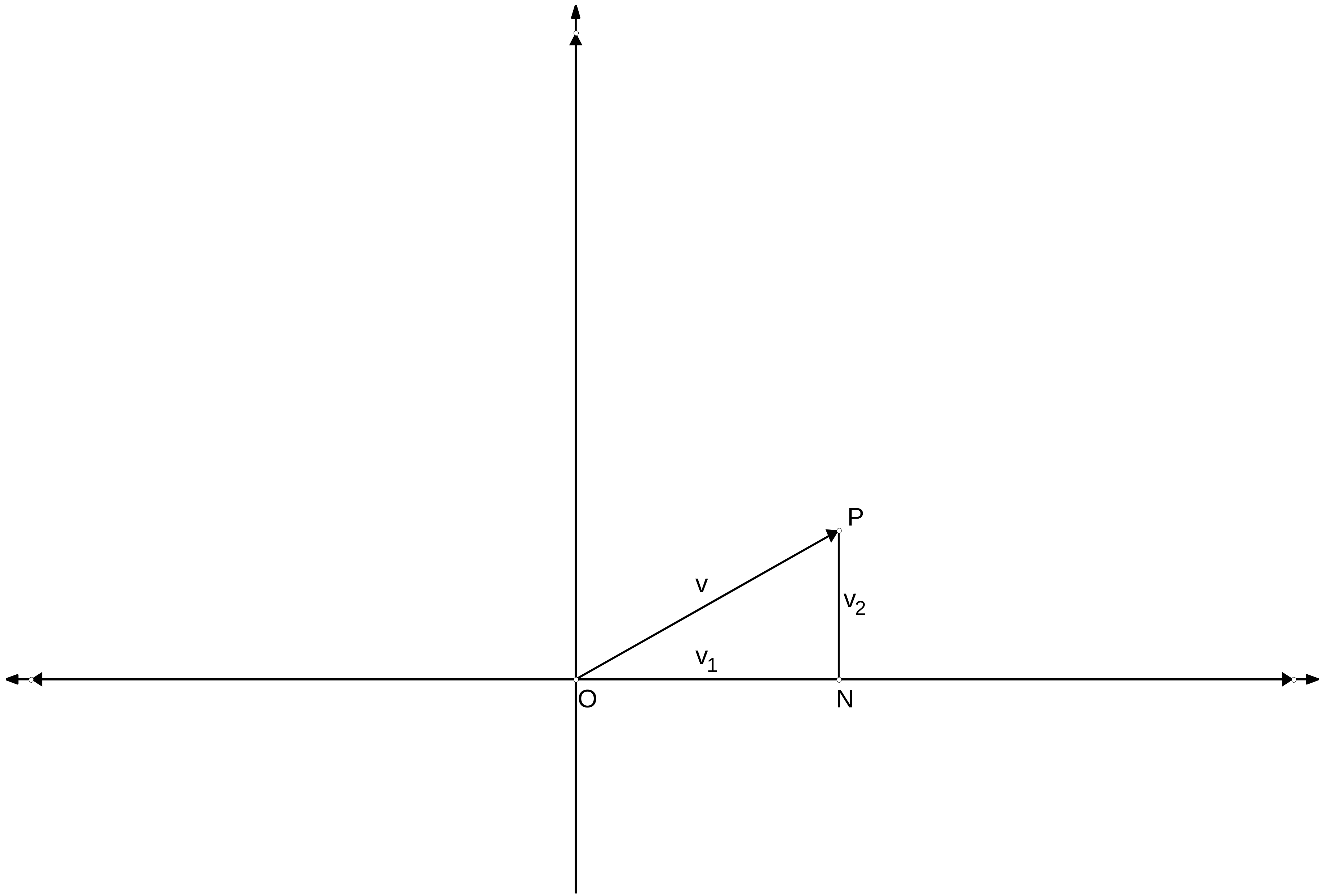
Lesson 1

Read the terminologies in the section 11.1


P
v
v_2
v_1
O
N

If the coordinates of P are (v_1, v_2) , the vector given by the directed line segment OP is $\overline{v} =< v_1, v_2 >$

the length $\|\overline{v}\|$of the vector $\overline{v}$ is $\sqrt{v_1^2 + v_2^2}$

A unit vector is a vector with length 1 unit.

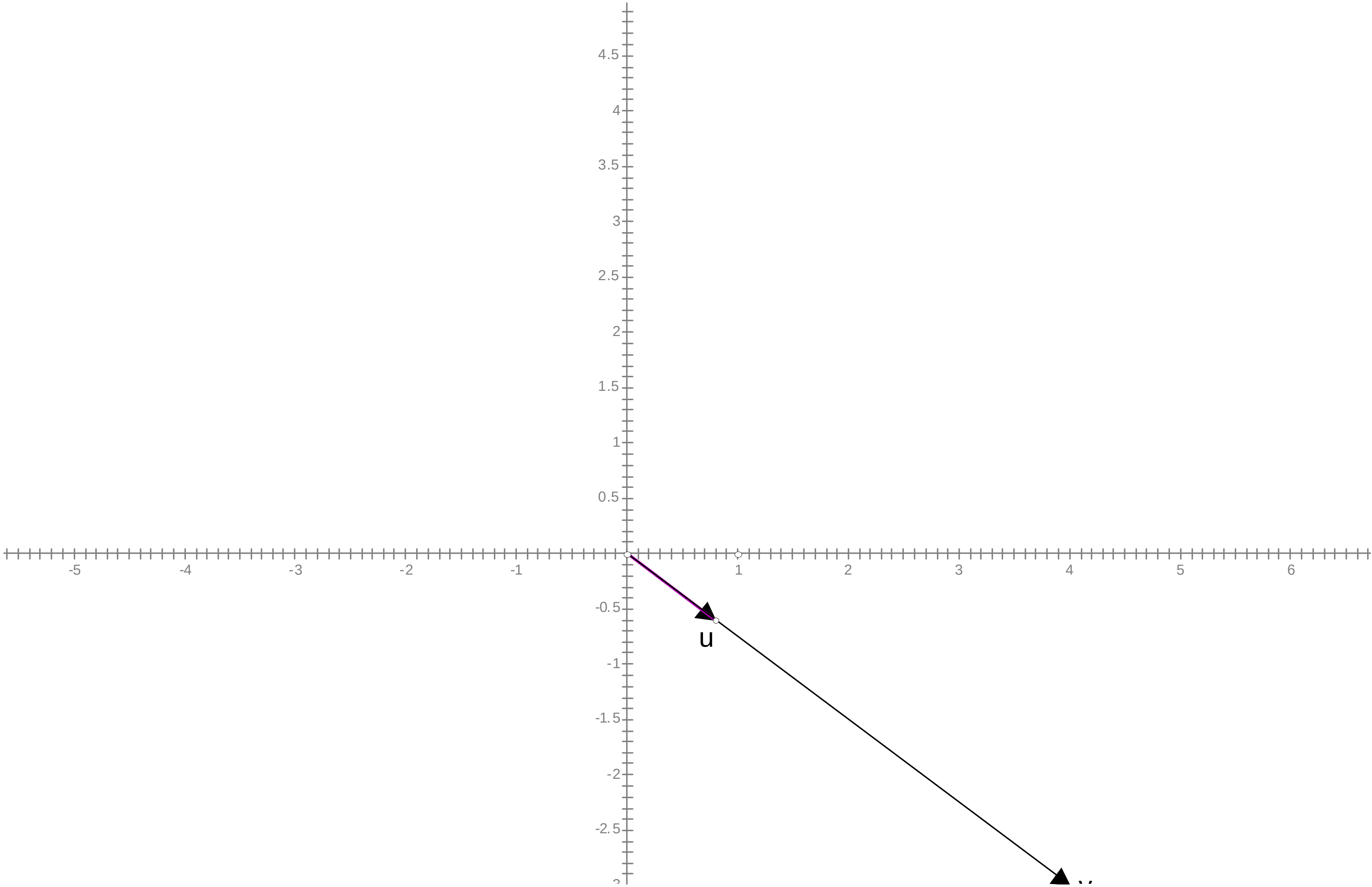
Example 1:

Find a unit vector in the direction of the vector $\overline{v} =< 4, -3 >$

A unit vector $\overline{u}$ in the direction of $\overline{v}$ is $\frac{1}{\|\overline{v}\|} \overline{v}$

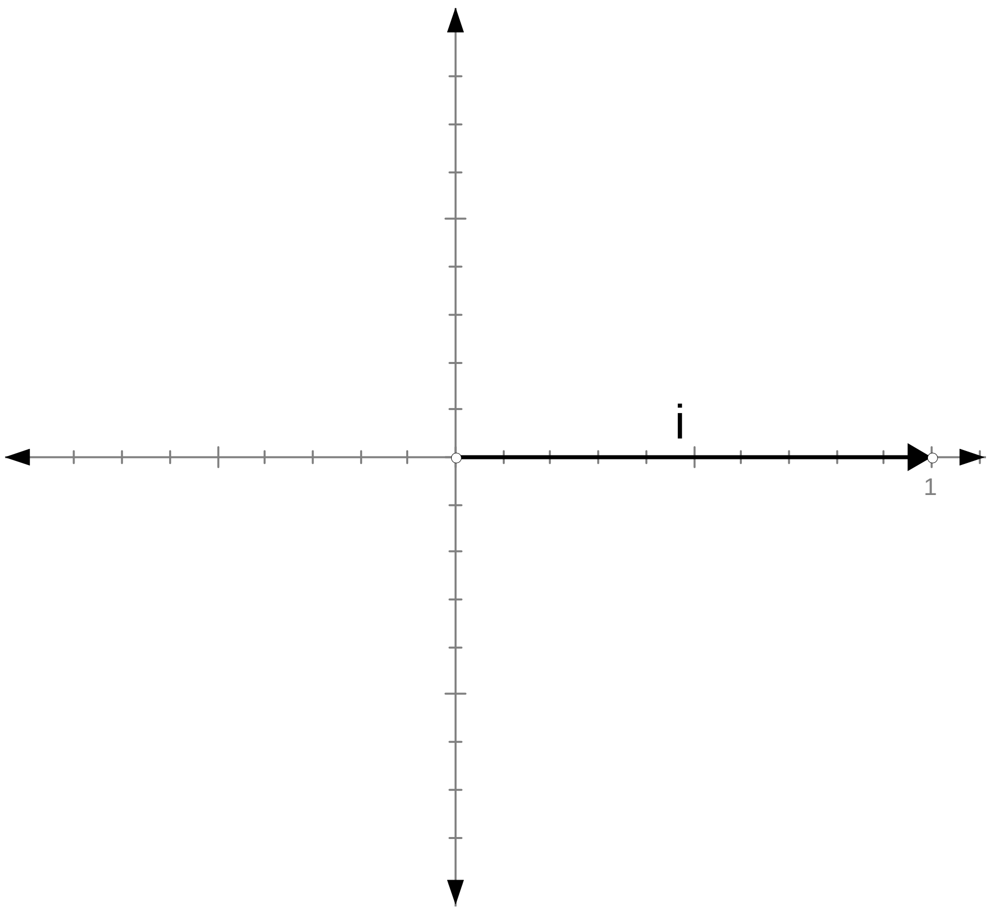
In this example, $\|\overline{v}\| = \sqrt{(3)^2 + (-4)^2} = 5$

Therefore $\overline{u} = \frac{1}{5} < 4, -3 >= \left\langle \frac{4}{5}, -\frac{3}{5} \right\rangle = \langle 0.8, -0.6 \rangle$

