uses the alamo in the presentation room of a large company 20% of the times. Jim forgets to pick the power chord of his laptop (after his presentation) 50% of the times. The other people forget to pick the power chord of his laptop (after presentation) 10% of the times. A power chord is found in the presentation room. Find the probability that it belongs to Jim.

J: Jim uses the room
O: Some one else uses the room
F: User forgets the power chord



P(F) = .10+.08=0.18

Here we have to find the probability P(J|F)

$$= \frac{P(J \cap F)}{P(F)} = \frac{0.10}{0.18} = \frac{10}{18}$$

Example 7:

A company orders one of the components of its product from four different suppliers (call them A,B,C,D.)

A supplies 80% of the products and 3% of their supplies are substandard
B supplies 12% of the products and 5% of their supplies are substandard
C supplies 5% of the products and 15% of their supplies are substandard
D supplies 3% of the products and 25% of their supplies are substandard

The company puts these components randomly into their products

Answer the following questions:

What is the probability that a component will be substandard

P(substandard)$=.80 \times .03 + .12 \times .05 + .05 \times .15 + .03 \times .25 = 0.045$

A component is reported as defective (given) , find the probability that it was supplied by C.

P(C|substandard)$=\frac{.05\times.15}{.80\times.03+.12\times.05+.05\times.15+.03\times.25} = 0.1666666667$

....

I am writing the following examples to help you review the computations that involve counting techniques.

Example 8:

How many ways can you select a President and a Vice President from a group of 5 people, such that one person can not hold more than one office.

Anil, Barb, Christi, Dan, Ejaz

AB	**BA**
AC	**CA**
AD	**DA**
AE	**EA**
BC	**CB**
BD	**DB**
BE	**EB**
CD	**DC**
CE	**EC**
DE	**ED**

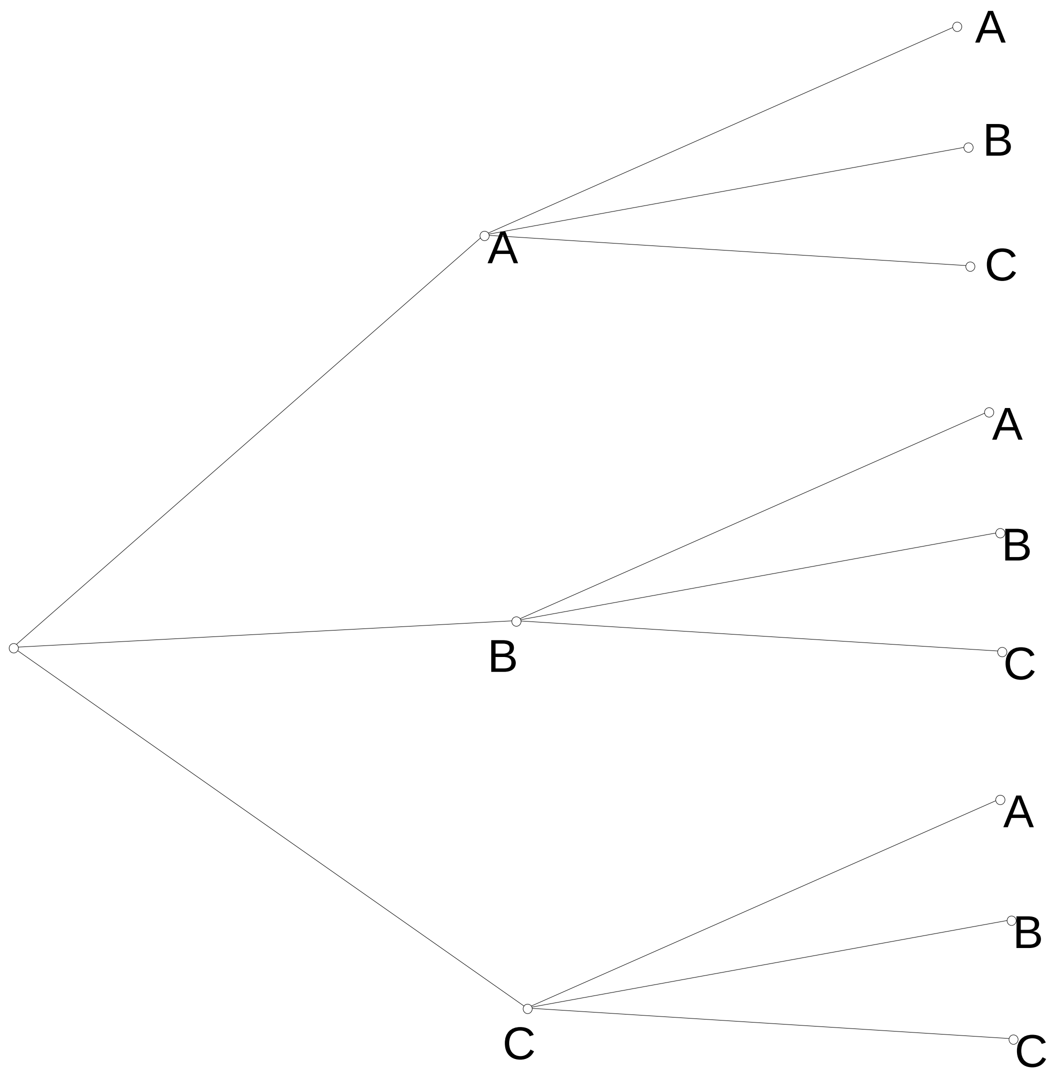
20 ways,
according to the multiplication principle:

$$5 \times 4 = 20$$

Example 9:
A password consists of 2 letters selected from A,B,C (not case sensitive.)
How many such passwords are possible.

