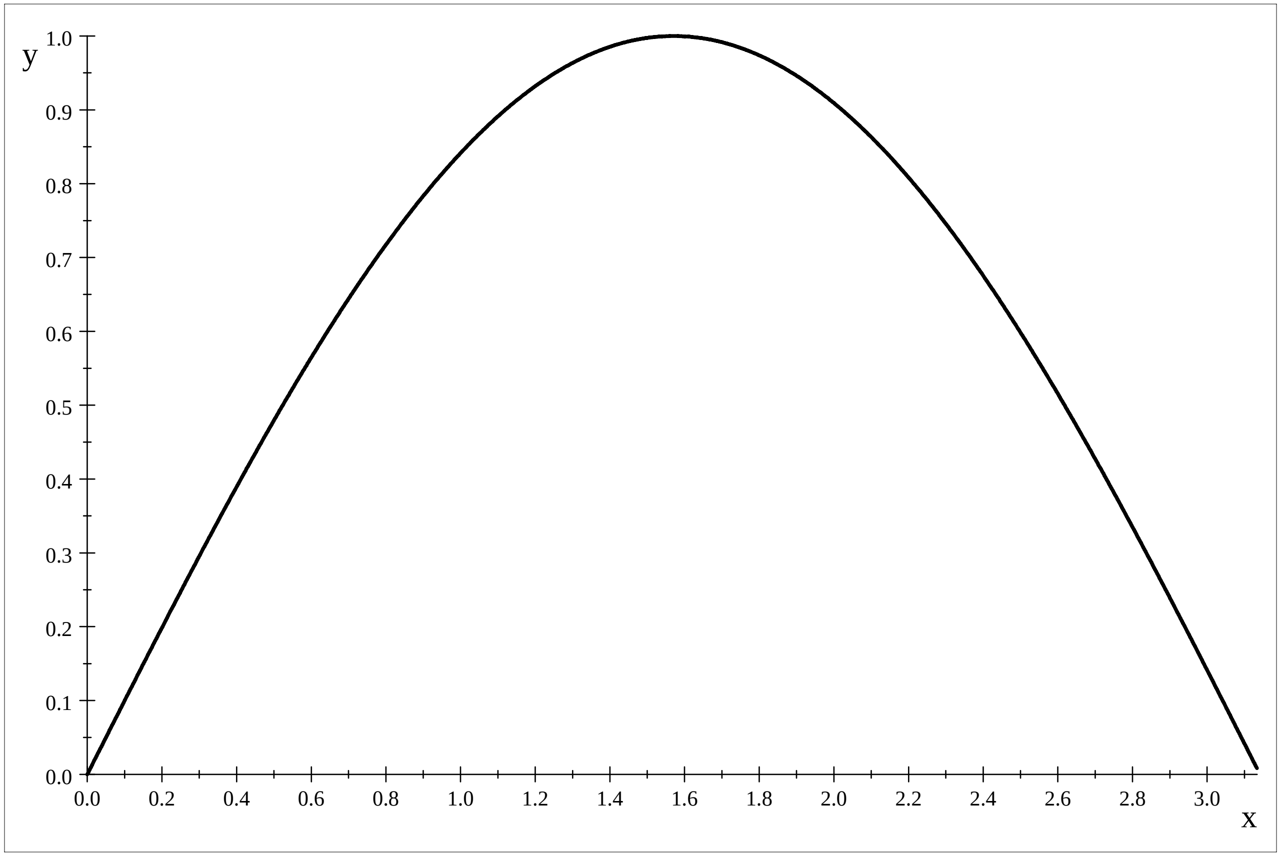
Consider the function

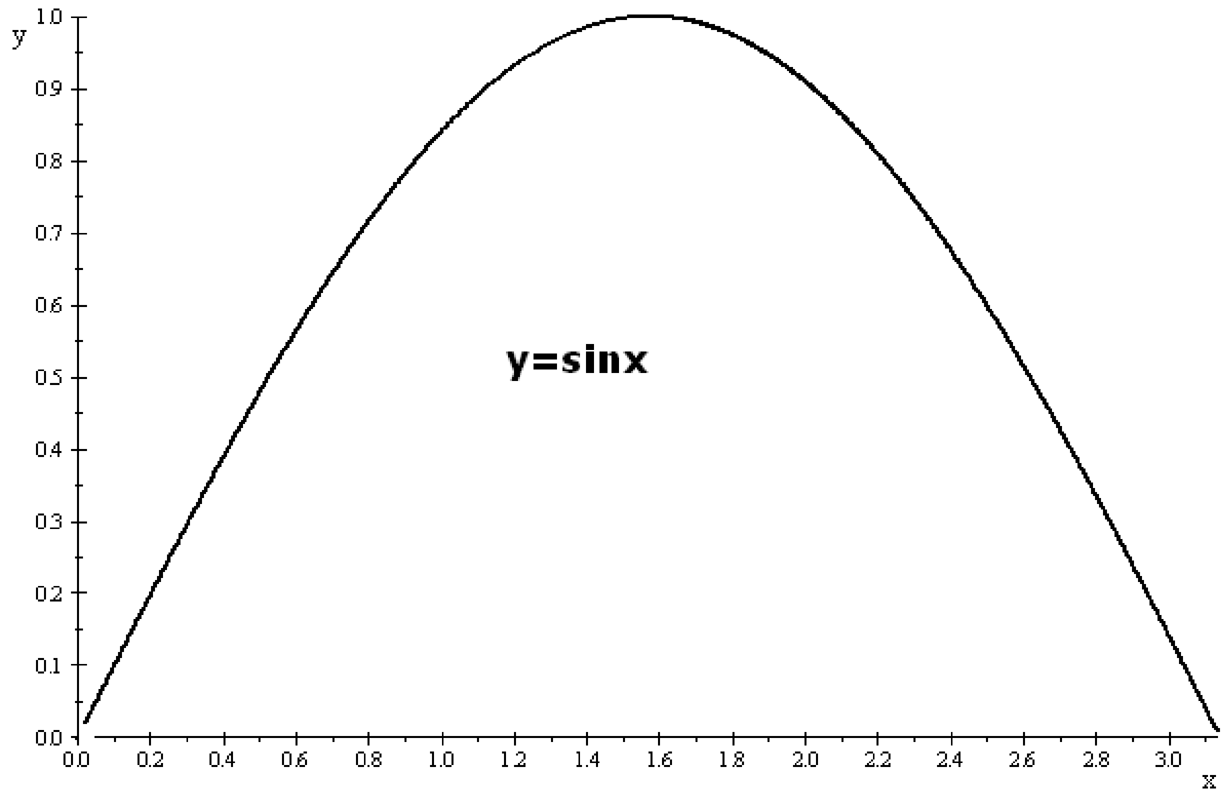
$\mathbf{f}(x) = \mathbf{2}^{\sin x}\mathbf{+1}$ $\qquad$ $\mathbf{x} \in (0, \pi)$



