21. Suppose that $N(t)$, the number of bacteria is a culture at time t is given by

$N(t) = 25 + te^{-t/20}$, where N is measured in millions of bacteria and t is in hours.

(a) At what time during the interval $[0,100]$ is the number of bacteria the smallest? What is the minimum number?

(b) At what time during the interval $[0,100]$ is the number of bacteria the largest? What is the maximum number?

(c) At what time during the interval $[0,100]$ is the rate of change of the number of bacteria a minimum?

$N(t) = 25 + te^{-t/20}$

a) when is the number of bacteria smallest?

$N'(t) = 0 + (1)e^{-t/20} + t(e^{-t/20})' \qquad (e^u)' = e^u u'$

$N'(t) = e^{-t/20} + te^{-t/20}\left(-\frac{1}{20}\right)$

$N'(t) = e^{-t/20} - \frac{t}{20}e^{-t/20}$

$N'(t) = \left(1 - \frac{t}{20}\right)e^{-t/20}$

$\left(1 - \frac{t}{20}\right)e^{-t/20} = 0 \to 1 - \frac{t}{20} = 0 \to t = 20$ hours

$t < 20$

$\left(1 - \frac{t}{20}\right)e^{-t/20} > 0 \qquad N(t) \nearrow$

$t > 20$

$\left(1 - \frac{t}{20}\right)e^{-t/20} < 0 \qquad N(t) \searrow$

$\nearrow \qquad \searrow$ local MAX

had to find the Min value on $[0,100]$

$N(0) = 25 + 0e^{-0/20} = 25$

$N(20) = 25 + 20e^{-20/20} = 32.357588823428846432$

$N(100) = 25 + 100e^{-100/20} = 25.67379469990854671$

min at $t = 0$, the min value is 25

b)

max at $t = 20$ max value is 32.358

c)

Rate of change is $N'(t) = \left(1 - \frac{t}{20}\right)e^{-t/20}$

to find, when is $N'(t)$ MIN on $[0, 100]$

$$(N'(t))' = N''(t) = \left(0 - \frac{1}{20}\right)e^{-t/20} + \left(1 - \frac{t}{20}\right)(e^{-t/20})'$$

$$N''(t) = \left(0 - \frac{1}{20}\right)e^{-t/20} + \left(1 - \frac{t}{20}\right)(e^{-t/20})\left(-\frac{1}{20}\right)$$

$$N''(t) = -\frac{1}{20}e^{-t/20} - \frac{1}{20}\left(1 - \frac{t}{20}\right)e^{-t/20}$$

$$N''(t) = -\frac{1}{20}e^{-t/20} - \frac{1}{20}\left(1 - \frac{t}{20}\right)e^{-t/20}$$

$$N''(t) = \left[-1 - \left(1 - \frac{t}{20}\right)\right]\frac{1}{20}e^{-t/20}$$

$$N''(t) = \left[-1 - 1 + \frac{t}{20}\right]\frac{1}{20}e^{-t/20}$$

$$N''(t) = \left[-2 + \frac{t}{20}\right]\frac{1}{20}e^{-t/20}$$

$N''(t) = 0$, $\frac{1}{20}e^{-t/20} \neq 0$, when $-2 + \frac{t}{20} = 0 \rightarrow t = 40$ hours

on $[0, 100]$

$$N'(t) = \left(1 - \frac{t}{20}\right)e^{-t/20}$$

$$N'(0) = \left(1 - \frac{0}{20}\right)e^{-0/20} = 1$$

$$N'(40) = \left(1 - \frac{40}{20}\right)e^{-40/20} = -0.135\,335\,283\,236\,612\,691\,89$$

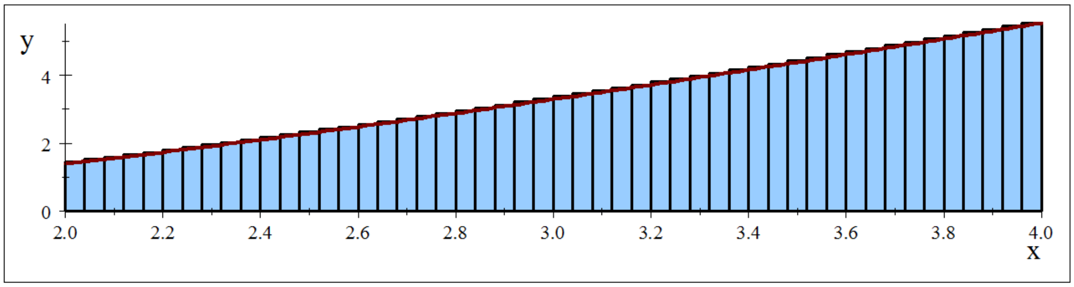
$$N'(100) = \left(1 - \frac{100}{20}\right)e^{-100/20} = -0.026\,951\,787\,996\,341\,868\,387$$