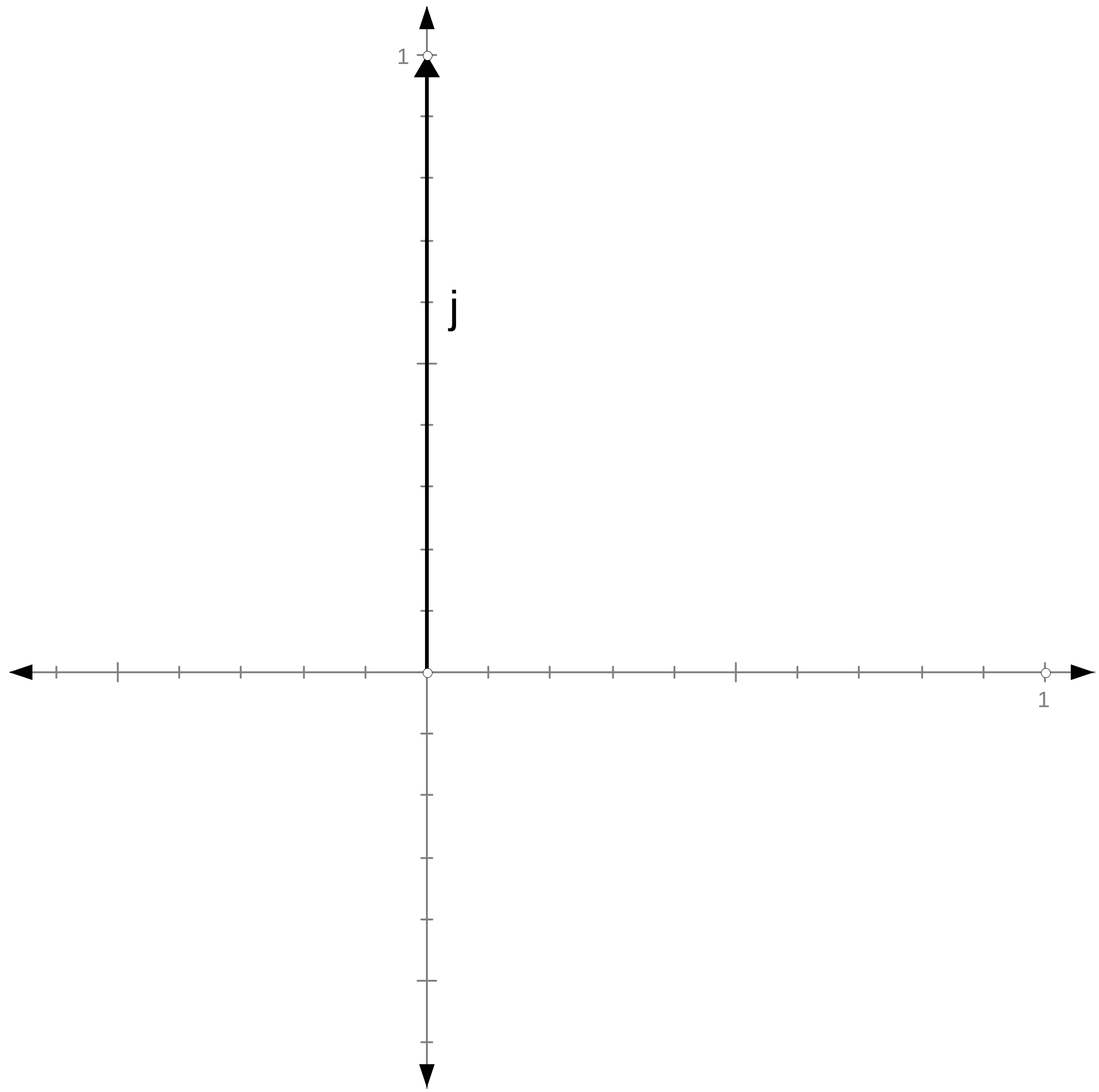

u

The standard unit vectors:

i $=\langle 1,0\rangle$

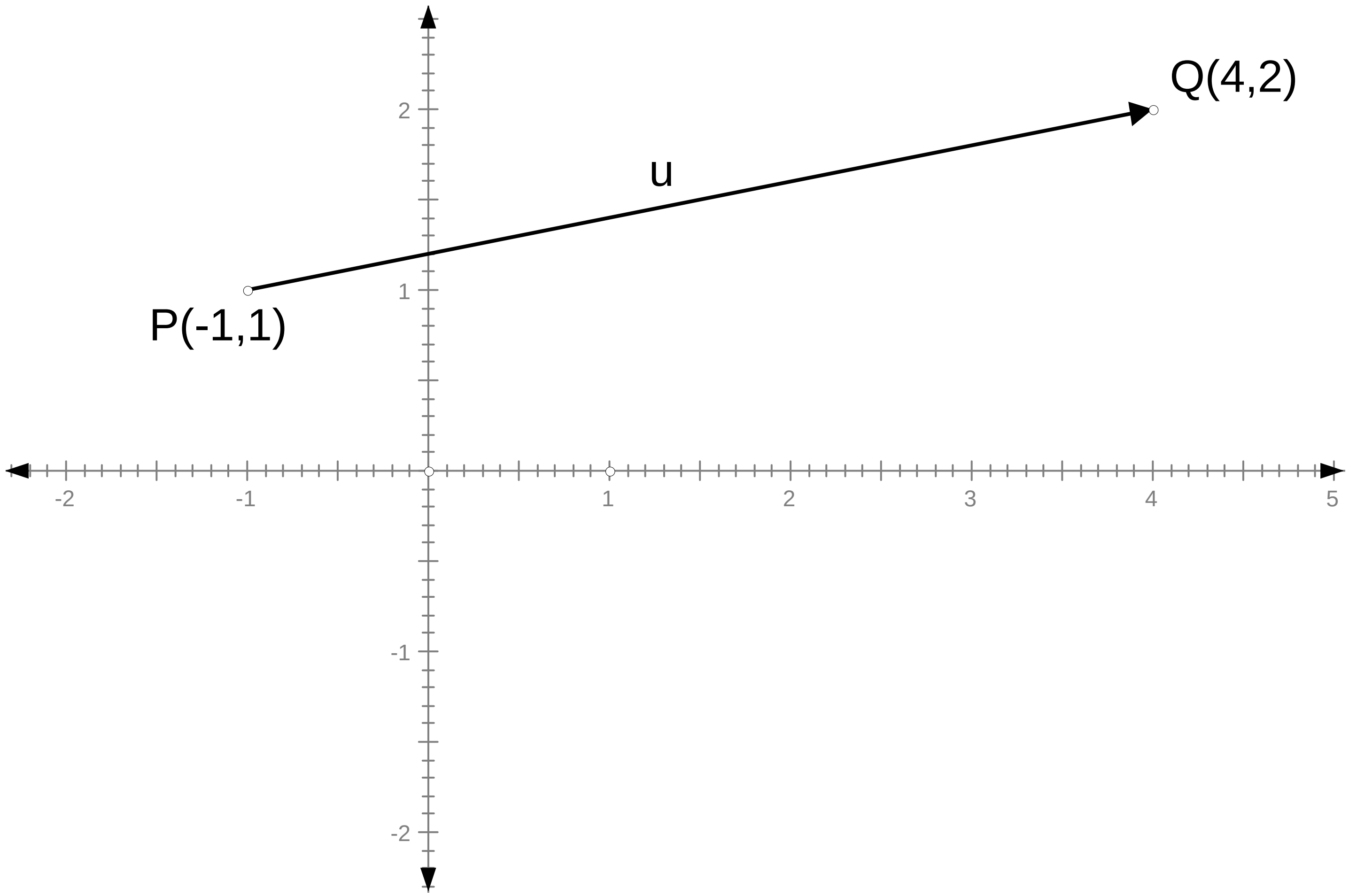


j $=\langle 0,1\rangle$

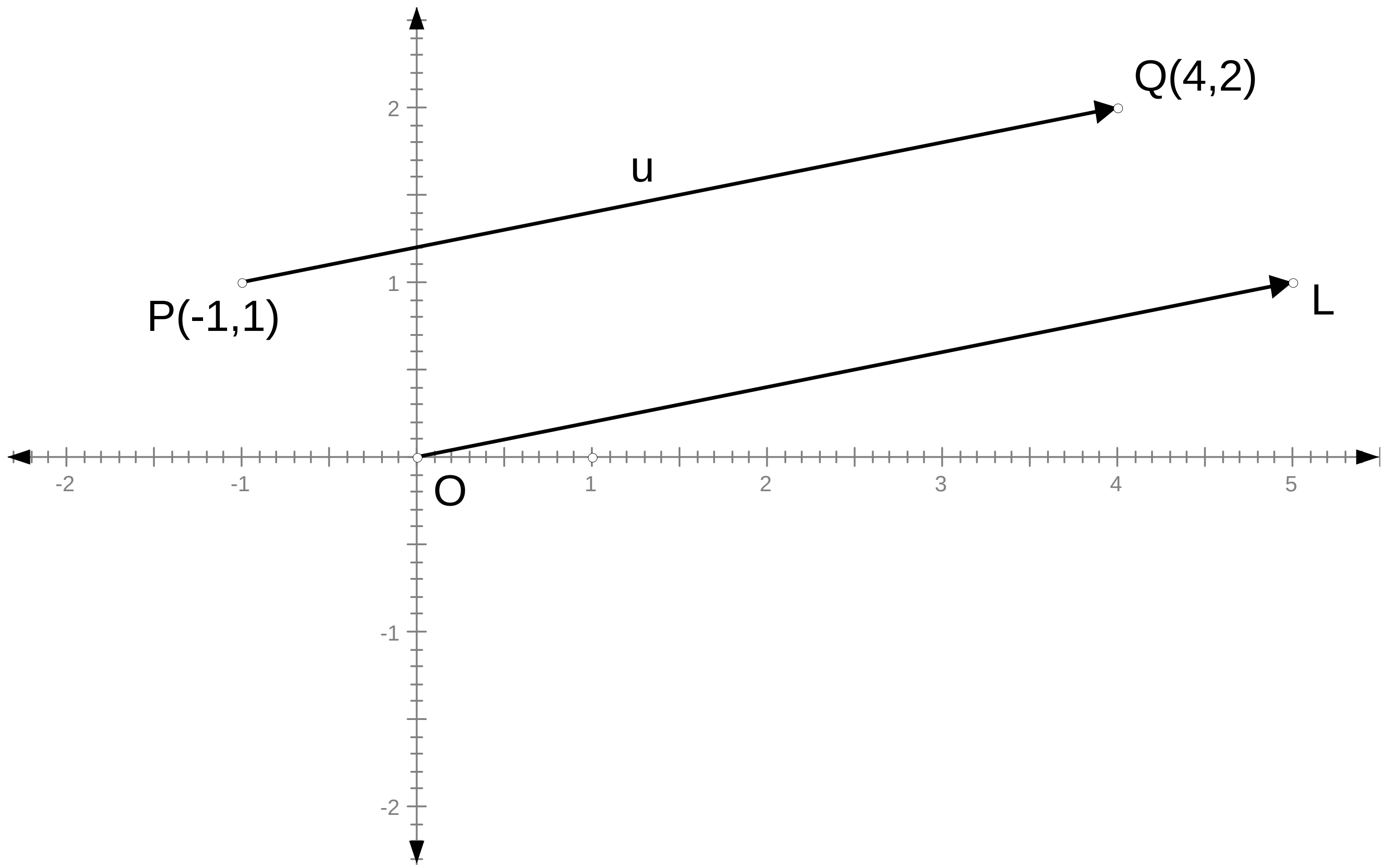

1
j
1

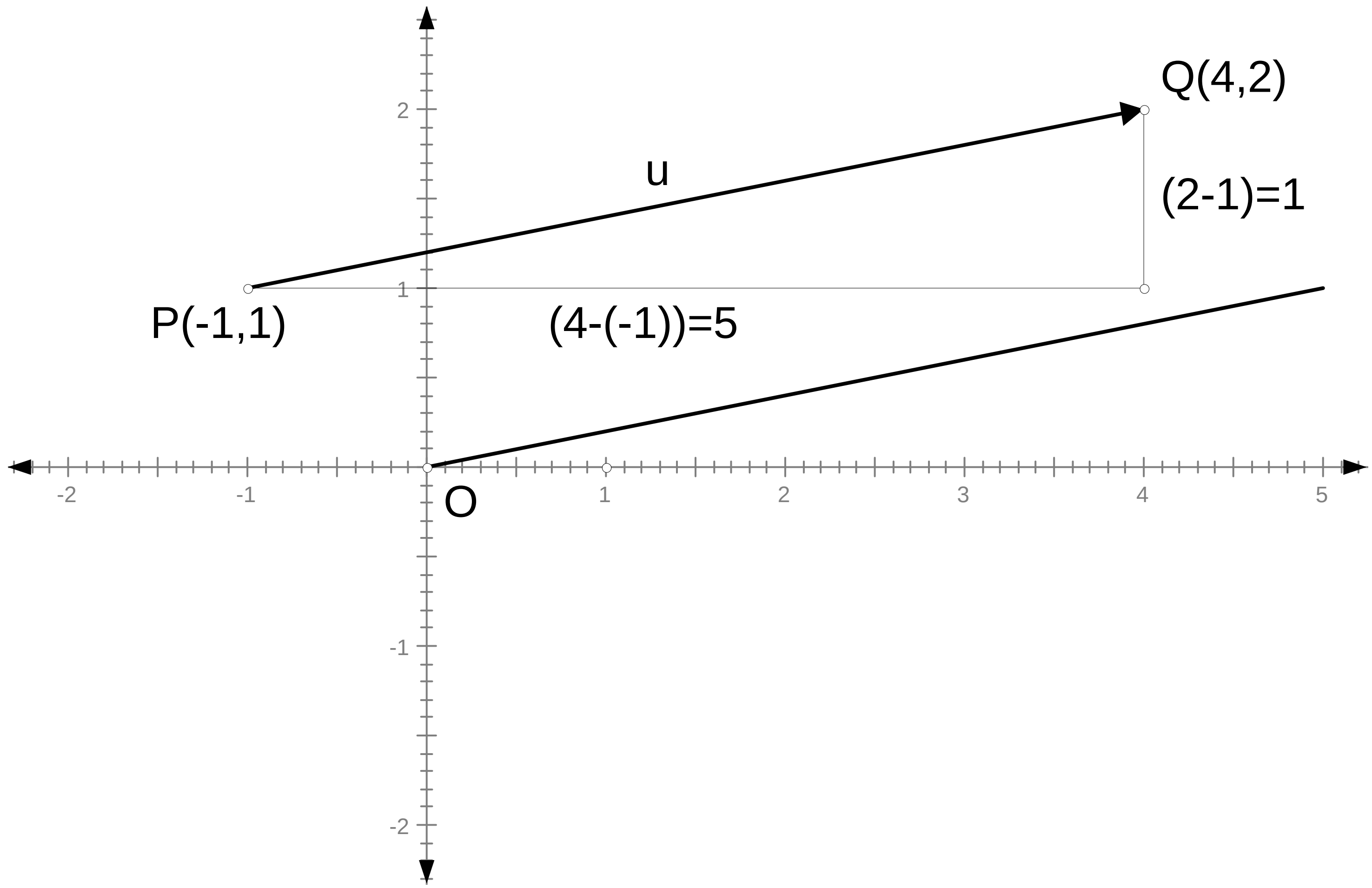
Example 1:

To write the vector $\overline{u}$ in component form



Note that the vector $\overline{OL}$ is equivalent to $\overline{u}$


Q(4,2)
u
P(-1,1)
L
O
2
1
-1
-2
-2
-1
1
2
3
4
5



The component form of $\overline{u}$ **is** $\langle 5,1 \rangle$

Example 2:

Given that $\overline{v_1} = \langle 4,3 \rangle$ and $\overline{v_2} = \langle 6,8 \rangle$, to find $\frac{1}{3}\overline{v_1} + \frac{3}{2}\ \overline{v_2}$

$\frac{1}{3}\overline{v_1} + \frac{3}{2}\ \overline{v_2}$
$=\frac{1}{3}\langle 4,3 \rangle + \frac{3}{2}\langle 6,8 \rangle$
$=\left\langle \frac{4}{3}, \frac{3}{3} \right\rangle + \left\langle \frac{3\times 6}{2}, \frac{3\times 8}{2} \right\rangle$
$=\left\langle \frac{4}{3}, 1 \right\rangle + \langle 9,12 \rangle$
$=\left\langle \frac{4}{3} + 9, 1 + 12 \right\rangle$
$=\left\langle \frac{31}{3}, 13 \right\rangle$

Example 3:

Find a vector $\overline{v}$ with magnitude $\|\overline{v}\| = 5$ and in the same direction as $\langle 2,1 \rangle$

Note that even though $5\langle 2,1 \rangle = \langle 10,5 \rangle$ has the same direction as that of $\langle 2,1 \rangle$

but its magnitude does not equal 1.