AA
AB
AC
BA

CA
BB
BC
CB
CC



$3 \times 3 = 9$

Example 8:

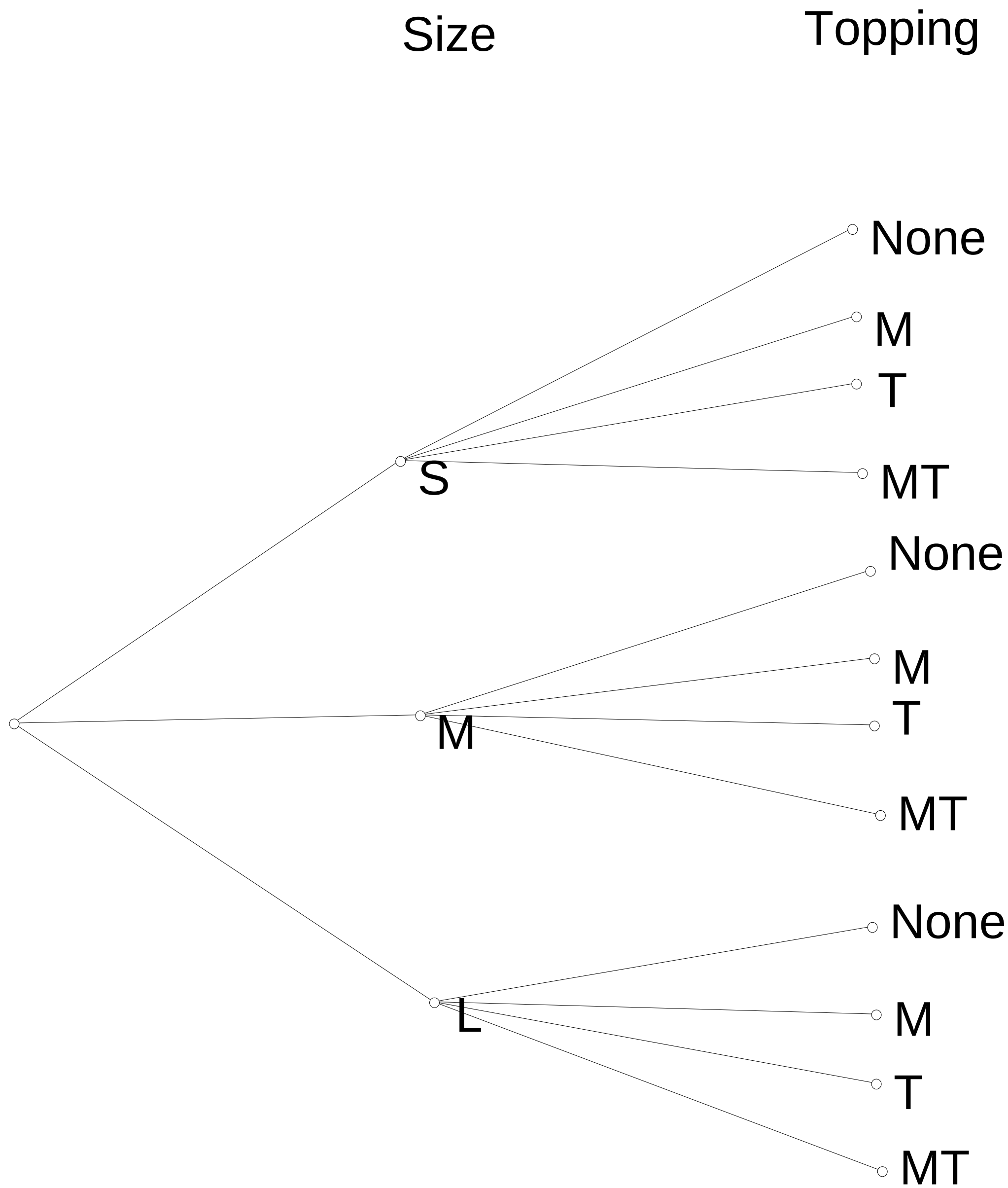
A password consists of four letters (selected from A-Z, case sensitive) followed by 6 digits taken from 0 thru 9. How many different passwords are possible.

$$52 \times 52 \times 52 \times 52 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 7311\,616\,000\,000$$

Example 10:

A pizza is available with 3 different sizes and two different toppings (Mushroom, Tomato.)
How many different Pizzas can be ordered if you may order a Pizza with any number of possible toppings including no topping.