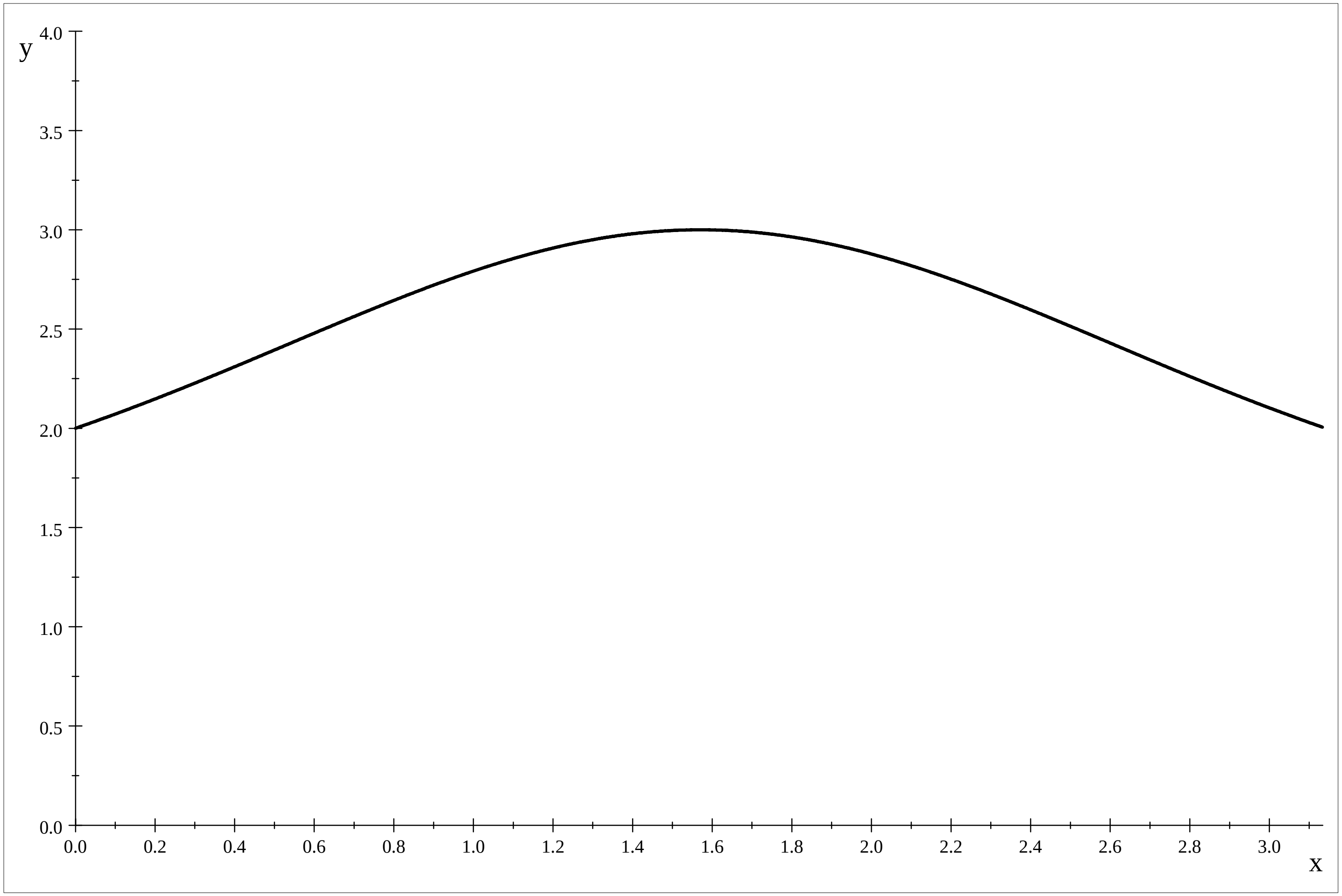
What is the range of this function?