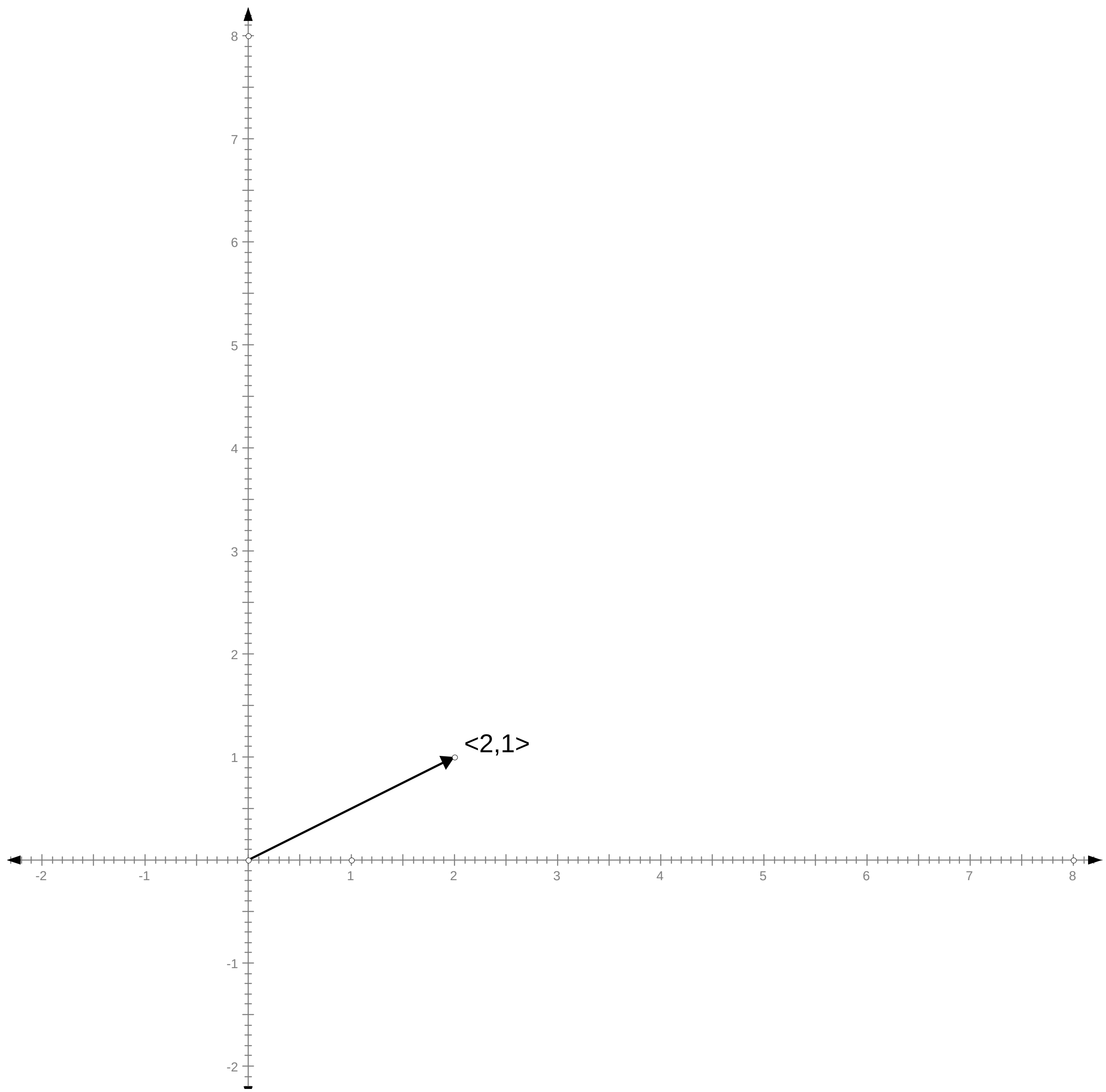
A good strategy will be to find a unit vector in the direction of $\langle 2,1 \rangle$ and 5 times that unit vector will is equivalent to $\overline{v}$

Recall that to obtain a unit vector equivalent to $\langle 2,1 \rangle$, we have to multiply the vector by the reciprocal of its magnitude.

The magnitude of $\langle 2,1 \rangle = \sqrt{2^2 + 1^2} = \sqrt{5}$

A unit vector in the direction of $\langle 2,1 \rangle$ is $\frac{1}{\sqrt{5}}\langle 2,1 \rangle$

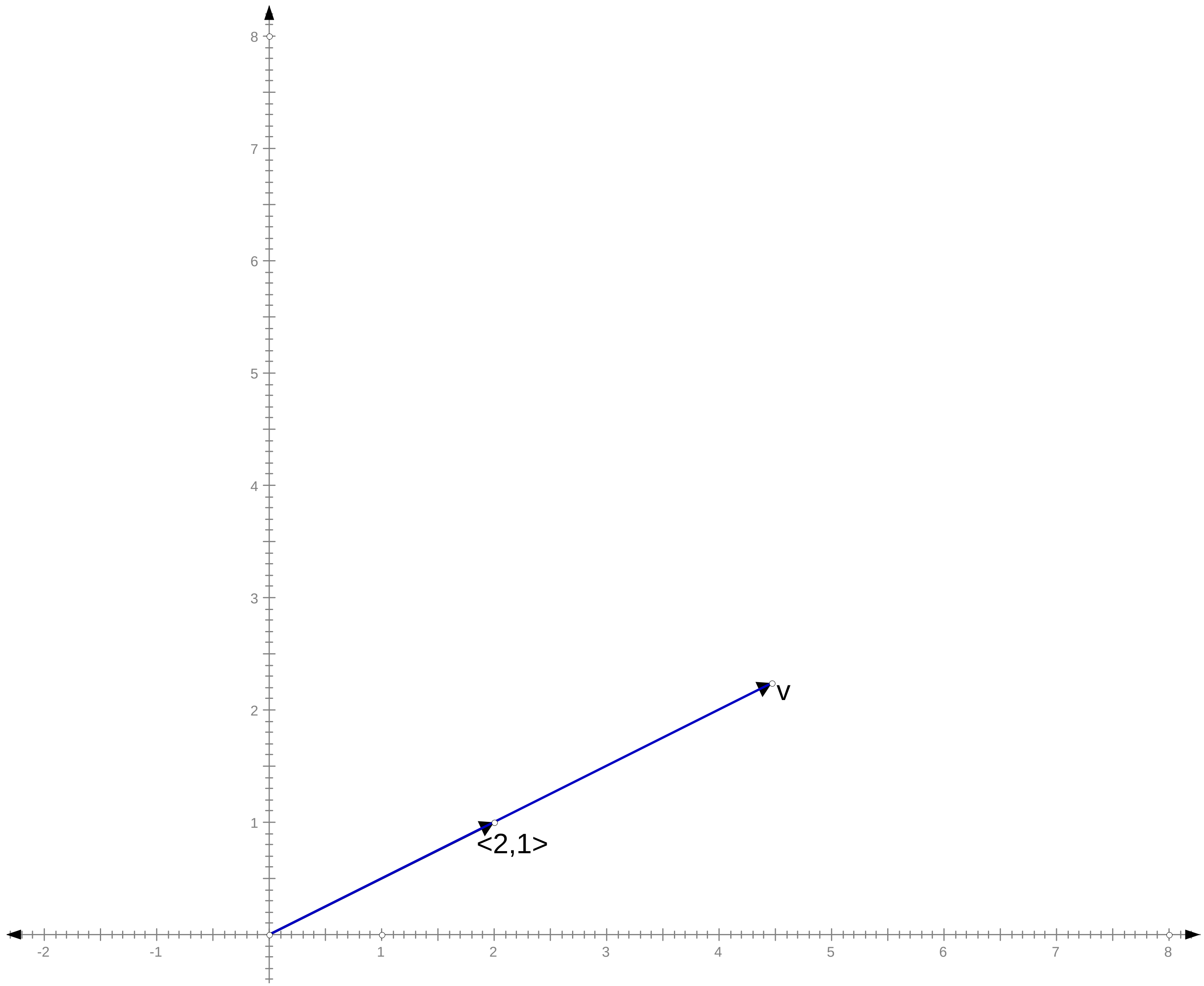
Therefore $\overline{v} = \|\overline{v}\| \frac{1}{\sqrt{5}}\langle 2,1\rangle = \frac{5}{\sqrt{5}}\langle 2,1\rangle = \sqrt{5}\langle 2,1\rangle = \left\langle 2\sqrt{5}, \sqrt{5}\right\rangle$


