18. Food is placed in a freezer. After t hours, the temperature of the food is changing at a rate of $R = 10e^{-0.2t}$ where R is in degrees F/hr. How much has the temperature dropped in the first two hours?

If the temperature is y degrees F after t hours

$R = 10e^{-0.2t}$ $\qquad R = \dfrac{dy}{dt}$

$$\frac{dy}{dt} = 10e^{-0.2t}$$

$$\int \frac{dy}{dt} dt = \int 10e^{-0.2t} dt$$

$$y = -50e^{-0.2t} + C$$

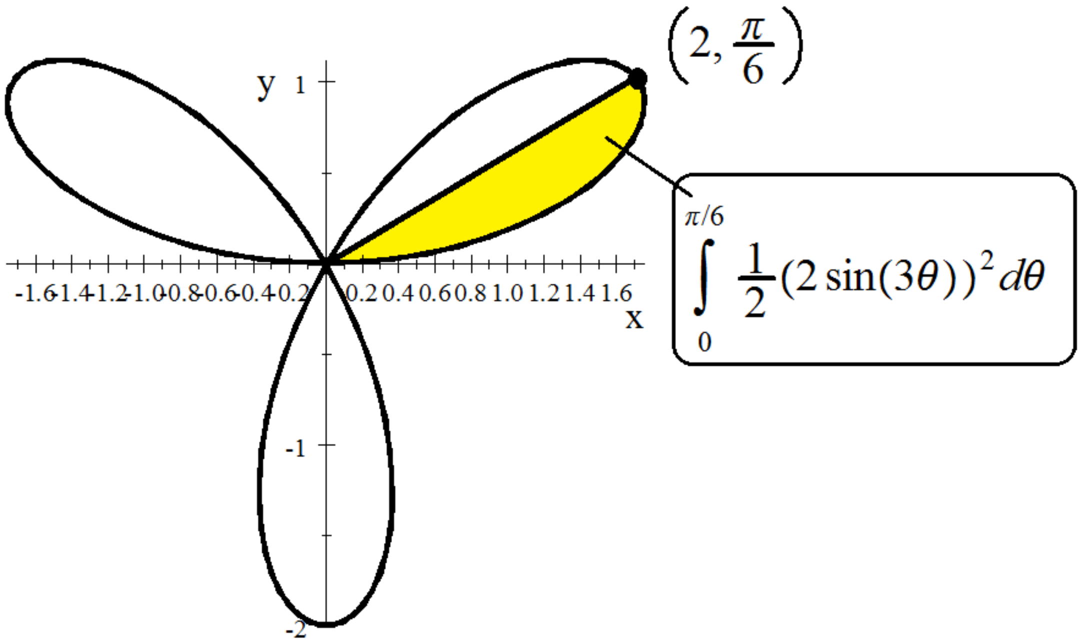
$$y(2) - y(0)$$

$$(-50e^{-0.2(2)} + C) - (-50e^{-0.2(0)} + C)$$

$$50e^{-0.2(0)} - 50e^{-0.2(2)} = 16.483997698218034963^\circ F$$

56.

To find the area inside of $r = 2\sin(3\theta)$



$$6\int_0^{\pi/6} \frac{1}{2}(2\sin(3\theta))^2 d\theta = 12\int_0^{\pi/6} \sin^2(3\theta)d\theta$$

$$\sin^2(3\theta) = \frac{1-\cos(6\theta)}{2}$$

$$12\int_0^{\pi/6} \frac{1-\cos(6\theta)}{2}d\theta$$

$$=6\int_0^{\pi/6} (1-\cos(6\theta))d\theta$$

$$=6\left(\theta - \frac{\sin 6\theta}{6}\right)\Bigg|_0^{\pi/6}$$

$$=6\left(\frac{\pi}{6} - \frac{\sin\pi}{6}\right)$$

$$=\pi$$

52. Find the Taylor Series for $e^{\frac{x}{2}}$ centered at $a = 2$ and use the Ratio Test to show that this series converges for all x.