$$3 \times 4 = 12$$

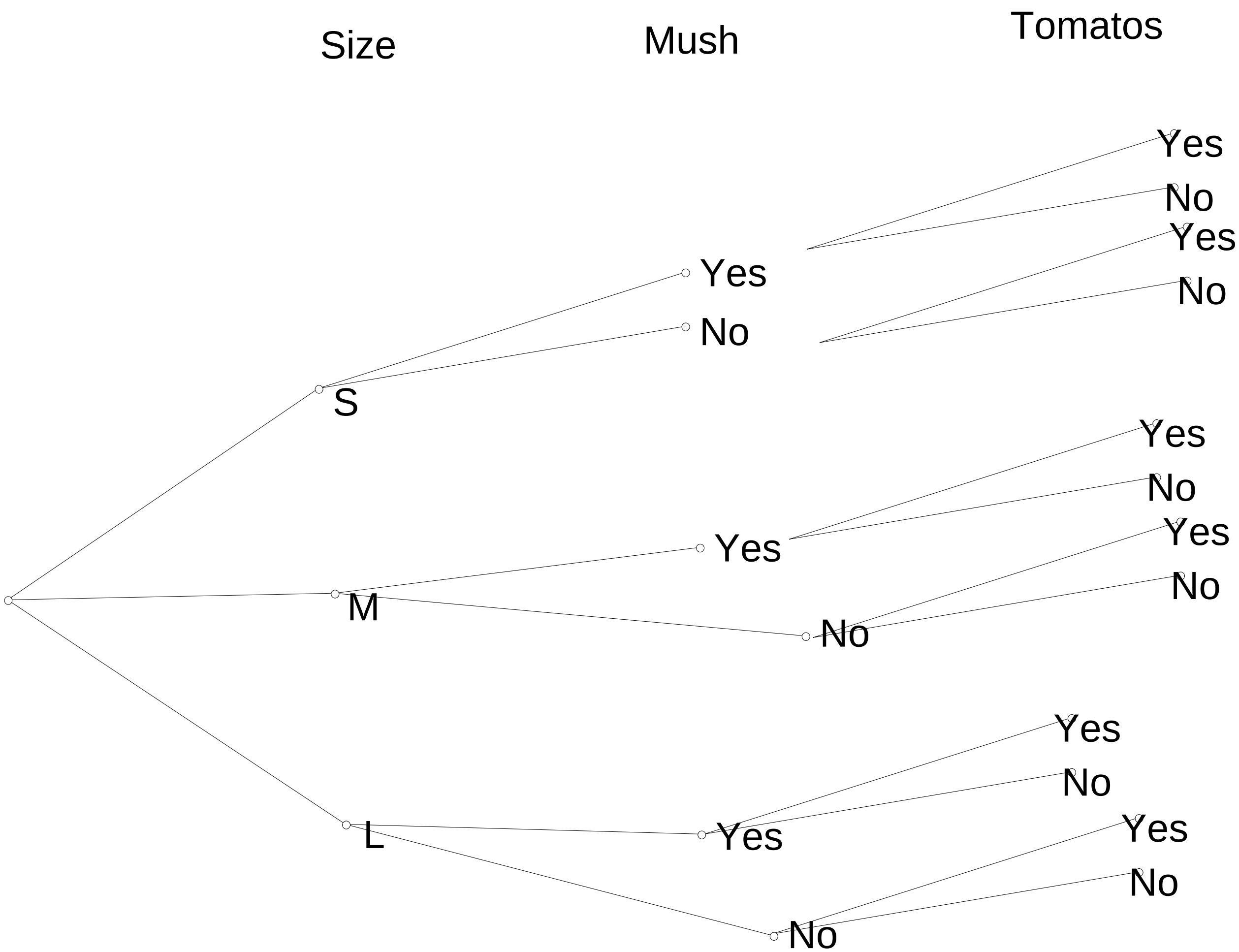

Size
Topping
None
M
T
S
MT
None
M
T
M
MT
None
L
M
T
MT

Think of the same thing in a difefferent style:

Al orders a Pizza from the drive in window.

the waitress tells him

There are three sizes, **Al has 3 choices**
She asks if Al would like Mushrooms for topping: **Al has 2 choices YES or NO**
She asks if Al would like Tomatos for topping: **Al has 2 choices YES or NO**



$$\mathbf{3 \times 2 \times 2 = 12}$$

Example 11:
A pizza is available with 3 different sizes, **4 different crusts**, **and 15 different toppings**.

How many different Pizzas can be ordered if you may order a Pizza with any number of possible toppings including no topping.

Size	Crust	Topping1	Topping2	.	.	.	.	.	.	.	.	.	.	.	.	Topping15
3	**4**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**

$3 \times 4 \times 2 \times 2 \times \ldots \times 2$ (**15 factors of 2**)

$= \mathbf{3} \times \mathbf{4} \times \mathbf{2}^{15}$

$= \mathbf{393\,216}$

Example 12. How many three letter passwords can be formed by using the letters from

MISSOURI

a) If the letters can be repeated and we are not case sensitive

there are 6 distinct choices

M I S O U R

$\mathbf{6} \times \mathbf{6} \times \mathbf{6} = \mathbf{216}$

b) If the letters can not be repeated and we are not case sensitive

$6 \times 5 \times 4 = 120$

Recall:

The number of ways can you select a President and a Vice President from a group of 5 people, such that one person can not hold more than one office.

Anil, **Barb**, **Christi**, **Dan**, **Ejaz**

AB	**BA**
AC	**CA**
AD	**DA**
AE	**EA**
BC	**CB**
BD	**DB**
BE	**EB**
CD	**DC**
CE	**EC**
DE	**ED**

20 ways,
according to the multiplication principle:

$$\mathbf{5 \times 4 = 20}$$

Example 13:

How many ways can we choose two of five people to send them to Switzerland this summer?

A B C D E

AB	**BA**
AC	**CA**
AD	**DA**
AE	**EA**

BC	**CB**
BD	**DB**
BE	**EB**
CD	**DC**
CE	**EC**
DE	**ED**

$$\frac{20}{2} = \mathbf{10}$$

Example 14:

How many ways can I select a committee of 3 people from out of a group of 11 people (note that for a committee appointment, the order will not matter.)

A B C D E F G H I J K

$$\frac{11 \times 10 \times 9}{6} = \mathbf{165}$$

ABC
ACB
BAC
BCA
CAB
CBA

\# **of repeats**
$\mathbf{3 \times 2 \times 1 = 6}$

Example 15:

How many ways can I select 6 numbers from 1 thru 54 such that the order of selection does not matter?

$$\frac{54 \times 53 \times 52 \times 51 \times 50 \times 49}{6 \times 5 \times 4 \times 3 \times 2 \times 1} = \mathbf{25\,827\,165}$$

....

$\mathbf{6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720}$

$0! = 1$

$\mathbf{1! = 1}$

$\mathbf{2! = 2}$

$\mathbf{3! = 6}$

$\mathbf{4! = 24}$

$\mathbf{5! = 120}$

$\mathbf{6! = 720}$

$\mathbf{7! = 7 \times 720 = 5040}$

$\mathbf{7! = 5040}$

$\mathbf{9! = 9 \times 8! = 9 \times 8 \times 7! = 362\,880}$

....

When we select r objects of of n total such that the order does not matter,
the number of possibilities are

$$\mathbf{C(n, r)} = \frac{n!}{r!(n-r)!}$$

combination of n objects taken r at a time $(n \geq r)$

....

$$C(8,5) = \frac{8!}{5! \times 3!} = \frac{8 \times 7 \times 6 \times 5!}{5! \times 3!} = \frac{8 \times 7 \times 6}{3!} = \frac{8 \times 7 \times 6}{6} = \mathbf{56}$$

$$C(8,5) = \mathbf{56}$$

Combination:

Combination of n objects taken r at a time is

$$C(n,r) = \frac{n!}{r!(n-r)!} \qquad n \geq r$$

We can apply this rule when we are selecting r objects out of n **total such that the order does not matter**.

