I am very sorry that I forgot the flash disk in the office.

I am going to rewrite the notes based on memory,
therefore, please do not get confused if our numbers differ.

Example 1: We did the following from the review sheet;

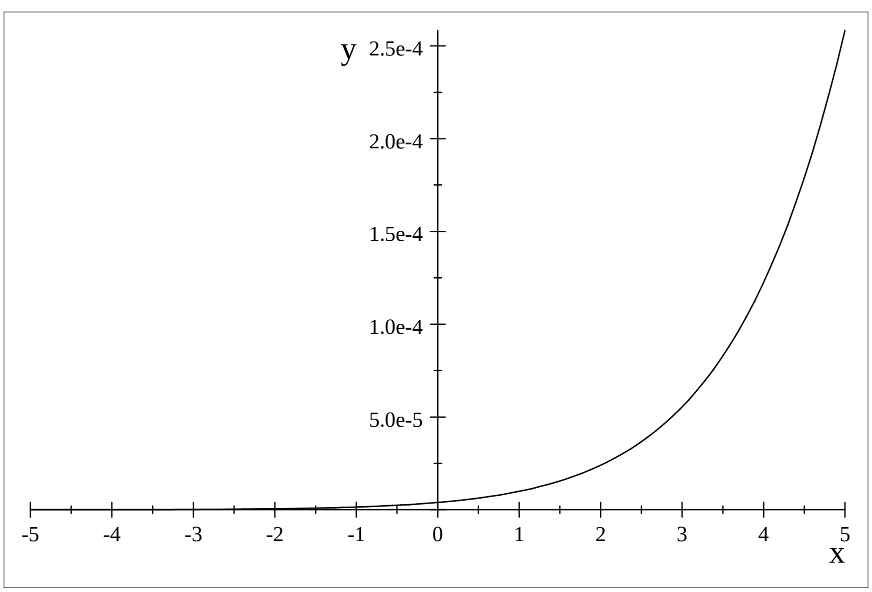
X is distributed normally with $\mu = 21.0$, $\sigma = 4.7$

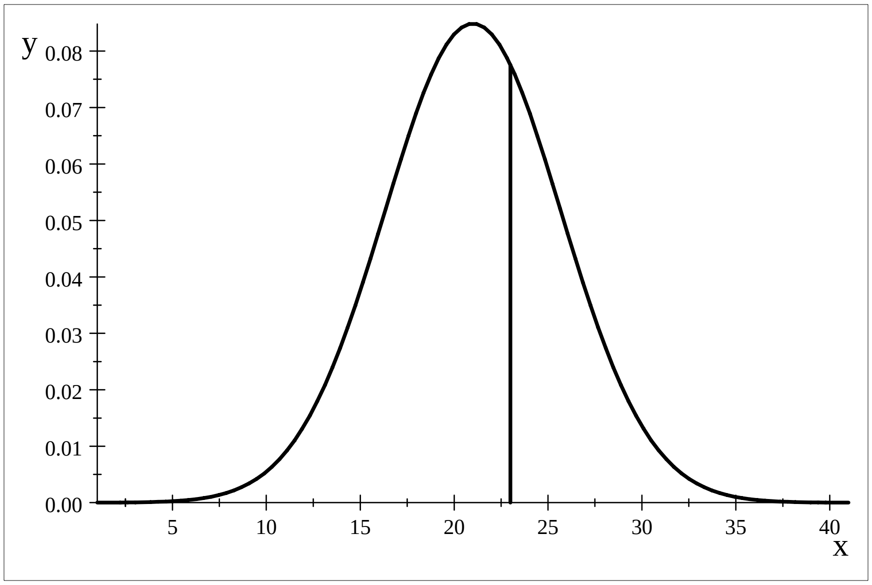
a) Probability that a **single** student scores 23 or higher

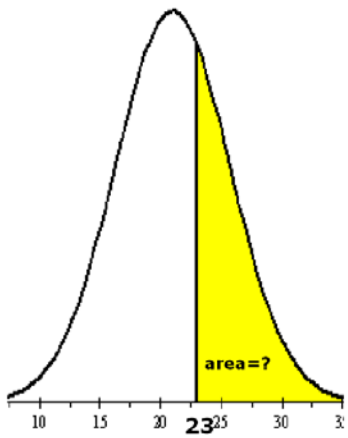
We have to find the area under the normal curve shown below