<2,1>
-2
-1
1
2
3
4
5
6
7
8

Note that

$2\sqrt{5} \cong 4.472\,135\,954\,999\,579\,392\,8$

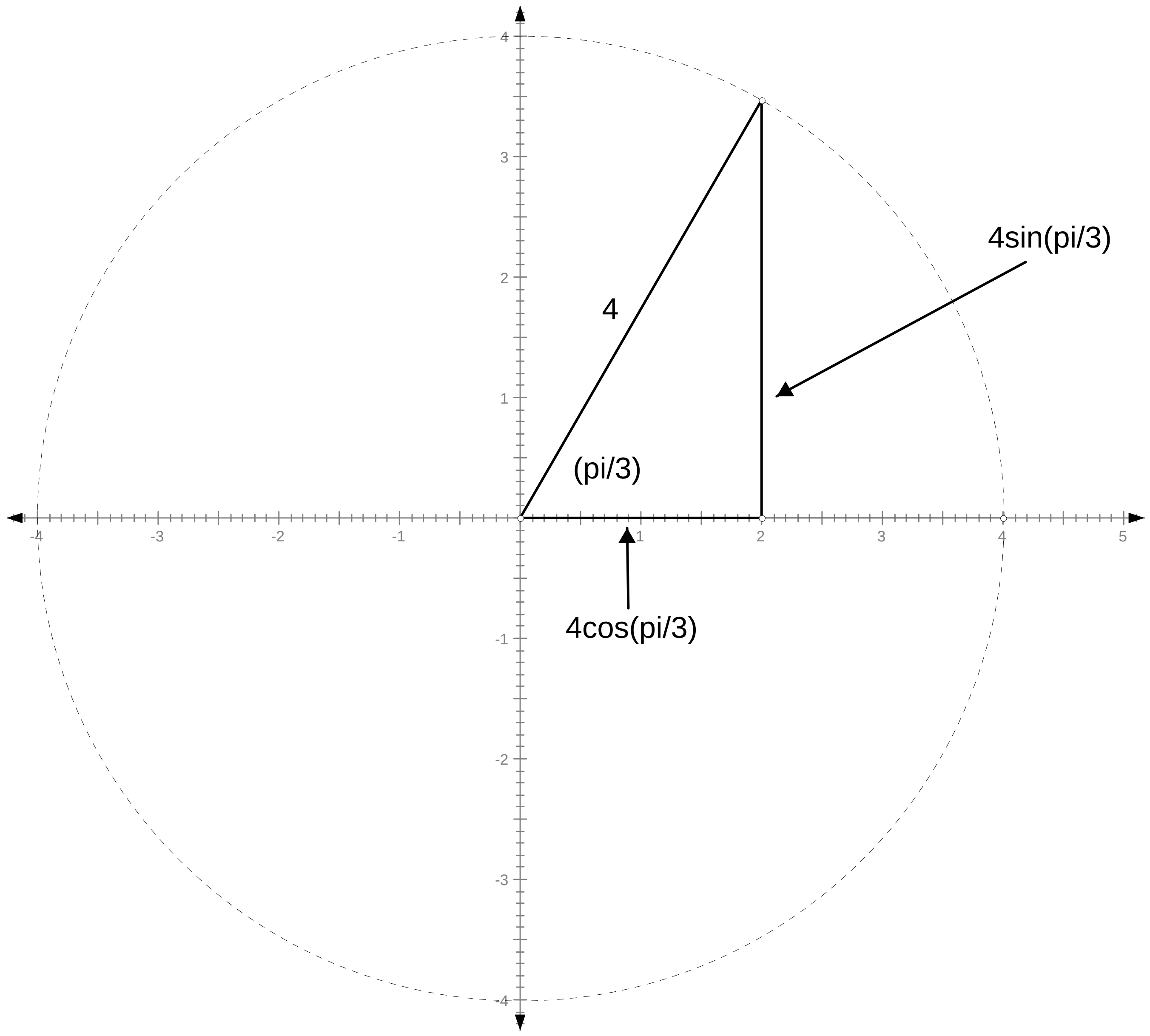
$\sqrt{5} = 2.236\,067\,977\,499\,789\,696\,4$



<2,1>
v

Example 4:

Find the component form of $\overline{v}$ **, where it is given that** $\|\overline{v}\| = 4$ **and it makes an angle** $\theta = \frac{\pi}{3}$ **radians with the positive x-axis.**


4sin(pi/3)
4
(pi/3)
4cos(pi/3)
4
3
2
1
-1
-2
-3
-4
-4
-3
-2
-1
1
2
3
4
5

The component form is $\left\langle 4\cos\left(\frac{\pi}{3}\right), 4\sin\left(\frac{\pi}{3}\right)\right\rangle = \left\langle 2, 2\sqrt{3}\right\rangle$

5. To find and sketch a unit vector along the tangent and along the normal to the graph of $f(x) = \tan^{-1}x$ **at** $\left(1, \frac{\pi}{4}\right)$

Let us find the slope of the tangent line at $\left(1, \frac{\pi}{4}\right)$

$\mathbf{f'(x)} = \frac{1}{1+x^2}$

$\mathbf{f'(1)} = \frac{1}{2}$

equation of the tangent line is

$\mathbf{y} - \frac{\pi}{4} = \frac{1}{2}(x-1)$

We already have the coordinates of one point already that are $\left(1, \frac{\pi}{4}\right)$