For $x \in (0, \pi)$ $\qquad$ $y = \sin x \in (0, 1]$

For $x \in (0,\pi)$ $y = \sin x \in (0,1]$



For $x \in (0,\pi)$ $f(x) = 2^{\sin x} + 1 \in (2,3]$



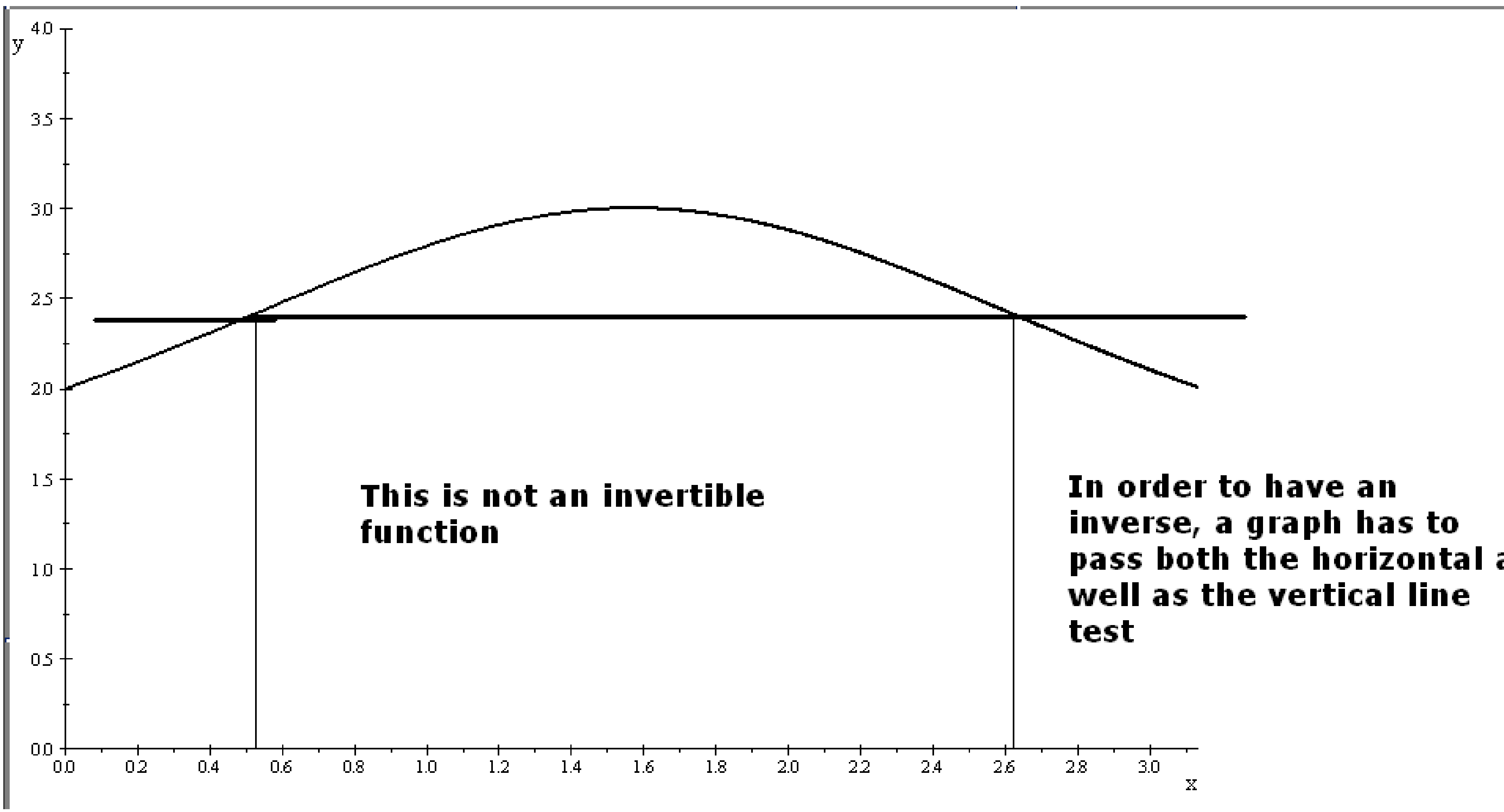
Is this function invertible?

$\mathbf{f}(x) = \mathbf{2}^{\sin x}\mathbf{+1}$

$$\mathbf{x} = \frac{\pi}{6} \in (0, \pi) \qquad \mathbf{f}\left(\frac{\pi}{6}\right) = \mathbf{2}^{\sin(\pi/6)} + \mathbf{1} = \mathbf{2}^{1/2} + \mathbf{1} = \sqrt{2} + \mathbf{1}$$

$$\mathbf{x} = \frac{5\pi}{6} \in (0, \pi) \qquad \mathbf{f}\left(\frac{5\pi}{6}\right) = \mathbf{2}^{\sin(5\pi/6)} + \mathbf{1} = \sqrt{2} + \mathbf{1}$$

Is not one-one OR not invertible



Fundamental Theorem of Calculus ?

If

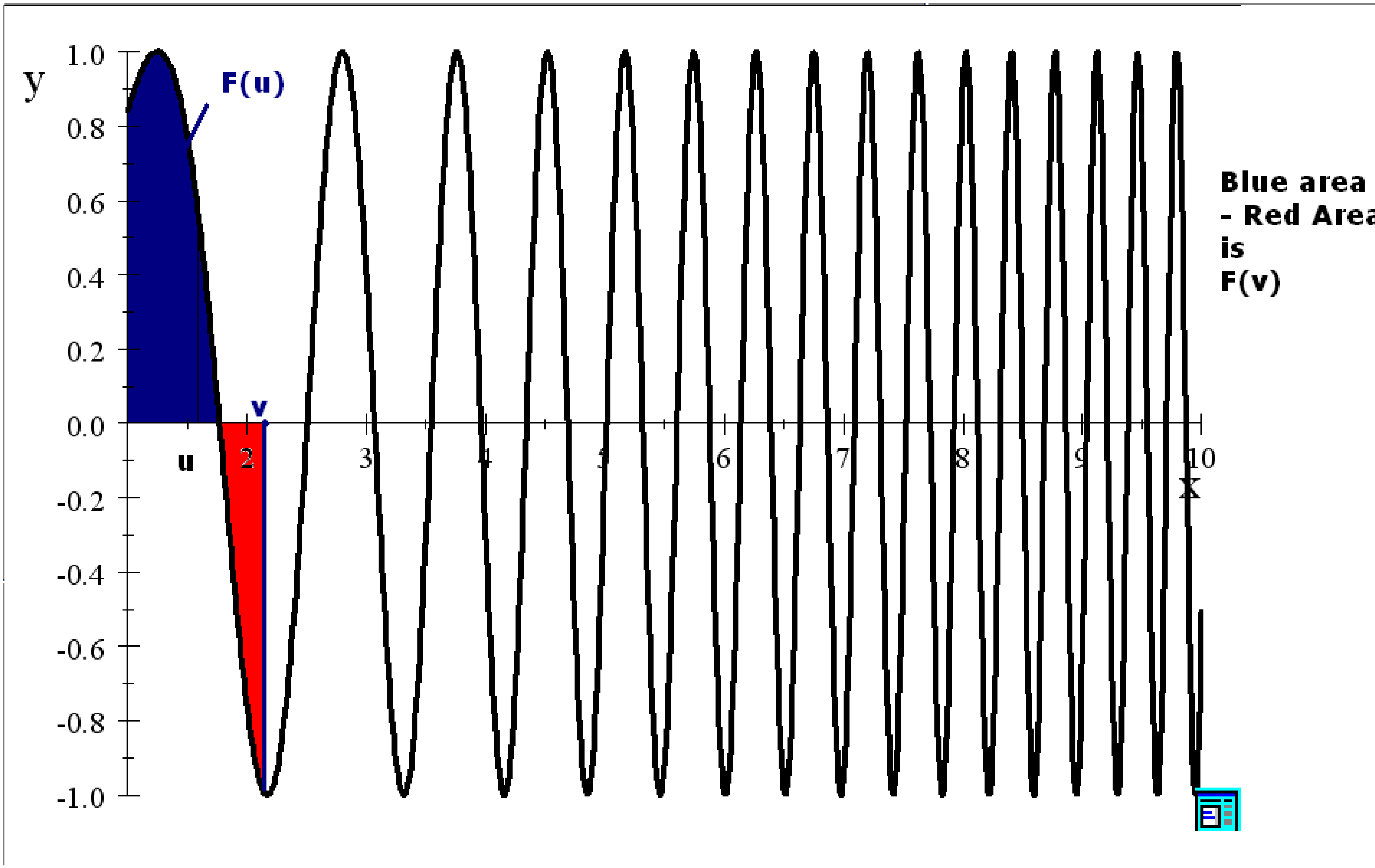
$F(x) = \int_a^x f(t)dt$ **for a continuous function** f

