Lesson2 Part1

The dot product of two vectors

Given a vector $u = \langle u_1, u_2, u_3 \rangle$ **and a vector** $v = \langle v_1, v_2, v_3 \rangle$

the dot product of these two vectors is defined to be

$\mathbf{u} \cdot \mathbf{v} = \mathbf{u}_1\mathbf{v}_1 + \mathbf{u}_2\mathbf{v}_2 + \mathbf{u}_3\mathbf{v}_3$

The output obtained from the dot product of two vectors is a scalar.

The dot product is often called scalar product.

Example 1:

Consider $u = \langle 5, -1, 3 \rangle$ **and** $v = \left\langle \sqrt{2}, 3, 2 \right\rangle$

$\mathbf{u} \cdot \mathbf{v} = (5)\left(\sqrt{2}\right) + (-1)(3) + (3)(2)$

$\mathbf{u} \cdot \mathbf{v} = \mathbf{5}\sqrt{2} - \mathbf{3} + \mathbf{6}$

$\mathbf{u} \cdot \mathbf{v} = \mathbf{5}\sqrt{2} + \mathbf{3}$

$\mathbf{u} \cdot \mathbf{v} = \mathbf{3} + \mathbf{5}\sqrt{2}$

IMPORTANT PROPERTY:

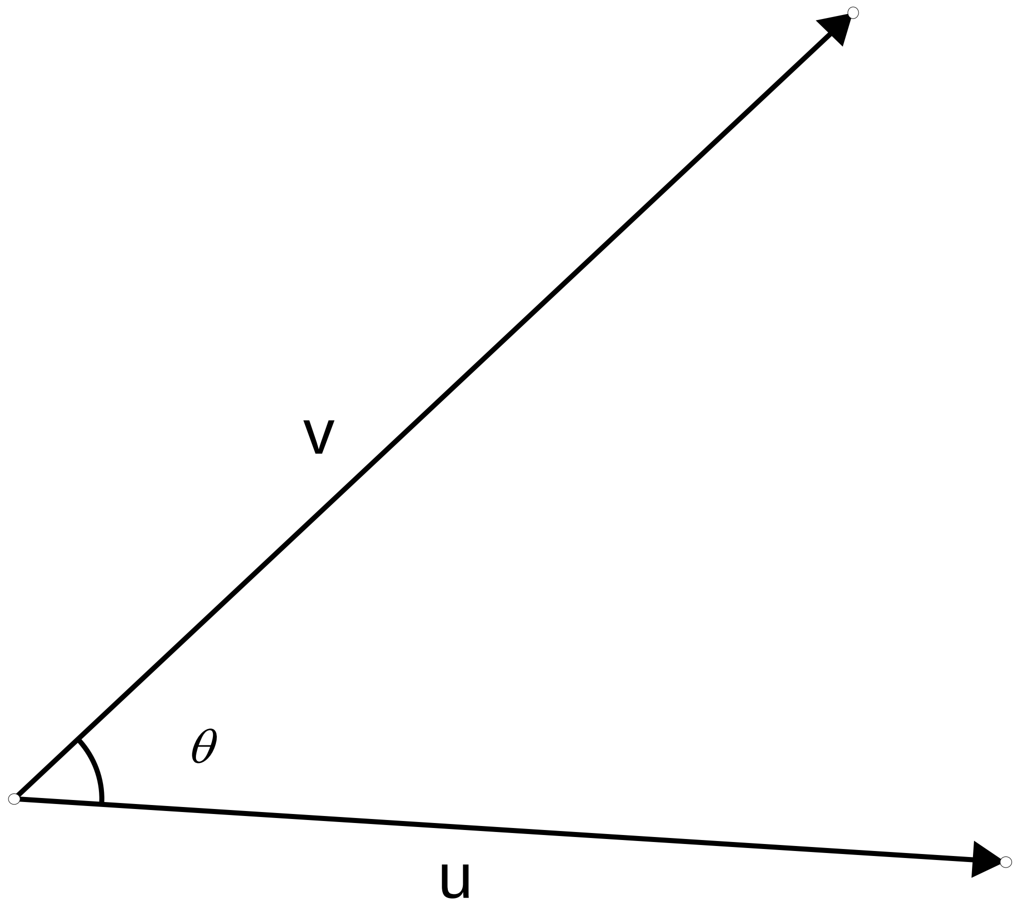
For $u = \langle u_1, u_2, u_3 \rangle$

Note that

$\mathbf{u} \cdot \mathbf{u} = \mathbf{u}_1\mathbf{u}_1 + \mathbf{u}_2\mathbf{u}_2 + \mathbf{u}_3\mathbf{u}_3$

$\mathbf{u} \cdot \mathbf{u} = \|u\|^2$

Read the Theorem 11.5 on the page 783, you shall see Law of Cosines from trigonometry help us relate the dot product to angle between two vectors in terms of the equation given below.