To expand $e^{x/2}$ in the powers of $(x-2)$

$$f(x) = f(2) + \frac{(x-2)}{1!}f^{(1)}(2) + \frac{(x-2)^2}{2!}f^{(2)}(2) + \ldots + \frac{(x-2)^{n-1}}{(n-1)!}f^{(n-1)}(2) + \ldots$$

$f^{(2)}(2)$ the k^{th} derivative of f at a

$$f(x) = e^{x/2} \to f^{(1)}(x) = \frac{1}{2}e^{x/2}$$

$$f^{(2)}(x) = \frac{1}{2^2}e^{x/2}$$

$$f^{(3)}(x) = \frac{1}{2^3}e^{x/2}$$

...

$$f^{(n-1)}(x) = \frac{1}{2^{n-1}}e^{x/2}$$

$$f^{(n-1)}(2) = \frac{1}{2^{n-1}}e^{2/2} = \frac{1}{2^{n-1}}e$$

$$e + \frac{(x-2)}{1! \times 2}e + \frac{(x-2)^2}{2! \times 2^2}e + \ldots + \frac{(x-2)^{n-1}}{(n-1)! \times 2^{n-1}}e + \ldots$$

Converges for all values of x because

$$\left| \frac{\dfrac{(x-2)^n}{n! \times 2^n}}{\dfrac{(x-2)^{n-1}}{(n-1)! \times 2^{n-1}}} \right| = \left| \frac{x-2}{2n} \right| \to 0 \text{ as } n \to \infty$$

#45

To determine whether

$$\sum_{n=2}^{\infty} \frac{3n}{n!2^{n+1}}$$ converges

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{\frac{3(n+1)}{(n+1)!2^{n+1+1}}}{\frac{3n}{n!2^{n+1}}} = \frac{3(n+1)}{3n} \cdot \frac{n!2^{n+1}}{(n+1)!2^{n+1+1}} = \frac{3n+3}{3n(n+1)2}$$

$$\frac{3n+3}{(n+1)6n} = \frac{3n+3}{6n^2+6n} \rightarrow 0 < 1$$ therefore converges

b)

$$\sum_{n=1}^{\infty} \frac{2n+1}{4n^2+n-1}$$

compare with $\sum_{n=1}^{\infty} \frac{1}{n}$ DIVERGENT

$$\frac{\frac{2n+1}{4n^2+n-1}}{\frac{1}{n}} = \frac{2n^2+n}{4n^2+n-1} \rightarrow \frac{2}{4}$$

$\sum_{n=1}^{\infty} \frac{1}{n}$ DIVERGENT→ $\sum_{n=1}^{\infty} \frac{2n+1}{4n^2+n-1}$ DIVERGENT c)

To test $\sum_{n=2}^{\infty} \frac{3}{n\sqrt[3]{\ln n}}$

For $x \geq 2$

$f(x) = \frac{3}{x\ \sqrt[3]{\ln x}} > 0$

$f(x) \searrow$

$f(x)$ is continuous on $[2, \infty)$

$$\int_2^{\infty} \frac{3}{x\ \sqrt[3]{\ln x}} dx$$

$$\int \frac{3}{x\ \sqrt[3]{\ln x}} dx \qquad u = \ln x \rightarrow du = \frac{1}{x} dx$$

$$\int \frac{3}{x\ \sqrt[3]{\ln x}} dx = 3 \int \frac{du}{u^{1/3}} = \frac{3u^{2/3}}{(2/3)} = \frac{9}{2} u^{2/3} = \frac{9}{2} (\ln x)^{2/3}$$

$$\int_2^{\infty} \frac{3}{x\ \sqrt[3]{\ln x}} dx = \lim_{x \rightarrow \infty} \frac{9}{2} (\ln x)^{2/3} - \frac{9}{2} (\ln 2)^{2/3} = \infty$$

The integral diverges, therefore the original series diverges

#3

$$\int \frac{5x+3}{x^3-2x^2-3x}dx$$

$$\frac{5x+3}{x^3-2x^2-3x} = \frac{5x+3}{x(x-3)(x+1)}$$

$$\frac{5x+3}{x(x-3)(x+1)} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{x+1}$$

$$\frac{5x+3}{x(x-3)(x+1)} = \frac{A(x-3)(x+1)+Bx(x+1)+Cx(x-3)}{x(x-3)(x+1)}$$

$$\frac{5x+3}{x(x-3)(x+1)} = \frac{A(x^2-2x-3)+B(x^2+x)+C(x^2-3x)}{x(x-3)(x+1)}$$

$$\frac{5x+3}{x(x-3)(x+1)} = \frac{Ax^2-2Ax-3A+Bx^2+Bx+Cx^2-3Cx}{x(x-3)(x+1)}$$

$$\frac{0x^2+5x+3}{x(x-3)(x+1)} = \frac{(A+B+C)x^2+(-2A+B-3C)x-3A}{x(x-3)(x+1)}$$