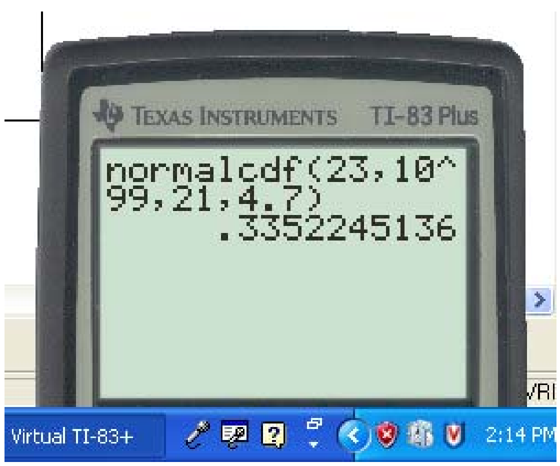
$$l(x) = \frac{1}{4.7\sqrt{2\pi}} e^{-\left((x-21)^2/\left(2\times 4.7^2\right)\right)}$$

$l(x)$









The probability is 0.3352

If using the z-table:

Recall

$$z = \frac{x - \text{mean}}{\text{standard deviation}}$$

$x \geq 23$

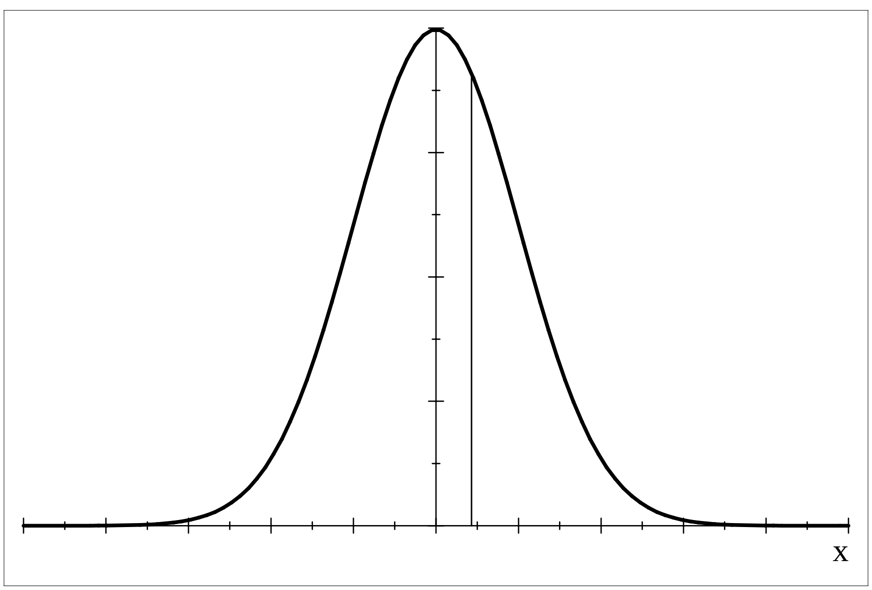
$$z \geq \frac{23 - 21}{4.7}$$

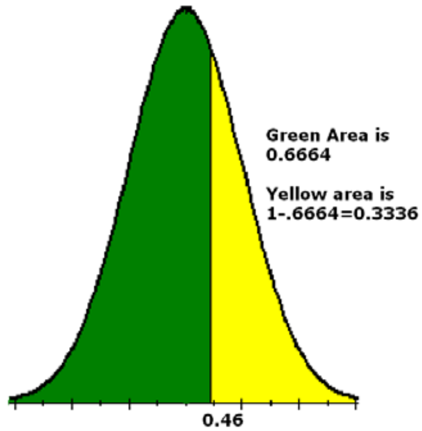
$z \geq 0.43$

Go to the z-table

$$\begin{array}{ccc} & & .03 \\ & & \downarrow \\ 0.4 & \rightarrow & .6664 \end{array}$$

$$\frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$





From the table, the answer is slightly different because of the rounding, the probability of an individual scoring 23 or higher is 0.3336.

b)

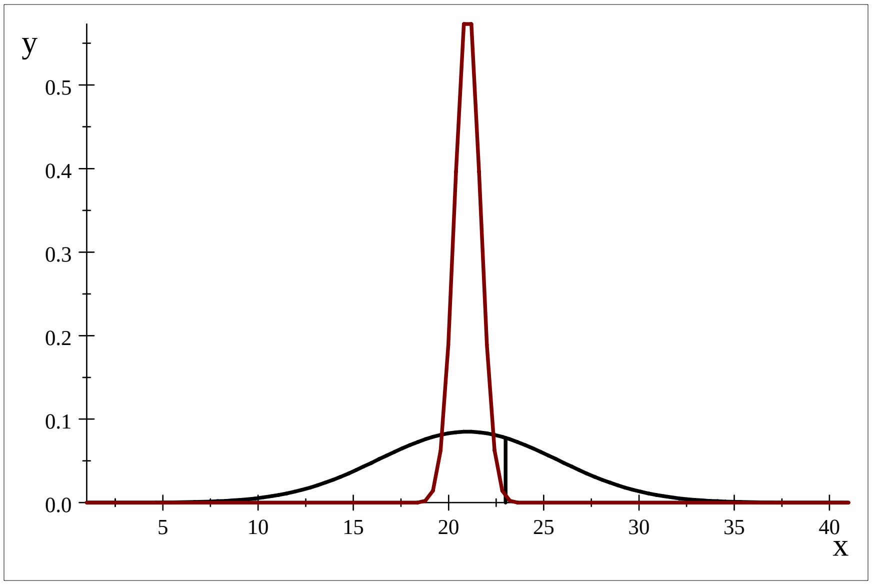
b) Probability that the **average for a group of 50 students** is 23 or higher – *this is now a sampling distribution question.*