$\cos\theta = \frac{u \cdot v}{\|u\|\|v\|}$ **, where** $0 \le \theta \le \pi$ **,** $\|u\| \ne 0$**,** $\|v\| \ne 0$

Consequently, If u **and** v **are orthogonal (the angle between the two vectors is** $\frac{\pi}{2}$**)**

then $u \cdot v = 0$

Example 2:

To find the angle between the vectors $u = < 4, 1, -2 >$ **and** $v = < 2, 1, 1 >$

If the angle between the vectors is θ

$$\cos\theta = \frac{4\times2+1\times1+(-2)(1)}{\sqrt{4^2+1^2+(-2)^2}\sqrt{2^2+1^2+1^2}}$$

$$\cos\theta = \frac{8+1-2}{\sqrt{16+1+4}\sqrt{4+1+1}}$$

$$\cos\theta = \frac{7}{\sqrt{21}\sqrt{6}}$$

$$\theta = \cos^{-1}\left(\frac{7}{\sqrt{21}\sqrt{6}}\right)$$

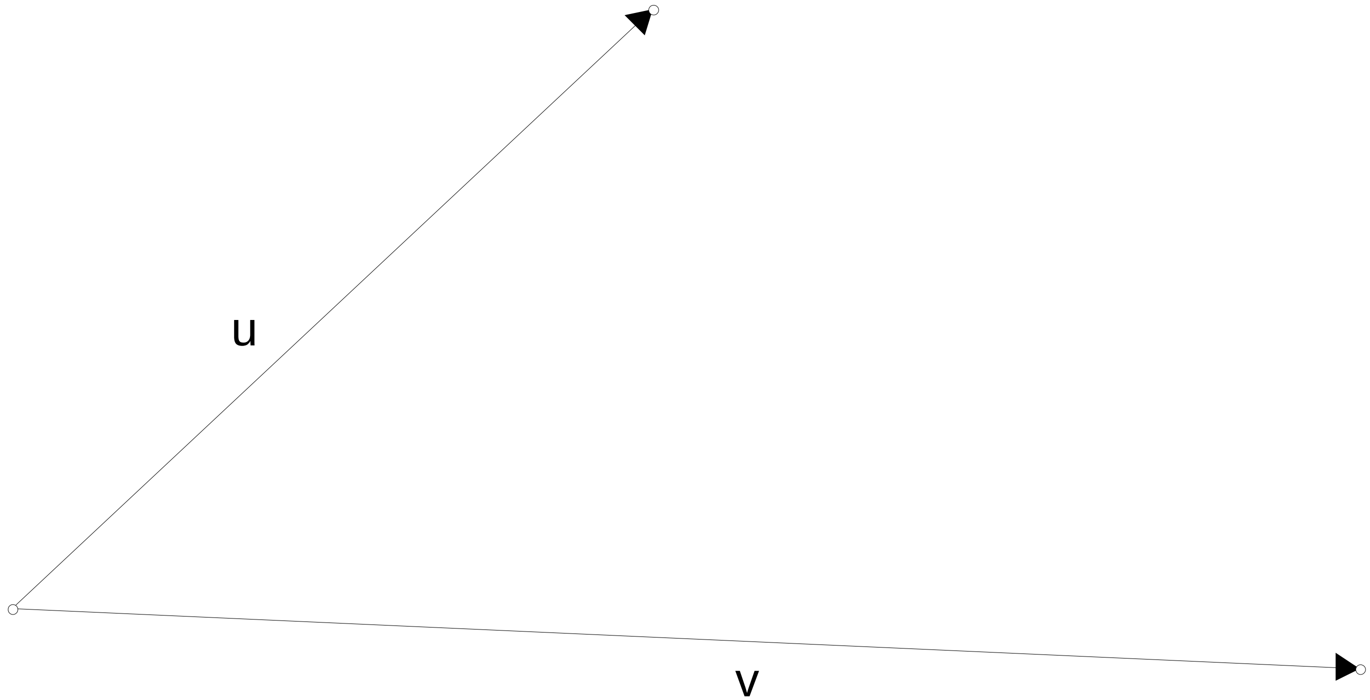
$$\cos^{-1}\left(\frac{7}{\sqrt{21}\sqrt{6}}\right) \cong 0.8974447095481337358$$ **radians**