then

$\frac{dF}{dx} = \mathbf{f}(x)$

Example:

$F(x) = \int_1^x \sin(t^2)dt$ $\qquad$ $f(t) = \sin t^2$

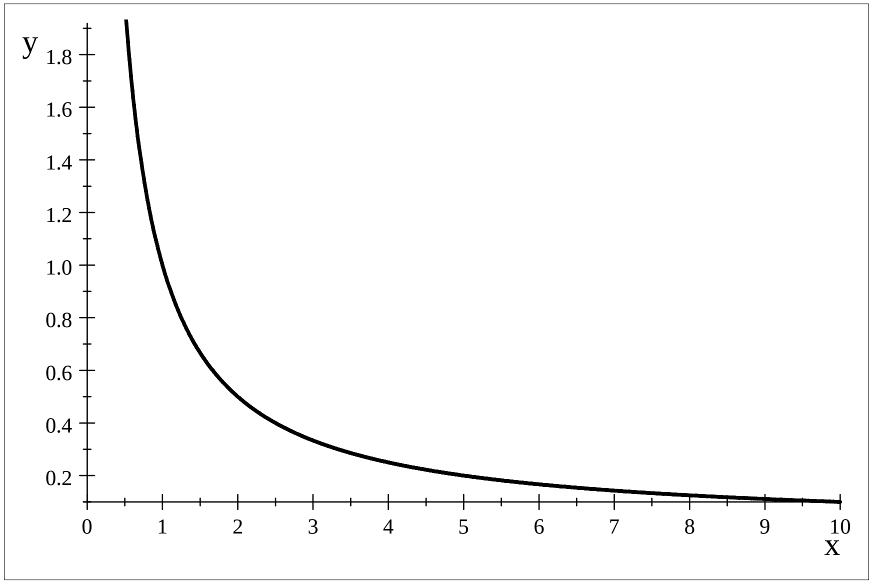

y
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
F(u)
v
u
2
3
4
5
6
7
8
9
10
x
Blue area
- Red Area
is
F(v)