$A+B+C=0$

$-2A+B-3C=5$

$-3A=3 \to A=-1$

$-1+B+C=0 \to B+C=1$

$-2(-1)+B-3C=5 \to B-3C=3$

$B+C=1$

$B-3C=3$

subtract the bottom from top

$4C=-2 \to C=-\frac{1}{2}$

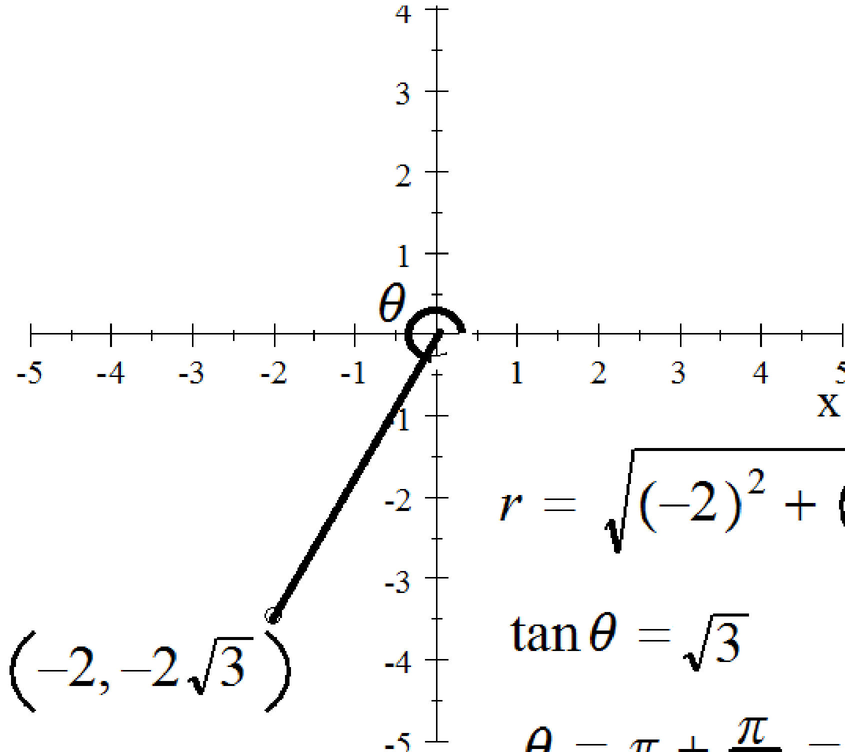
$B-\frac{1}{2}=1 \to B=\frac{3}{2}$

$$\frac{5x+3}{x(x-3)(x+1)} = \frac{3}{2(x-3)} - \frac{1}{2(x+1)} - \frac{1}{x}$$

$$\int \frac{5x+3}{x(x-3)(x+1)}dx = \int \frac{3}{2(x-3)}dx - \frac{1}{2}\int \frac{dx}{x+1} - \int \frac{dx}{x}$$

$$\int \frac{5x+3}{x(x-3)(x+1)}dx = \frac{3}{2}\ln|x-3| - \frac{1}{2}\ln|x+1| - \ln|x| + C$$