Let us get the coordinates of another point,

set $x = 3$

$\mathbf{y} - \frac{\pi}{4} = \frac{1}{2}(3-1)$

$\mathbf{y} - \frac{\pi}{4} = \mathbf{1}$

$\mathbf{y} = \mathbf{1} + \frac{\pi}{4}$

The coordinates of another point are $\left(3, 1+\frac{\pi}{4}\right)$

a vector along the tangent may be taken as a vector with

$\left(1, \frac{\pi}{4}\right)$ **as the initial point and** $\left(3, 1+\frac{\pi}{4}\right)$ **as the terminal point,**

one such vector is $\left\langle 3-1, 1+\frac{\pi}{4}-\frac{\pi}{4}\right\rangle = \langle 2, 1\rangle$

A unit vector along $\langle 2, 1\rangle$ **is** $\frac{1}{\sqrt{5}}\langle 2, 1\rangle$

Normal Line: The slope of the line normal to the graph of $f(x) = \tan^{-1}x$ **is**

$-\frac{1}{\left(\frac{1}{2}\right)} = -\mathbf{2}$

An equation of the normal line at $\left(1, \frac{\pi}{4}\right)$ **is**

$\mathbf{y} - \frac{\pi}{4} = -\mathbf{2}(x-1)$

take $x = 2$

$\mathbf{y} - \frac{\pi}{4} = -\mathbf{2}(2-1)$

$\mathbf{y} - \frac{\pi}{4} = -\mathbf{2}$

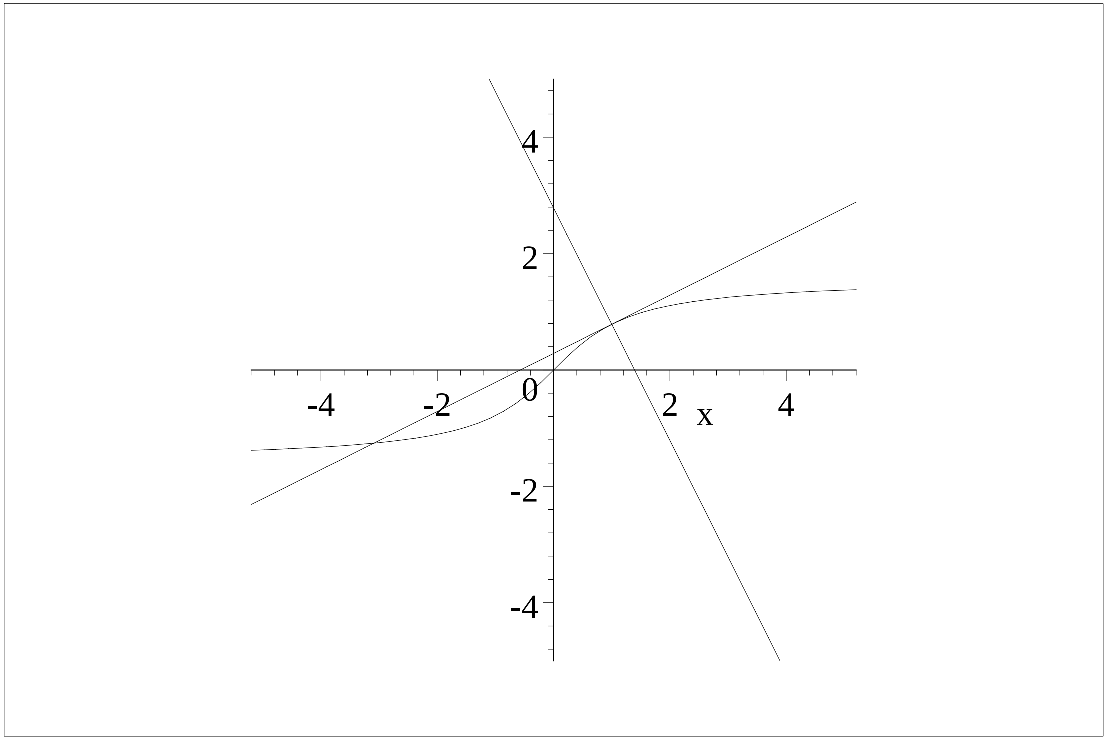
$\mathbf{y} = -\mathbf{2} + \frac{\pi}{4}$

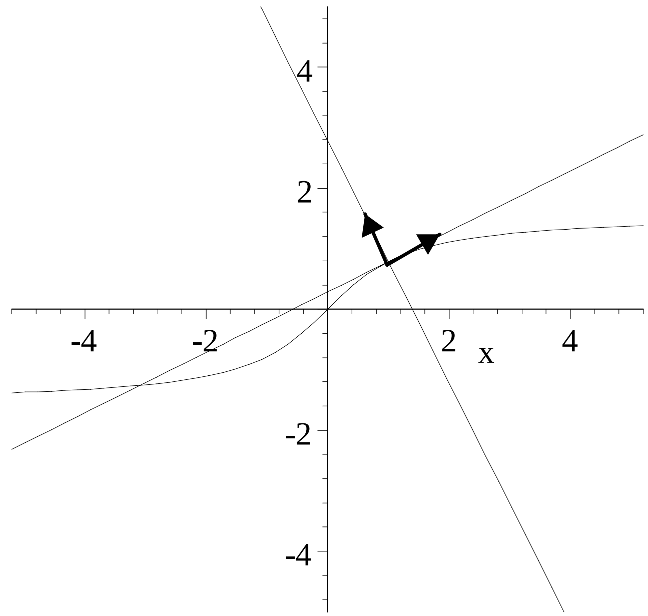
therefore $\left(2, -2 + \frac{\pi}{4}\right)$ **show the coordinates of a point on** $y - \frac{\pi}{4} = -2(x - 1)$

A vector along the normal may be taken as a vector with initial point $\left(1, \frac{\pi}{4}\right)$ **and terminal point** $\left(2, -2 + \frac{\pi}{4}\right)$