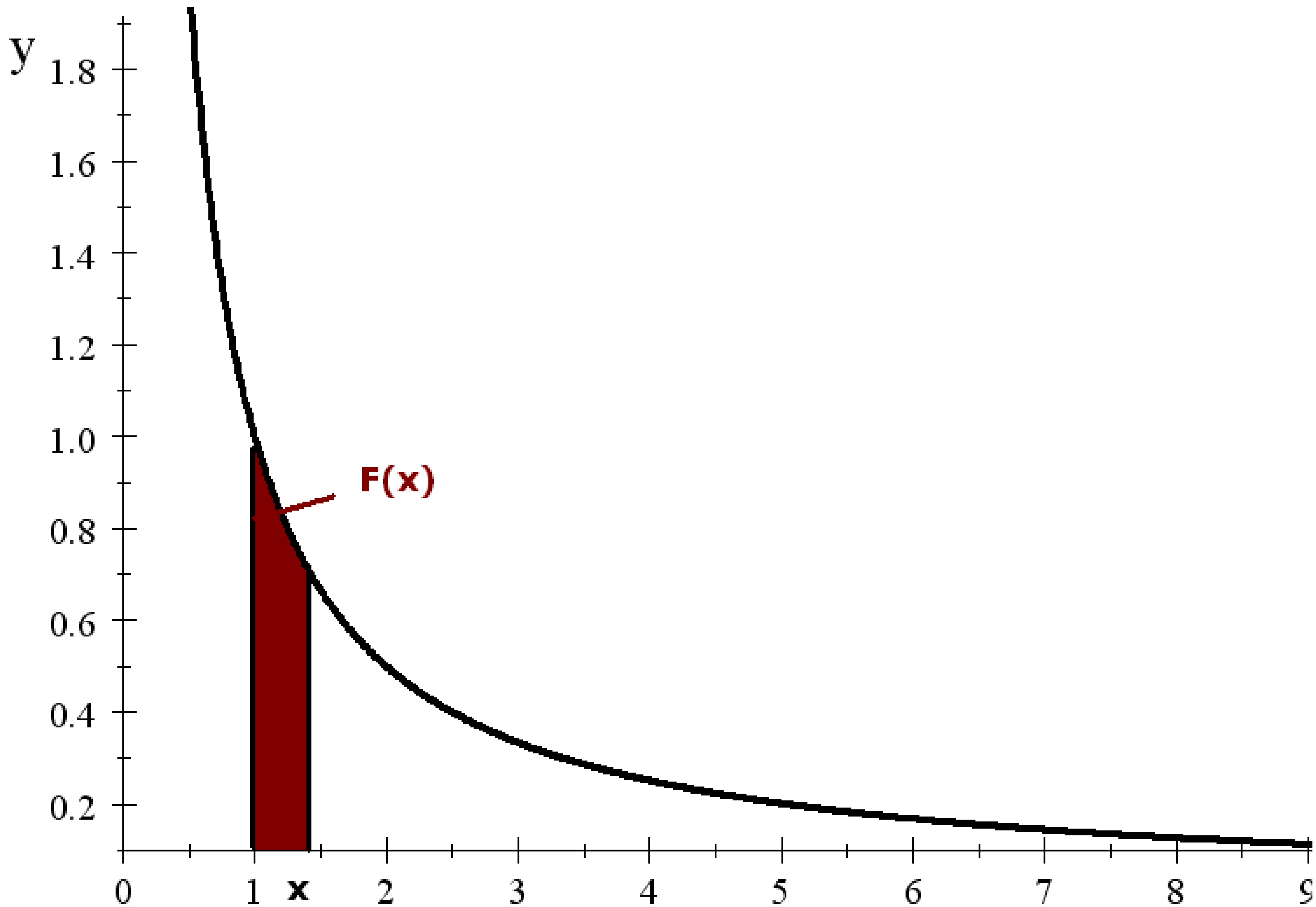
$$\frac{dF}{dx} = \sin \mathbf{x}^2$$

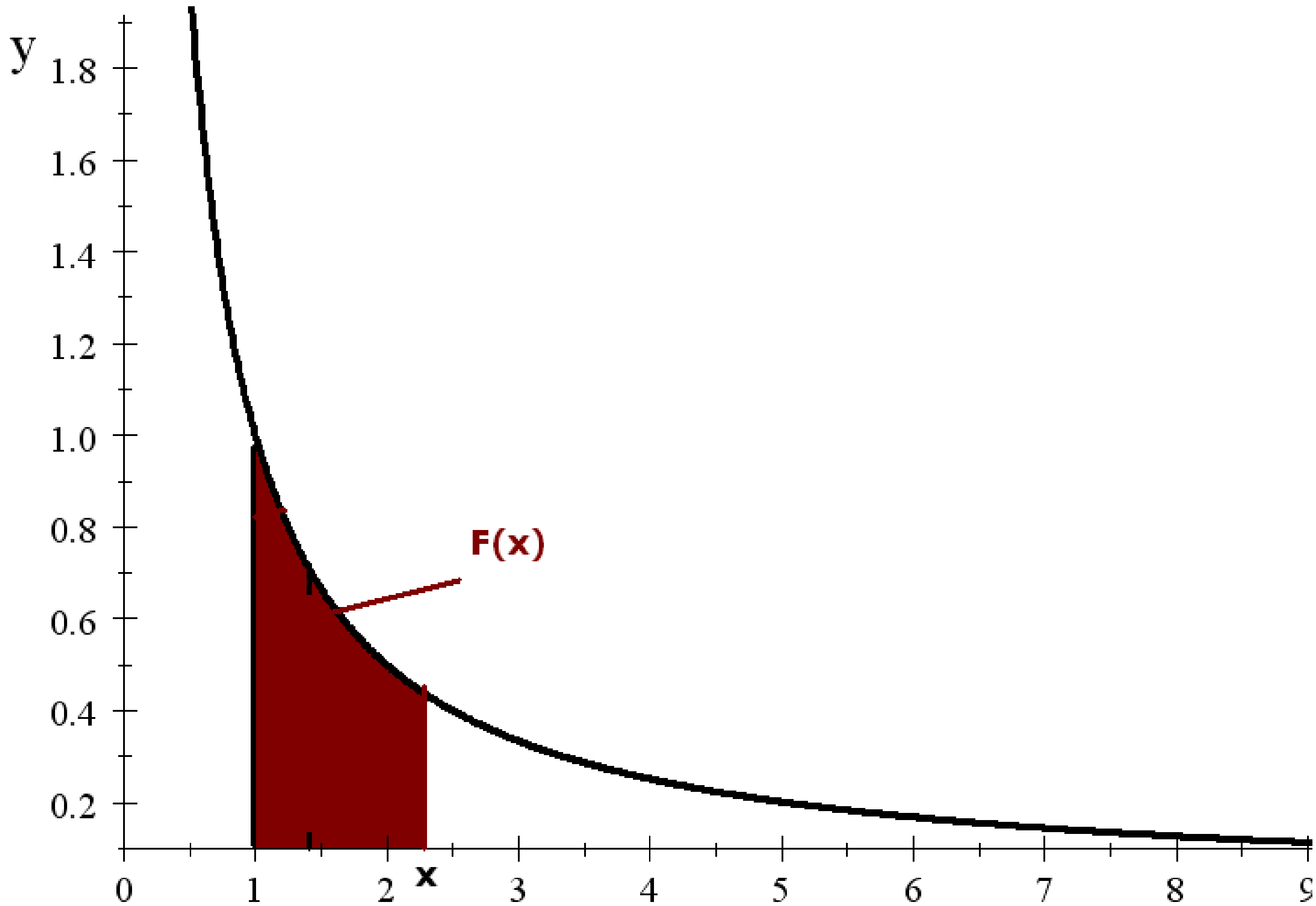
Example 2:

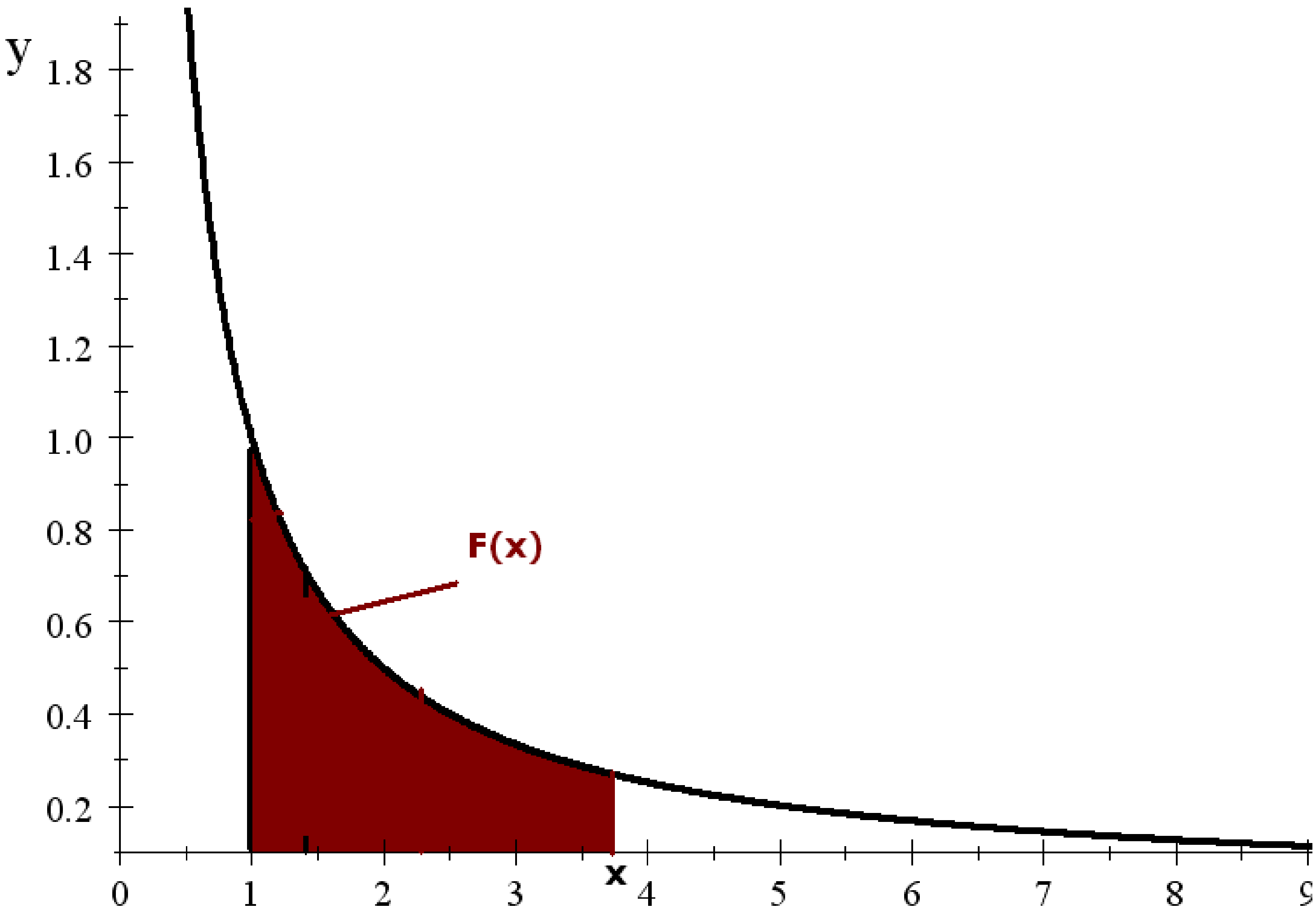
Define

$$\mathbf{F}(x) = \int_1^x \frac{1}{t}\,\mathbf{dt} \qquad \mathbf{x} > \mathbf{0}$$