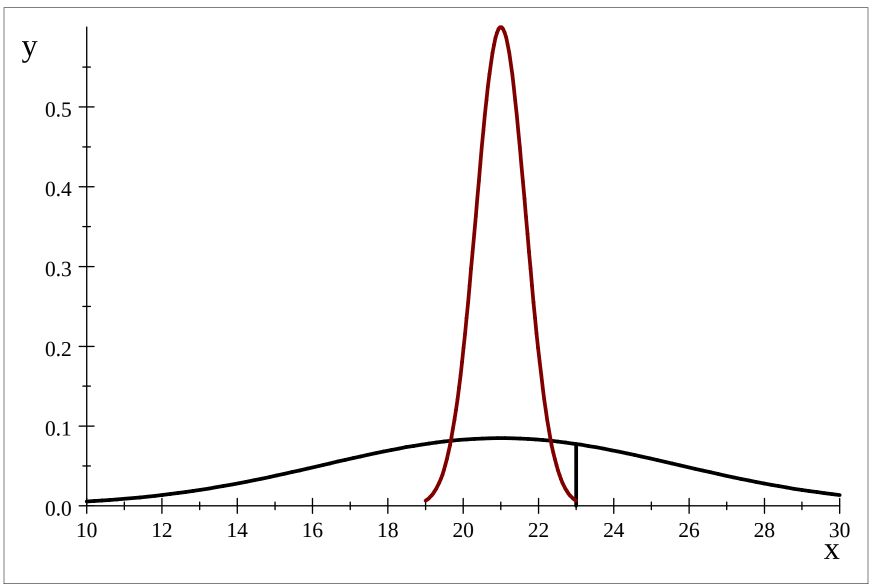
We are assuming a simple random sample of size 50

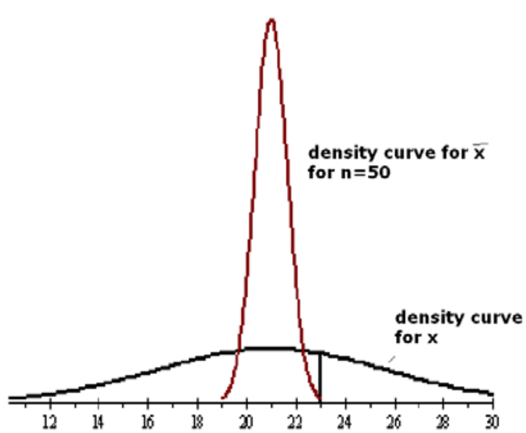
We have to find the probability that

$\overline{x} \geq 23$ for a simple random sample of size 50

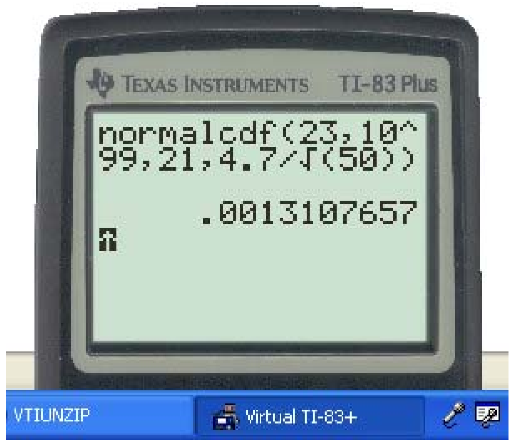
Since the population has a normal distribution, $\overline{x}$ also has a normal distribution, and has mean 21 and standard deviation $\frac{4.7}{\sqrt{50}}$


y
0.5
0.4
0.3
0.2
0.1
0.0
5
10
15
20
25
30
35
40
x


y
0.5
0.4
0.3
0.2
0.1
0.0
10
12
14
16
18
20
22
24
26
28
30
x


density curve for $\bar{x}$
for n=50
density curve
for x
12
14
16
18
20
22
24
26
28
30

We would like find the the area to the right of 23 for the red curve,



The probability that $\overline{x} \geq 23$ is 0.0013

If we are using the table,

$$\overline{x} \geq 23$$
$$\rightarrow$$
$$z > \frac{23-21}{\left(\frac{4.7}{\sqrt{50}}\right)}$$
$$z > 3.01$$

z-table

area to the left of $z = 3.01$ is 0.9987

$$\begin{array}{ccc} & & .01 \\ & & \downarrow \\ 3.0 & \rightarrow & 0.9987 \end{array}$$