Example 16: **In how many ways can I select five people out of 15 total** (**such that the order does not matter**) **people**?

$$C(15,5) = \frac{15!}{5!10!} = \frac{15\times14\times13\times12\times11\times10!}{5!\times10!} = \frac{15\times14\times13\times12\times11}{5!} = \frac{15\times14\times13\times12\times11}{5\times4\times3\times2\times1} = \mathbf{3003}$$

$$C(15,5) = \mathbf{3003}$$

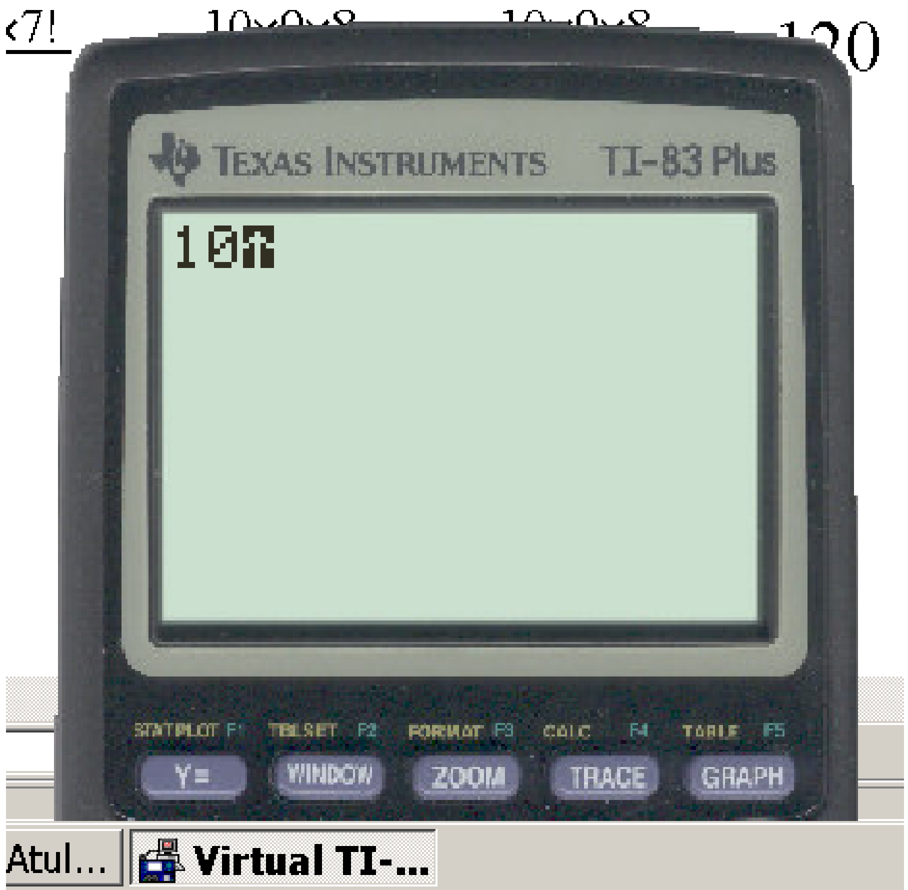
Example 17:

Sara has choose 3 kinds of desert from 10 total choices to be served at a banquet. **How many ways can she do that**?

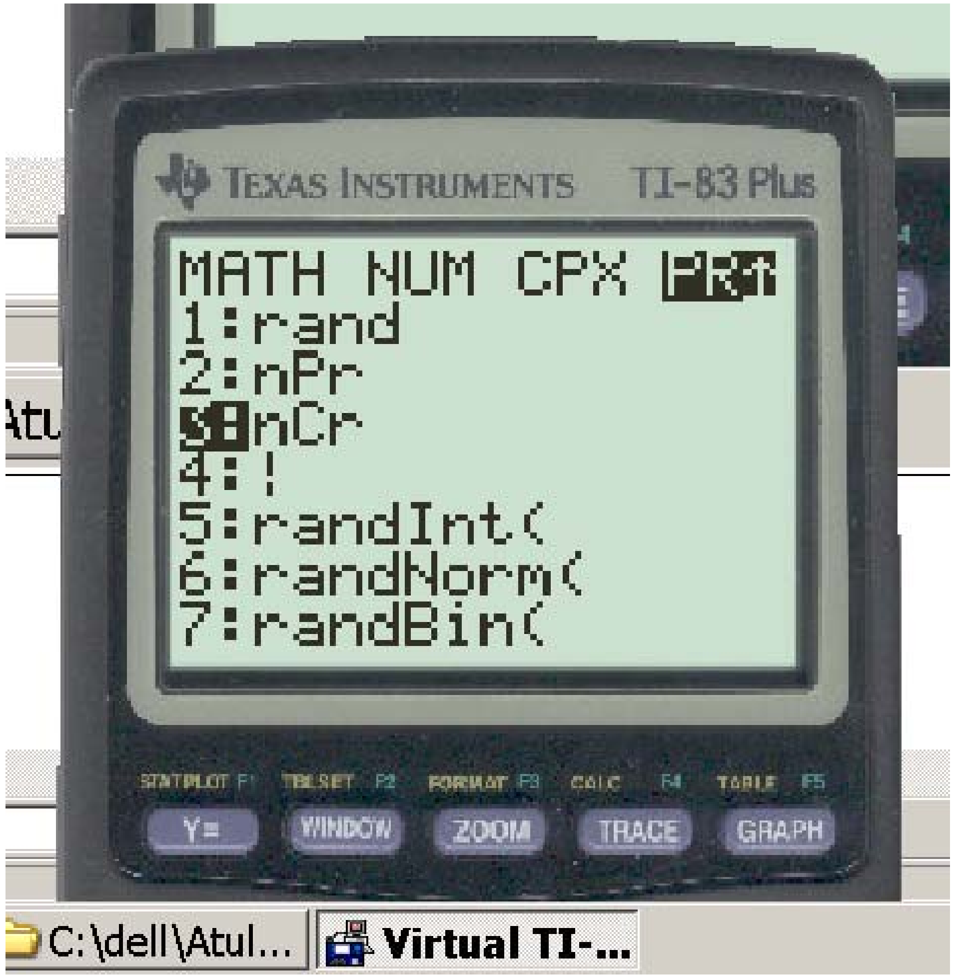
$$C(10,7) = \frac{10!}{3!\times7!} = \frac{10\times9\times8\times7!}{3!\times7!} = \frac{10\times9\times8}{3!} = \frac{10\times9\times8}{3\times2\times1} = \mathbf{120}$$