or

$\mathbf{0.8974447095481337358}(\frac{180}{\pi}) \cong \mathbf{51.42}^\circ$

Projection of a vector u **onto a vector** v

Consider


u
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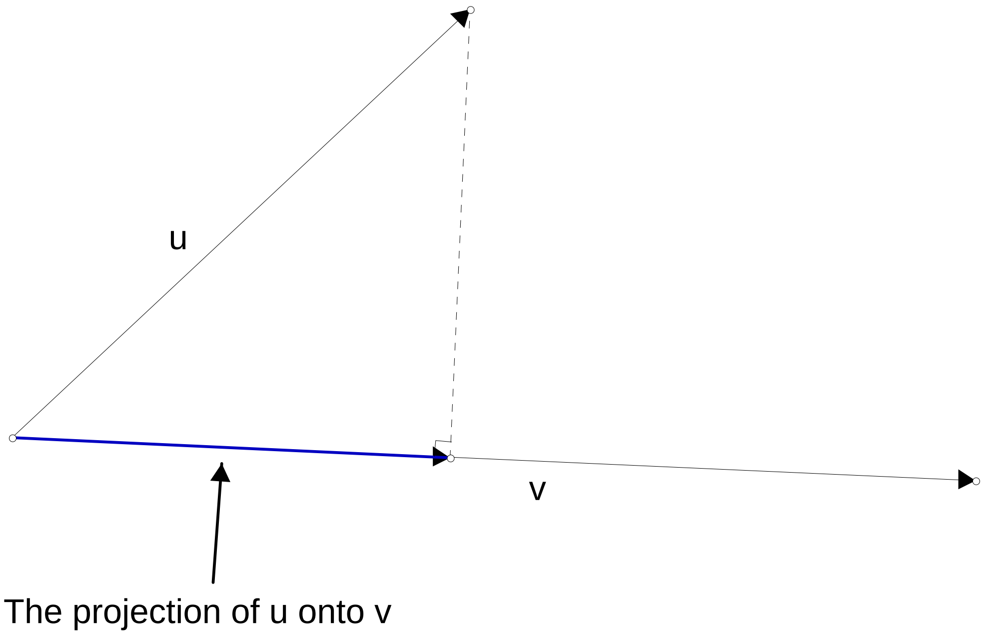
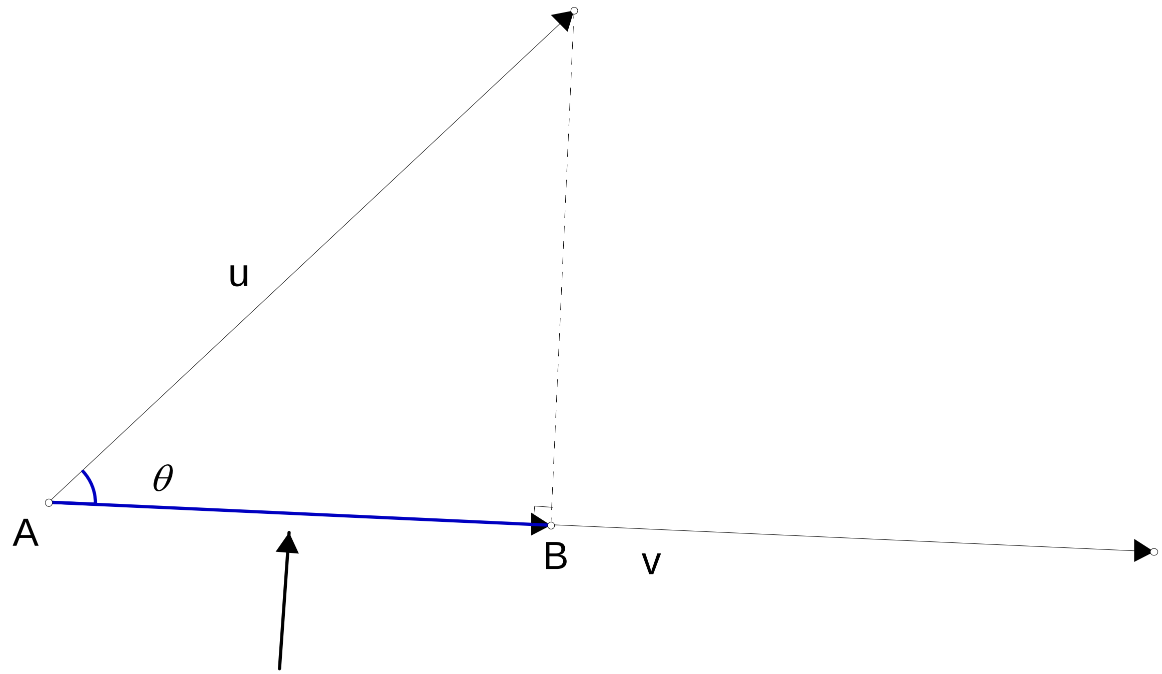