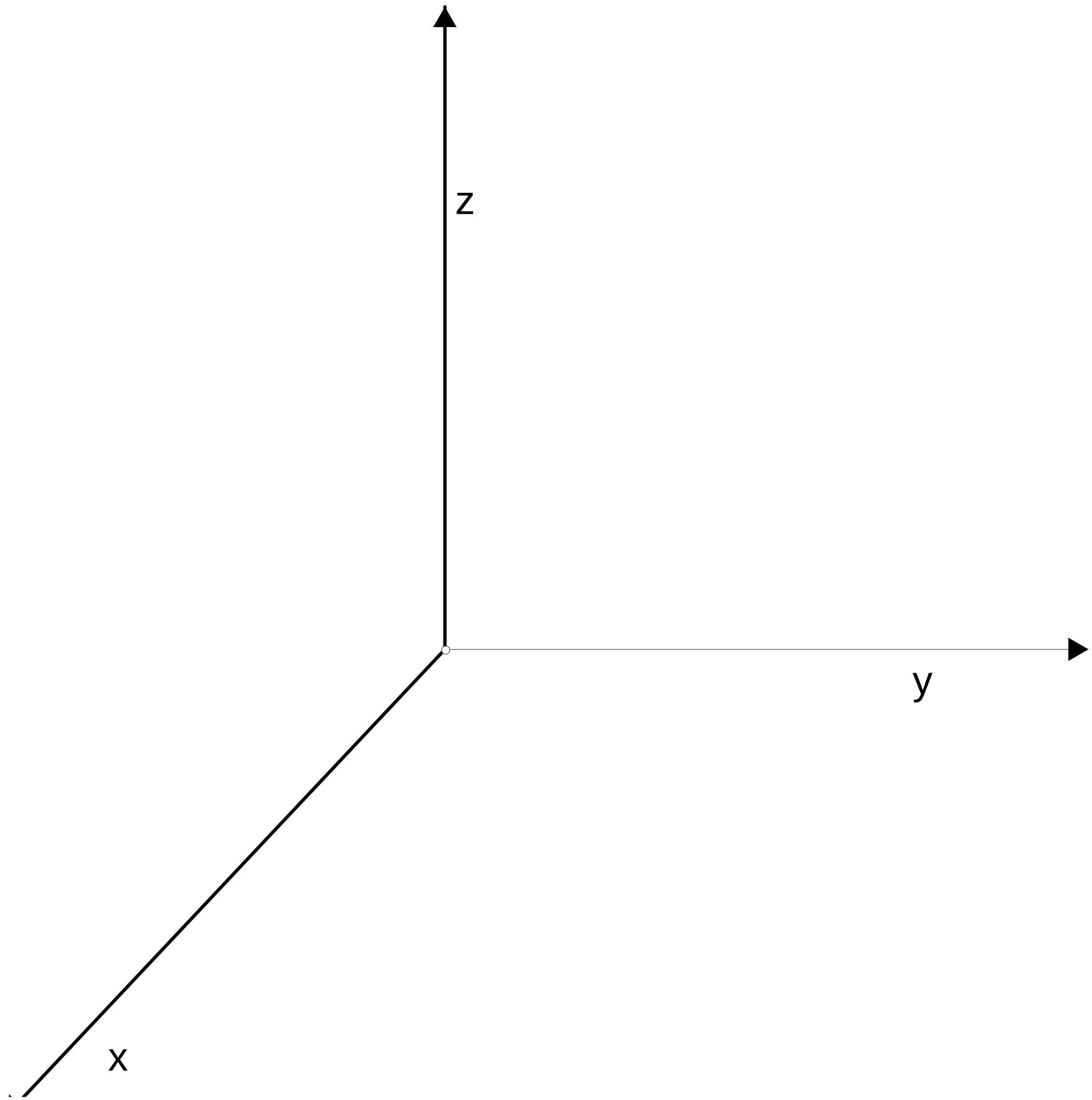
that is $\left\langle 2 - 1, -2 + \frac{\pi}{4} - \frac{\pi}{4} \right\rangle = \langle 1, -2 \rangle$

a unit vector along the normal line to the graph of $f(x) = \tan^{-1} x$ **is** $\frac{1}{\sqrt{5}} \langle 1, -2 \rangle$





Vectors in Space


z
y
x

Please read the pages 774-775 in the text book,

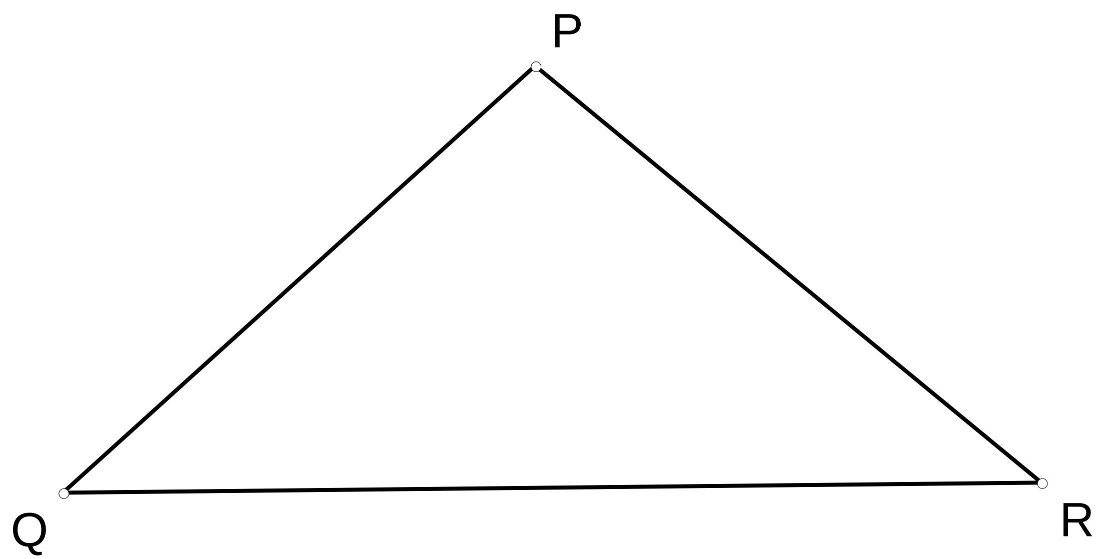
Some worked out examples from the section 11.2

Page 778:

30

Let $P(5,3,4)$**,** $Q(7,1,3)$**,** $R(3,5,3)$

|PQ| $=\sqrt{(7-5)^2+(1-3)^2+(3-4)^2}=$ **3**
|QR| $=\sqrt{(3-7)^2+(5-1)^2+(3-3)^2}=$ $\mathbf{4}\sqrt{2}$
|PR| $=\sqrt{(3-5)^2+(5-3)^2+(3-4)^2}=$ **3**



This is an isoceles triangle.

#40.

$4x^2+4y^2+4z^2-4x-32y+8z+33=0$

First, let us divide the entire equation by 4

$x^2+y^2+z^2-x-8y+2z+\frac{33}{4}=0$

Rearrange the terms before completing the square

$x^2-x+y^2-8y+z^2+2z+\frac{33}{4}=0$

$$\left(x^2 - x + \tfrac{1}{4}\right) + (y^2 - 8y + 16) + (z^2 + 2z + 1) + \tfrac{33}{4} - \tfrac{1}{4} - \mathbf{16} - \mathbf{1} = \mathbf{0}$$

$$\left(x - \tfrac{1}{2}\right)^2 + (y - 4)^2 + (z + 1)^2 + \tfrac{32}{4} - \mathbf{16} - \mathbf{1} = \mathbf{0}$$

$$\left(x - \tfrac{1}{2}\right)^2 + (y - 4)^2 + (z + 1)^2 - \mathbf{9} = \mathbf{0}$$

$$\left(x - \tfrac{1}{2}\right)^2 + (y - 4)^2 + (z + 1)^2 = \mathbf{9}$$

The sphere has center at $\left(\tfrac{1}{2}, 4, -1\right)$ **and radius** 3

Suggested Practice:

Section 11.1: 1 thru 93 Odd Numbered
Section 11.2: 1 thru 113 Odd Numbered