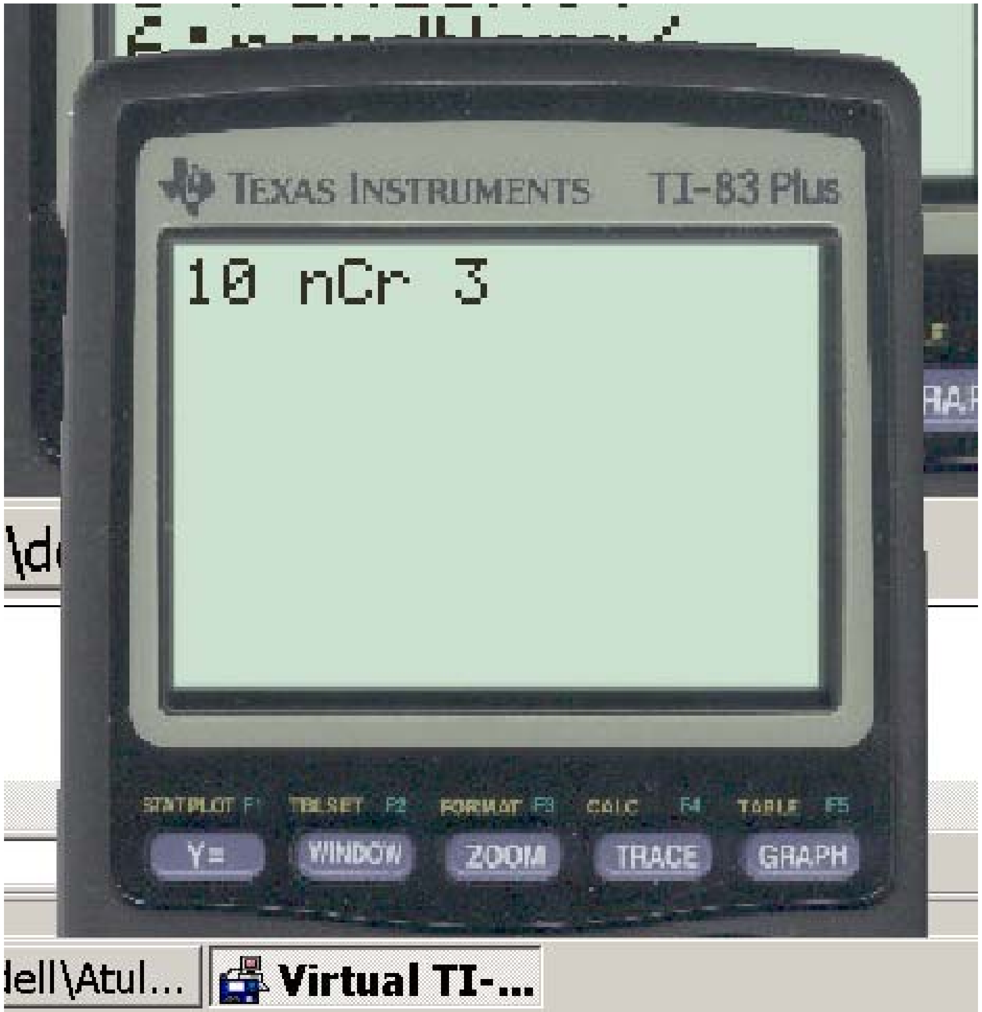
$C(10, 7) = 120$

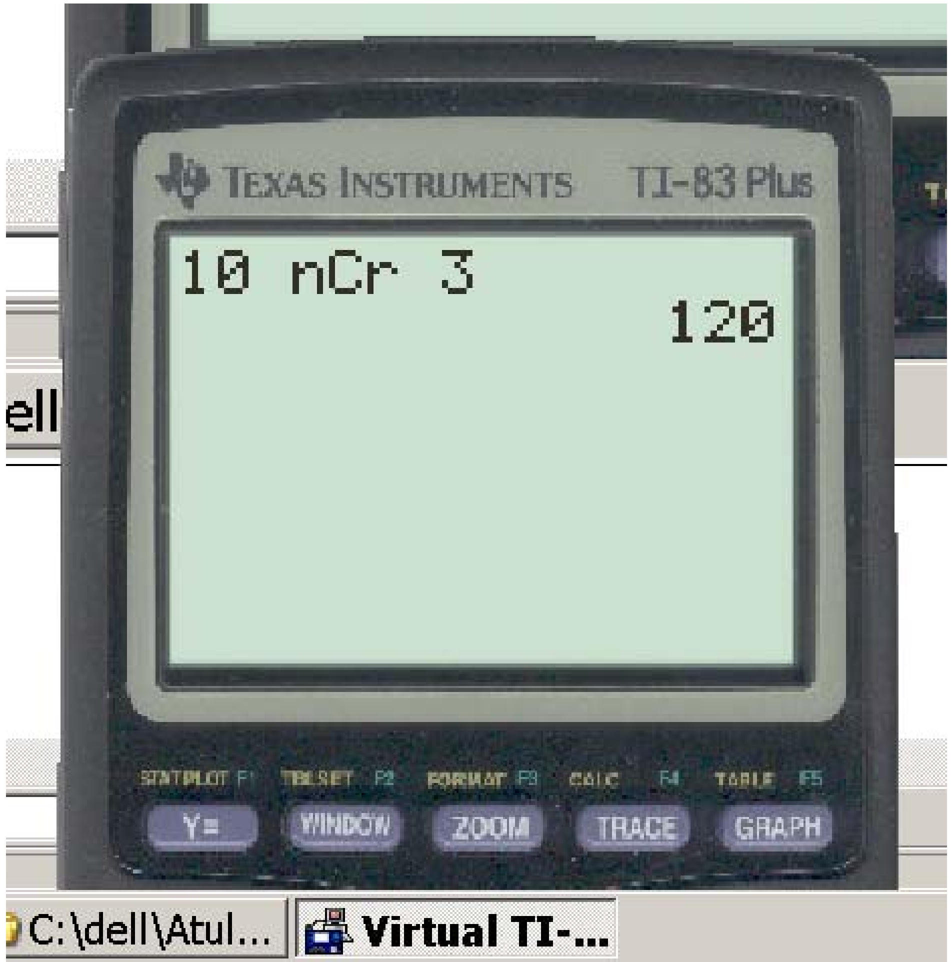
With a TI83plus



MATH→


TEXAS INSTRUMENTS
TI-83 Plus
MATH NUM CPX PRB
1:rand
2:nPr
3:nCr
4:!
5:randInt(
6:randNorm(
7:randBin(
STATPLOT F1
TBLSET F2
FORMAT F3
CALC F4
TABLE F5
Y=
WINDOW
ZOOM
TRACE
GRAPH
C:\dell\Atul...
Virtual TI-...


TEXAS INSTRUMENTS
TI-83 Plus
10 nCr 3
STATPLOT F1
TBLSET F2
FORMAT F3
CALC F4
TABLE F5
Y=
WINDOW
ZOOM
TRACE
GRAPH
dell\Atul...
Virtual TI-...



Example 18:

A five member committee has to be chosen from a group of 20 people consisting of 7 women and 13 men. **How many selections will have 3 women and 2 men**?

Selecting 3 women **Selecting 2 men**

$\mathbf{C(7,3)} \quad \times \quad \mathbf{C(13,2)}$

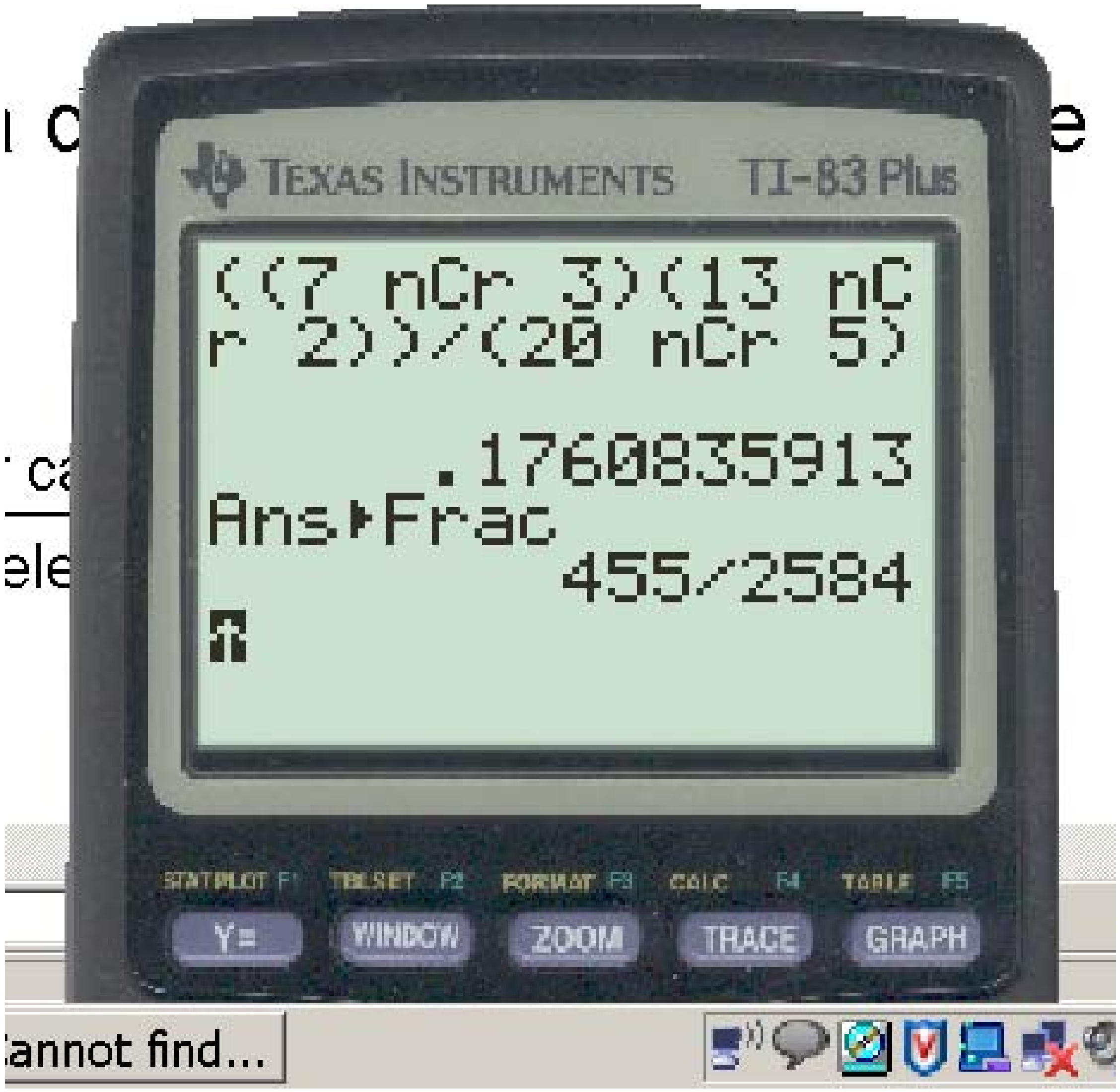
$\mathbf{C(7,3) \times C(13,2) = 2730}$

Example 19: **A five member committee has to be chosen from a group of 20 people consisting of 7 women and 13 men**. **Find the probability that 3 women and 2 men will be selected**.