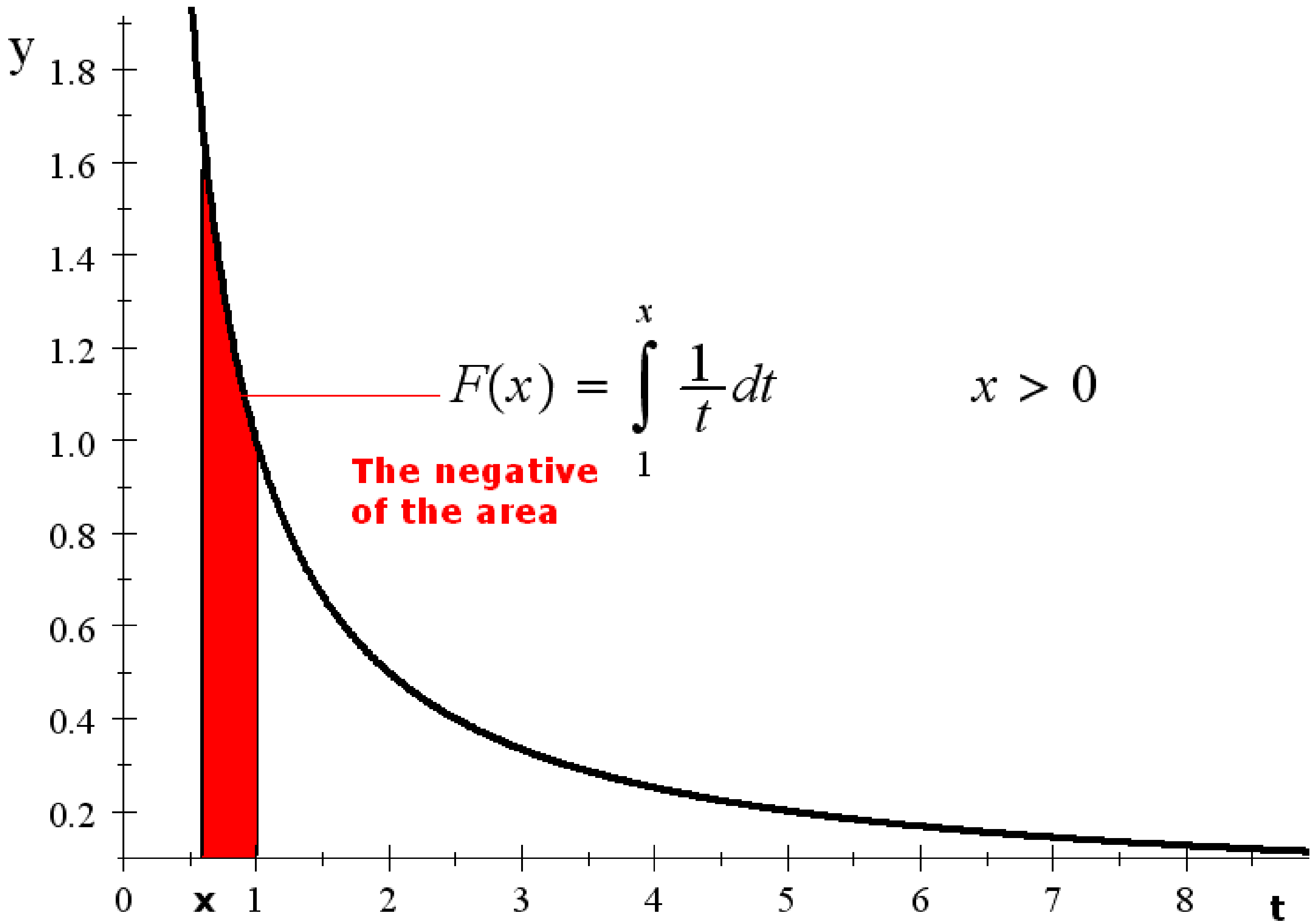



y
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
F(x)
0
1
x
2
3
4
5
6
7
8
9


y
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
F(x)
0
1
2
x
3
4
5
6
7
8
9



y
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
F(x)
x
0
1
2
3
4
5
6
7
8


y
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
x
1
2
3
4
5
6
7
8
t
$F(x) = \int_1^x \frac{1}{t} dt \qquad x > 0$
The negative
of the area

Define

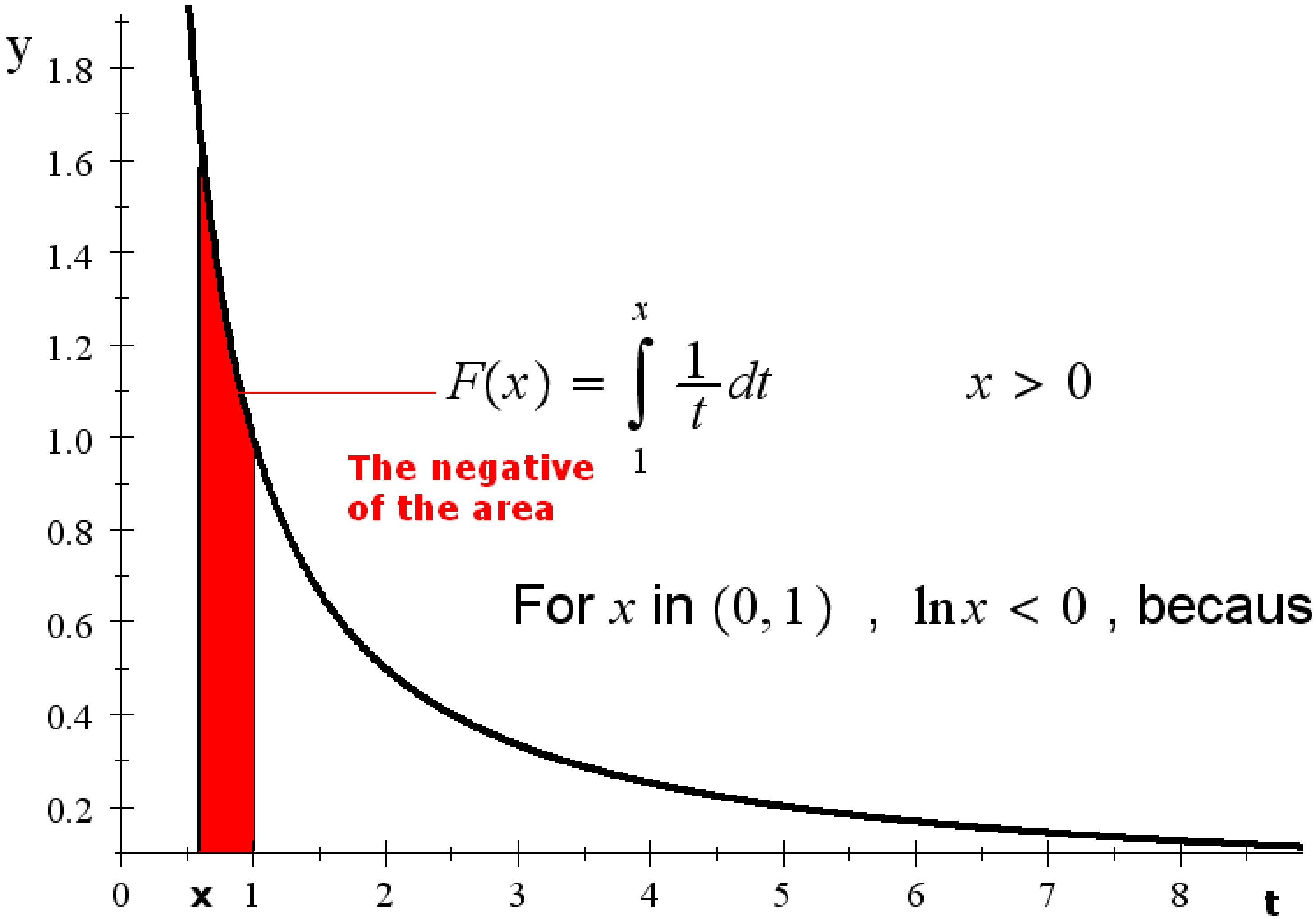
$$\ln x = F(x) = \int_{1}^{\infty} \frac{1}{t}\, dt \quad , \quad x > 0$$