Therefore the area to the right of z=3.01 is
$1 - .9987 = 0.0013$

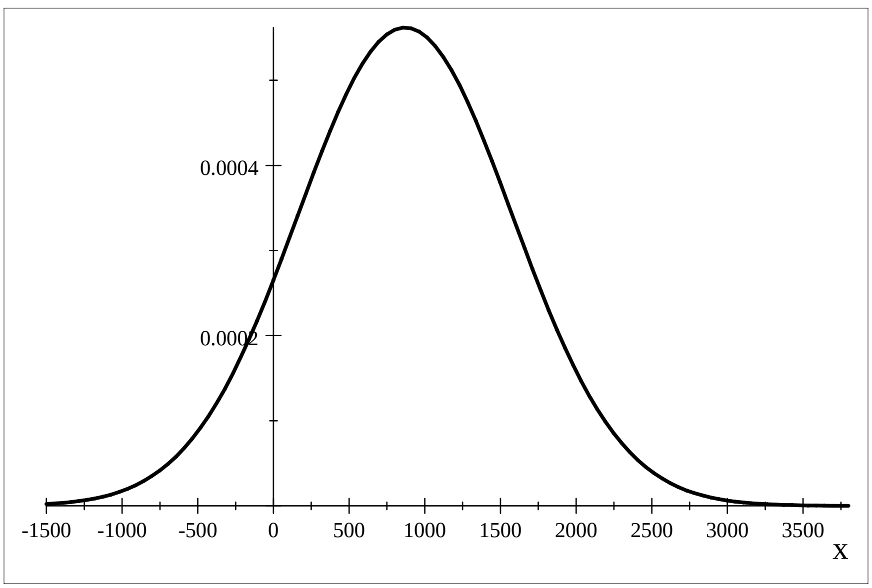
Another Example:

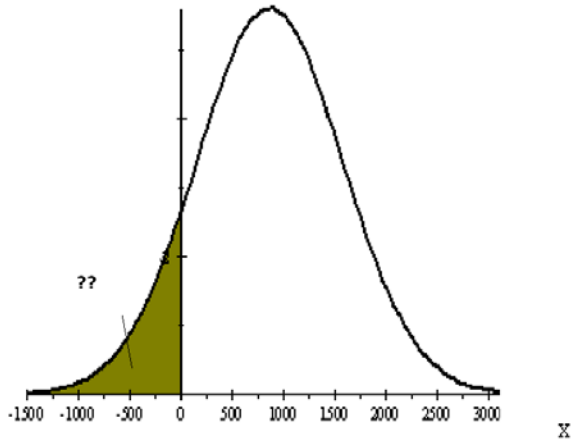
The claims for auto collisions that are paid by the insurance companies in a certain region has a mean \$870 and standard deviation \$710. Find the probability that the mean $\overline{x}$ of 100 collisions (treat as an SRS) will be more than \$900.

First, let us examine whether the population of the claims have a normal distribution OR not.

Note that a normal distribution with mean 870 and standard deviation 710 looks like

$$\frac{1}{710\sqrt{2\pi}}e^{-\left((x-870)^2/\left(2\times 710^2\right)\right)}$$



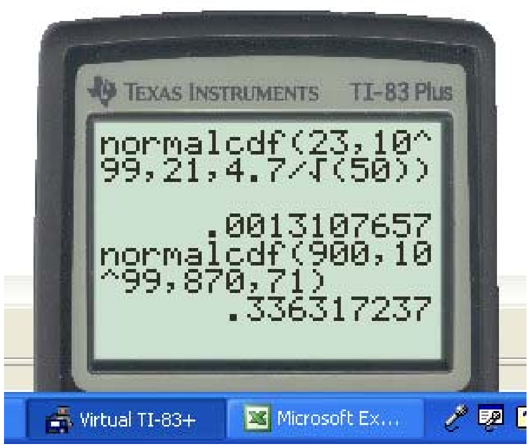


This can not be applied to this context.
Also, some claims are very high that will skew the distribution to the right.

Now even though the population does not have a normal distribution, the sample means for simple random samples of size 100
will have a normal distribution because of the Central Limit Theorem.

The mean of $\overline{x}$ is 870 and standard deviation is $\frac{710}{\sqrt{100}} = 71$

The probability that $\overline{x} \geq 900$ is 0.3363 as shown below



another question:

Find the probability that the total of 100 claims (Simple Random Sample) will exceed a total of $110000?

Total exceeds 110000

$\rightarrow$

the mean $\overline{x}$ exceeds $\frac{110000}{100} = 1100$

The probability is approximately 0.0006 as shown below