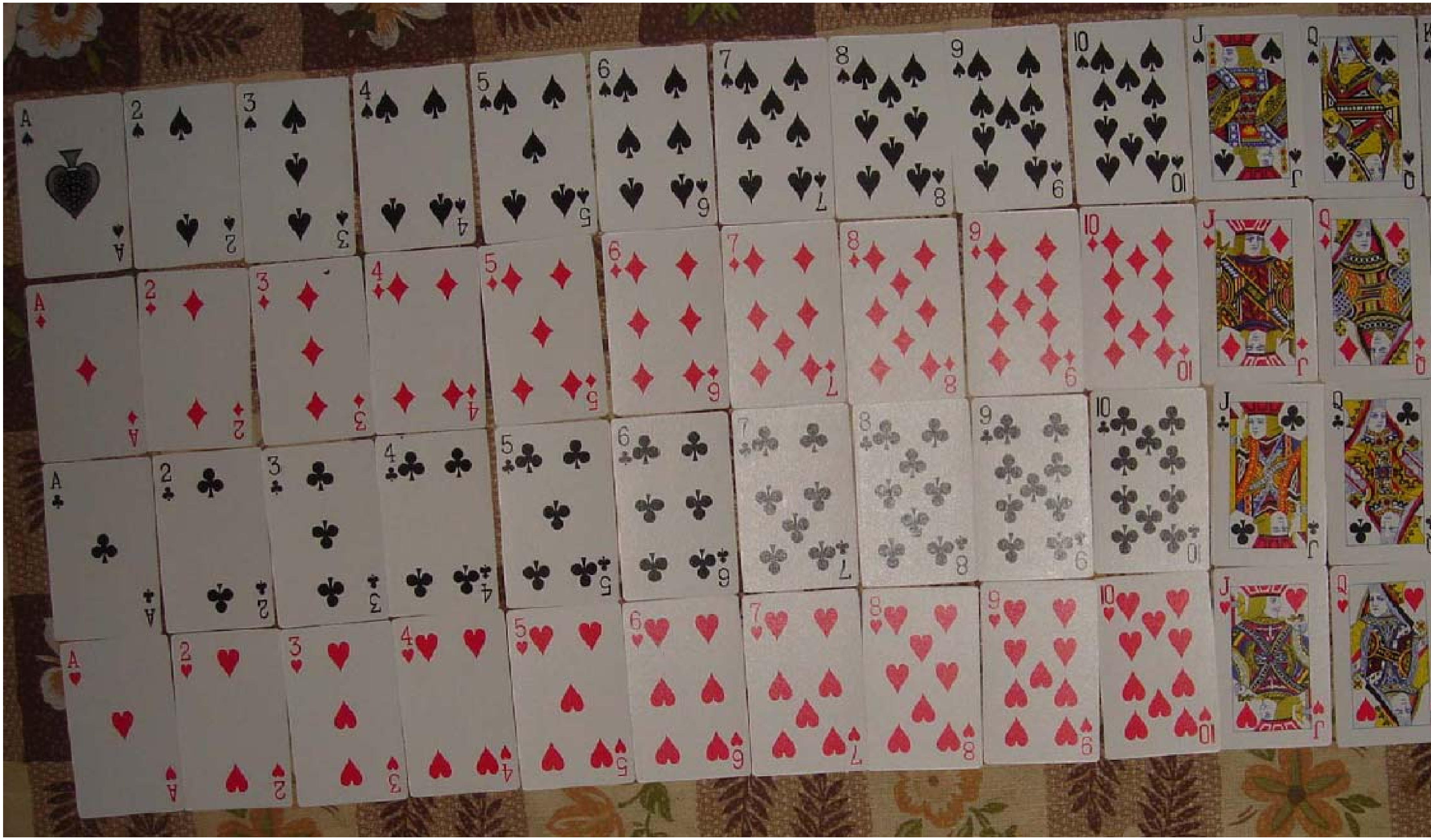
$$\frac{\text{The number of ways 3 women and 2 men can be selected}}{\text{The total number of ways that 5 out of 20 people can be selected}}$$

$$= \frac{C(7,3) \times C(13,2)}{C(20,5)}$$

$$= \frac{455}{2584}$$


TEXAS INSTRUMENTS
TI-83 Plus
((7 nCr 3)(13 nCr 2))/(20 nCr 5)
.1760835913
Ans▸Frac
455/2584
Y=
WINDOW
ZOOM
TRACE
GRAPH
Cannot find...

Example 18:

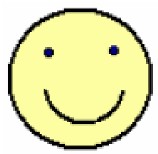


4 card hand is randomly selected from a deck of 52 cards. Find the probability that EXACTLY one of them is an Ace.

$$\frac{\text{Number of ways we can select ONE ace when picking four cards at random}}{\text{Total number of ways 4 out of 52 cards can be selected}}$$
$$=\frac{C(4,1)C(48,3)}{C(52,4)}$$
$$=\frac{69\,184}{270\,725}$$

Example 29:

A game charges $1 to play and lets a player draw 4 cards from a well shuffled pack of 52-cards and gives back the number of dollars equal the number of aces in those 4-cards. Find the expected value of this game.

Outcome	Amount gained	Probability
All 4 Aces 	$3	$\frac{C(4,4)C(48,0)}{C(52,4)} = \frac{1}{270\,725}$
Exactly 3 aces 	$2	$\frac{C(4,3)C(48,1)}{C(52,4)} = \frac{192}{270\,725}$
Exactly 2 aces 	$1	$\frac{C(4,2)C(48,2)}{C(52,4)} = \frac{6768}{270\,725}$
Exactly 1 Ace	$0	$\frac{C(4,1)C(48,3)}{C(52,4)} = \frac{69\,184}{270\,725}$
No Ace 	–$1	$\frac{C(4,0)C(48,4)}{C(52,4)} = \frac{38\,916}{54\,145}$

$$\mathbf{3} \times \frac{1}{270\,725} + \mathbf{2} \times \frac{192}{270\,725} + \mathbf{1} \times \frac{6768}{270\,725} + \mathbf{0} \times \frac{69\,184}{270\,725} - \mathbf{1} \times \frac{38\,916}{54\,145} = -\$\frac{9}{13}$$

Example 20:

How many ways can we select a committee of 5 people consisting of 2 men and 3 women from a group of 15 people consisting of 9 men and 6 women?