u
v
The projection of u onto v



Note that the $|\text{AB}| = \|u\| \cos\theta$ **consider** θ **to be acute for the moment**

The projection, call it w_1 **has** $\|w_1\| = |\text{AB}| = \|u\| \cos\theta$

and the direction the same as that of v

Therefore

$\mathbf{w}_1 = \|u\| \cos\boldsymbol{\theta}\left(\text{a unit vector in the direction of } v\right)$

or

$$\mathbf{w}_1 = \|u\| \cos\boldsymbol{\theta}\left(\frac{v}{\|v\|}\right)$$

Remeber that $\cos\theta = \frac{u \cdot v}{\|u\|\|v\|}$

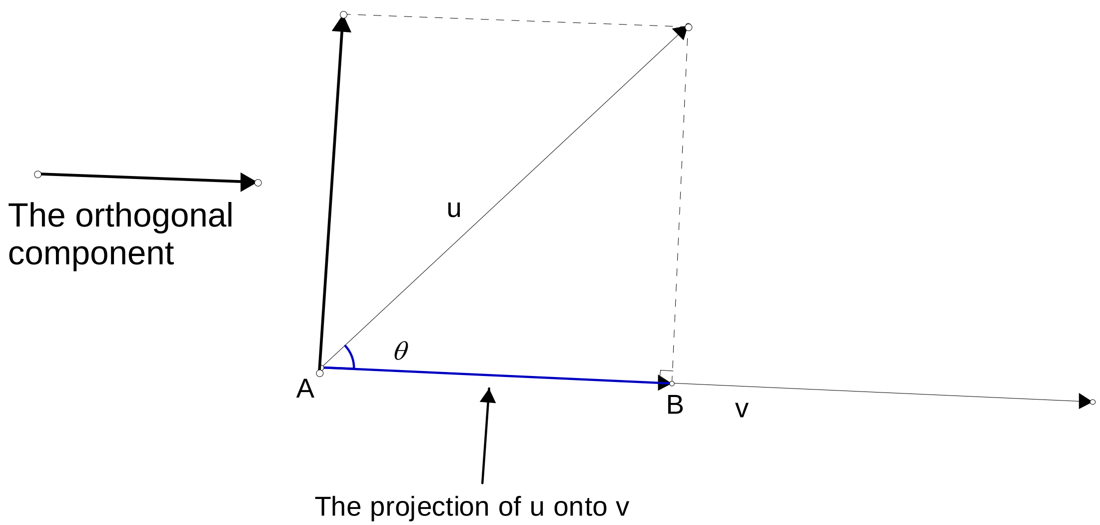
therefore

$$\mathbf{w}_1 = \|u\| \frac{u \cdot v}{\|u\|\|v\|}\left(\frac{v}{\|v\|}\right)$$

that is

$$\mathbf{w}_1 = \frac{(u \cdot v)}{\|v\|^2}\mathbf{v}$$

the component of u **that is orthogonal to** v **is**



$\mathbf{w}_2 = \mathbf{u} - \frac{(u \cdot v)}{\|v\|^2}\mathbf{v}$

Example 3:

Find the projection of $u = 2i + j + 2k$ **onto** $v = i + j - k$ **and orthogonal to** v

Note that $v \cdot v = 1 \times 1 + 1 \times 1 + (-1) \times (-1) = 3$

$\mathbf{u} \cdot \mathbf{v} = \mathbf{2} \times \mathbf{1} + \mathbf{1} \times \mathbf{1} + \mathbf{2} \times (-1) = \mathbf{1}$

Onto $v = \frac{(u \cdot v)}{\|v\|^2} v = \frac{1}{3}(i + j - k)$

Orthogonal to $v = 2i + j + 2k - \frac{1}{3}(i + j - k) = \frac{5}{3}i + \frac{2}{3}j + \frac{7}{3}k$

Please work on the practice problems

Section 11.3
7,9,13,15,23,25,29,33,37,39,41,45,49,59,71,81,87