#53 b) Write the polar coordinates (r,θ) of $\left(-2,-2\sqrt{3}\right)$ such that $r>0$ and $0\le\theta<2\pi$



$$r=\sqrt{(-2)^2+\left(-2\sqrt{3}\right)^2}=4$$

$$\tan\theta=\sqrt{3}$$

$$\theta=\pi+\frac{\pi}{3}=\frac{4\pi}{3}$$

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+2)!}$$

To test for convergence

$$\left| \frac{\frac{(-1)^{n+1} x^{2n+2}}{(2n+4)!}}{\frac{(-1)^n x^{2n}}{(2n+2)!}} \right| = \left| \frac{x^2}{(2n+4)(2n+3)} \right| \to 0 < 1$$

converges for all values of x

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+2)!} = \frac{1}{2!} - \frac{x^2}{4!} + \frac{x^4}{6!} - \frac{x^6}{8!} + \ldots$$

$$f(0) = \frac{1}{2}$$

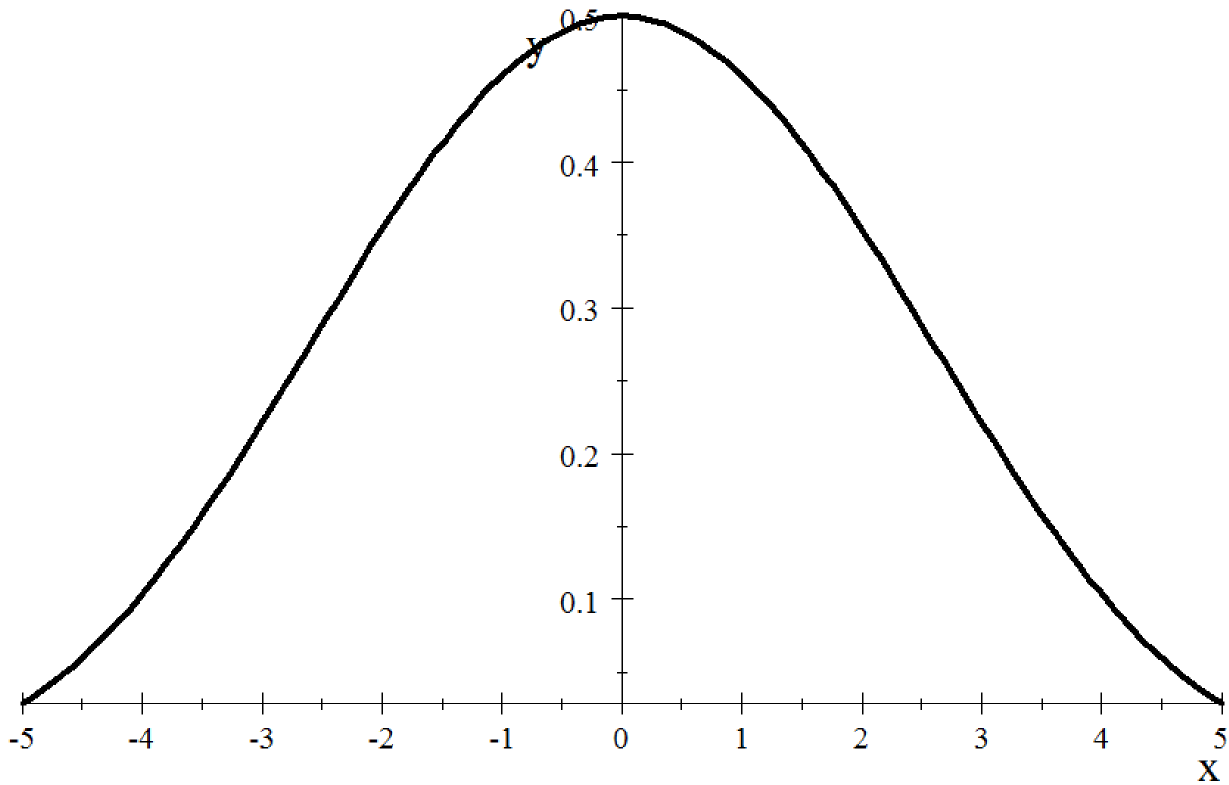
$$f'(x) = -\frac{2x}{4!} + \frac{4x^3}{6!} - \frac{6x^5}{8!} + \ldots$$

$f'(0) = 0$ $\quad$ 0 is a critical number

$$f''(x) = -\frac{2}{4!} + \frac{4 \cdot 3x^2}{6!} - \frac{6 \cdot 5x^4}{8!} + \ldots$$

$$f''(0) = -\frac{1}{12}$$

MAX at $x = 0$


y
0.5
0.4
0.3
0.2
0.1
-5
-4
-3
-2
-1
0
1
2
3
4
5
x