2 men out of 9	3 women out of 6	2men and 3 women on the committee
$\mathbf{C}(\mathbf{9},\mathbf{2})$	$\mathbf{C}(\mathbf{6},\mathbf{3})$	$\mathbf{C}(\mathbf{9},\mathbf{2}) \times \mathbf{C}(\mathbf{6},\mathbf{3})$
$\mathbf{C}(\mathbf{9},\mathbf{2}) = \mathbf{36}$	$\mathbf{C}(\mathbf{6},\mathbf{3}) = \mathbf{20}$	$\mathbf{C}(\mathbf{9},\mathbf{2}) \times \mathbf{C}(\mathbf{6},\mathbf{3}) = \mathbf{720}$

Example 21:

A committee of 5 members is selected from a group of 15 people consisting of 9 men and 6 women. What is the probability that there will be 2 men and 3 women on this committee?

$$\frac{\text{The number of ways 2 men and 3 women are possible}}{\text{Total number of ways of selecting 5 out of 15 total}}$$

$\mathbf{C}(\mathbf{15},\mathbf{5}) = \mathbf{3003}$

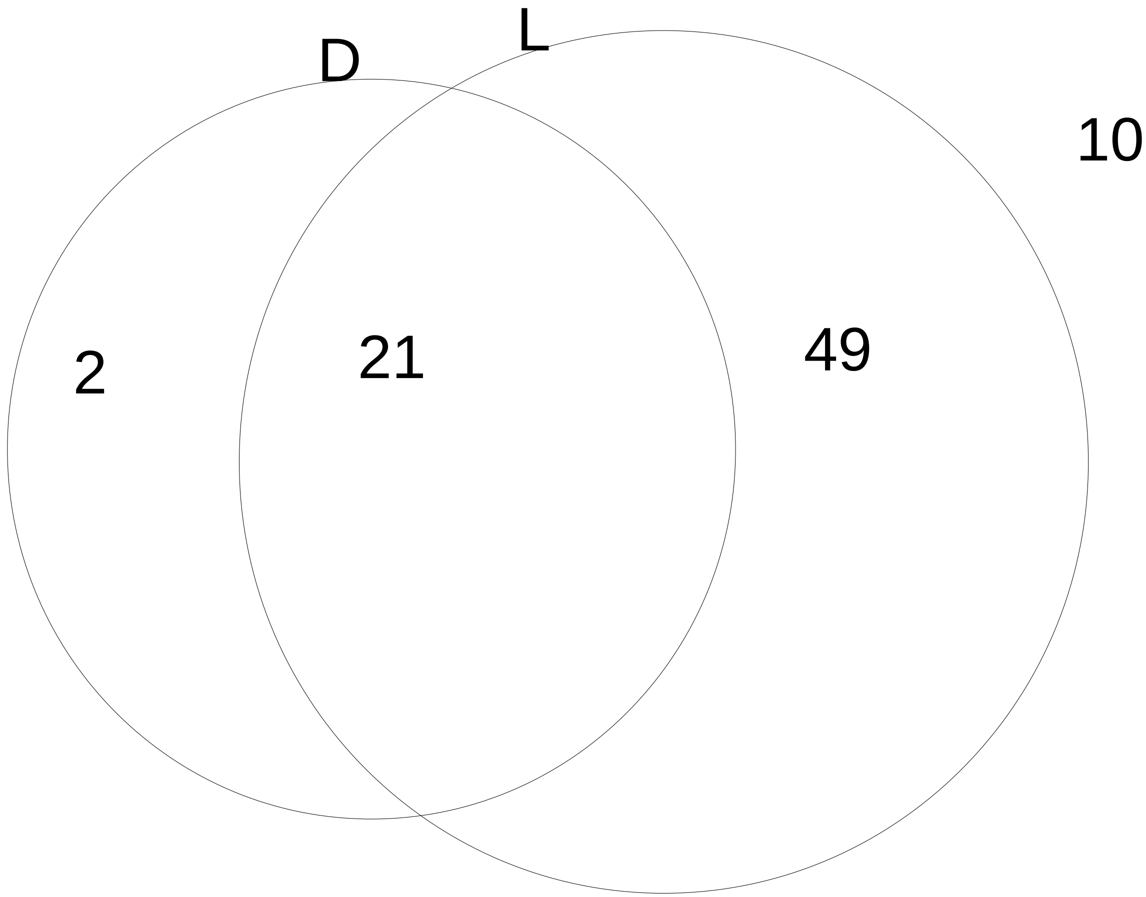
$$\frac{C(9,2) \times C(6,3)}{C(15,5)} = \frac{720}{3003}$$

Example 22:

Gina has taken two courses from a tardy professor. She has been keeping a record of the number of days the professor came to the class late. Sometimes, the professor blamed the late arrival on traffic delays. Here are Gina's records, where is has noted the number days that the traffic was delayed alongwith the number of days that the professor arrived late.

L: Late **D: Traffic Delay**

	L	**L'**	**Total**
D	**21**	**2**	**23**
D'	**49**	**10**	**59**
	70	**12**	**82**



A student is currently taking a course taught by this professor and happens to have Gins'a records.

The traffic is reported to be delayed, find the (empirical) probability that the professor will be late.

P(L|D)$=\frac{21}{23}$

Example 23:

Recall:

$$\binom{n}{r} = \mathbf{C}(\mathbf{n},\mathbf{r}) = \frac{n!}{r!(n-r)!}$$

There are 11 balls in a bag out of which 5 are blue and 6 are red.

4 balls are taken out of this bag randomly.

Find the probability that at least 3 of these 4 will be red.

$$\frac{\text{\# of ways we can get at least 3 (3 or 4) red balls in a set of 4 chosen balls}}{\text{\# of ways 4 balls can be drawn out of 11 total}}$$

$$= \frac{\text{\# of ways 3 red out of 4} + \text{\# of ways 4 red out of 4}}{\text{\# of ways 4 balls can be drawn out of 11 total}}$$

$$= \frac{\text{\# of ways 3 red and 1 blue out of 4} + \text{\# of ways 4 red out of 4}}{\text{\# of ways 4 balls can be drawn out of 11 total}}$$

$$= \frac{C(6,3)C(5,1) + C(6,4)}{C(11,4)}$$

$$= \frac{23}{66}$$