$\ln x$ is the natural log of x

$$\ln \mathbf{1} = \int_{1}^{1} \frac{1}{t}\, \mathbf{dt} = \mathbf{0}$$

For x **in** $(1, \infty)$, $\ln x > 0$

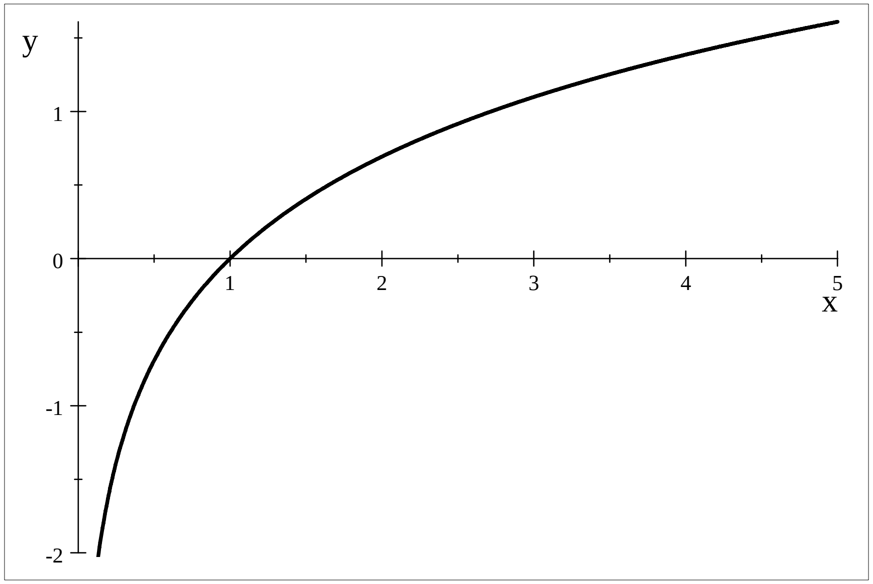
For x **in** $(0,1)$, $\ln x < 0$, **because** $\int_{1}^{x} \frac{1}{t}\, dt = -\int_{x}^{1} \frac{1}{t}\, dt$



$$F(x) = \int_1^x \frac{1}{t}\,dt \qquad x > 0$$

For x in $(0,1)$, $\ln x < 0$, because $\int_1^x \frac{1}{t}\,dt = -\int_x^1$

$\ln x$



The above is a graph of $y = \ln x$

$$\ln \mathbf{x} = \int_1^x \frac{1}{t}\,\mathbf{dt}$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

The Fundamental Theorem of Calculus

In general, for a differentiable function u **of** x

$$\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$$

Example 1:

Find $\frac{dy}{dx}$ **if** $y = \ln(1 + x^2)$

$$\frac{dy}{dx} = \frac{1}{1 + x^2}(2x) \qquad \boxed{u = 1 + x^2}$$

$$\frac{dy}{dx} = \frac{2x}{1 + x^2}$$

Example 2:

Find $\frac{dy}{dx}$ **if** $y = \ln|\tan x|$

$$\frac{dy}{dx} = \frac{1}{\tan x}(\sec^2 x) = \frac{\sec^2 x}{\tan x} = \frac{1}{\cos^2 x \tan x} = \frac{1}{\cos x \sin x} = \frac{2}{\sin 2x} = \mathbf{2}\csc \mathbf{2x}$$

Properties

For $M > 0 \qquad N > 0$

$$\ln(MN) = \ln \mathbf{M} + \ln \mathbf{N}$$

$$\ln\left(\frac{M}{N}\right) = \ln \mathbf{M} - \ln \mathbf{N}$$

$$\ln \mathbf{M}^p = \mathbf{p} \ln \mathbf{M}$$

Example 3:

Find $\frac{dy}{dx}$ **if** $y = \frac{(1 + 4x^2)\sqrt{1 + x^2}}{5 + x^6}$

$$\mathbf{y} = \frac{(1 + 4x^2)\sqrt{1 + x^2}}{5 + x^6}$$

$$\ln \mathbf{y} = \ln\left(\frac{(1 + 4x^2)\sqrt{1 + x^2}}{5 + x^6}\right)$$

$$\ln \mathbf{y} = \ln\left((1+4x^2)\sqrt{1+x^2}\right) - \ln(5+x^6)$$
$$\ln \mathbf{y} = \ln(1+4x^2) + \ln\left(\sqrt{1+x^2}\right) - \ln(5+x^6)$$
$$\ln \mathbf{y} = \ln(1+4x^2) + \ln(1+x^2)^{1/2} - \ln(5+x^6)$$
$$\ln y = \ln(1+4x^2) + \frac{1}{2}\ln(1+x^2) - \ln(5+x^6)$$

Differentiate wrt x

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{1+4x^2}(8x) + \frac{1}{2}\bullet\frac{1}{1+x^2}(2x) - \frac{1}{5+x^6}(6x^5)$$
$$\frac{1}{y}\frac{dy}{dx} = \frac{8x}{1+4x^2} + \frac{x}{1+x^2} - \frac{6x^5}{5+x^6}$$
$$\frac{dy}{dx} = \left(\frac{8x}{1+4x^2} + \frac{x}{1+x^2} - \frac{6x^5}{5+x^6}\right)\mathbf{y}$$
$$\frac{dy}{dx} = \left(\frac{8x}{1+4x^2} + \frac{x}{1+x^2} - \frac{6x^5}{5+x^6}\right)\left(\frac{(1+4x^2)\sqrt{1+x^2}}{5+x^6}\right)$$

Using the antiderivative on

$$\frac{d}{du}(\ln|u|) = \frac{1}{u}$$
$$\boxed{\int \frac{1}{u}\,du = \ln|u| + C}$$

Example:

$$\int_0^{\pi} \frac{\cos x}{3+\sin x}\,\mathbf{dx}$$

$$\mathbf{3} + \sin \mathbf{x} = \mathbf{u} \rightarrow \cos \mathbf{x} \mathbf{dx} = \mathbf{du}$$

$$\mathbf{x} = \mathbf{0} \rightarrow \mathbf{u} = \mathbf{3} + \sin \mathbf{0} = \mathbf{3}$$

$$\mathbf{x} = \boldsymbol{\pi} \rightarrow \mathbf{u} = \mathbf{3} + \sin \boldsymbol{\pi} = \mathbf{3}$$

$$\int_0^\pi \frac{\cos x}{3 + \sin x} \mathbf{dx} = \mathbf{0}$$

$$\frac{\cos x}{3 + \sin x}$$