smallest, when, $t = 100$

smallest rate of change when $t = 40$

#24

$$\int_2^4 x \ln x dx$$



we have 50 rectangles

Each rectangle is $\frac{4-2}{50} = 0.04$ units wide

the k^{th} rectangle has area $.04((2+.04k)\ln(2+.04k))$

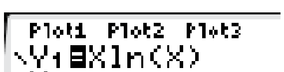
we have 50 rectangles

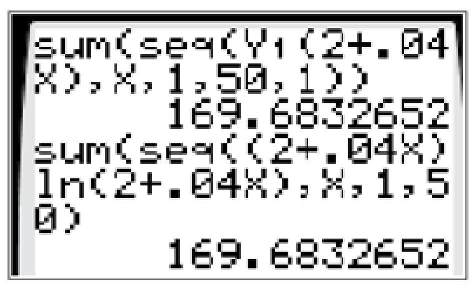
Each rectangle is $\frac{4-2}{50} = 0.04$ units wide

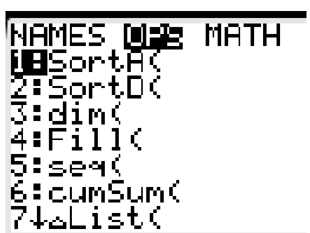
the k^{th} rectangle has area $.04((2+.04k)\ln(2+.04k))$

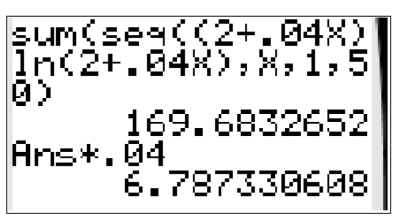
the sum the areas of all the 50 rectangles is

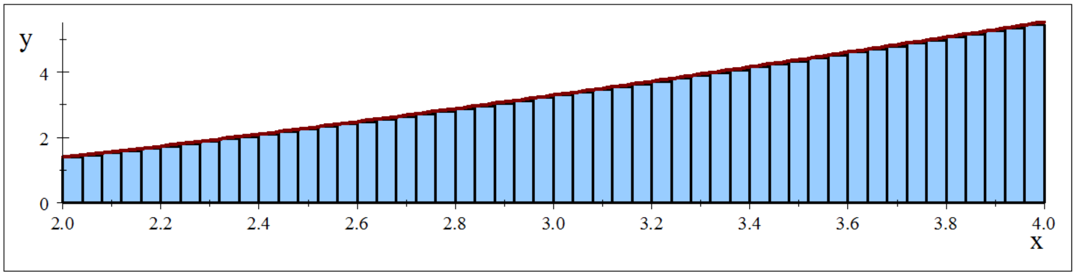
$$\sum_{k=1}^{50} .04((2+.04k)\ln(2+.04k)) = 6.787330608463883707$$



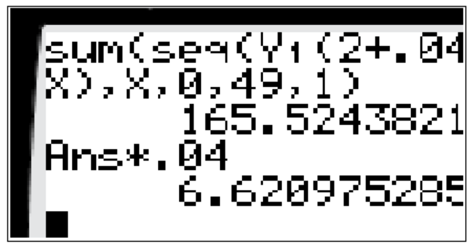








$$\sum_{k=0}^{49} .04((2+.04k)\ln(2+.04k)) = 6.6209752851294964965$$



$$y = (2x^3 + 5x^2 - 6x - 4)^5$$
$$y' = 5(2x^3 + 5x^2 - 6x - 4)^4(2x^3 + 5x^2 - 6x - 4)'$$
$$y' = 5(2x^3 + 5x^2 - 6x - 4)^4(6x^2 + 10x - 6)$$

$$y = \tan^3(5x)$$
$$y = (\tan(5x))^3$$
$$y' = 3(\tan(5x))^2(\tan(5x))'$$
$$y' = 3(\tan(5x))^2(\sec^2(5x))(5x)'$$
$$y' = 3(\tan(5x))^2(\sec^2(5x))5$$

c)

$$y = \sqrt{\frac{x}{3x+1}}$$

$$(\sqrt{u})' = \frac{1}{2\sqrt{u}}u'$$

$$y' = \frac{1}{2\sqrt{\frac{x}{3x+1}}}\left(\frac{x}{3x+1}\right)'$$

$$y' = \frac{\sqrt{3x+1}}{2\sqrt{x}}\left(\frac{1(3x+1)-x(3)}{(3x+1)^2}\right)$$

23. The acceleration of gravity near the surface of Mars is 3.72m/sec^2. If a rock is blasted straight up from the surface with an initial velocity of 93 m/sec, how high does it go?

$$\frac{d^2s}{dt^2} = -3.72$$

$$v = \frac{ds}{dt}$$

$$\frac{dv}{dt} = -3.72$$

The rocket is highest, when $v = 0$

To calculate, when $(t = ?)$ is $v = 0$

$$\frac{dv}{dt} = -3.72$$

$$\int \frac{dv}{dt}dt = \int -3.72dt$$

$$v = -3.72t + C$$

when $t = 0,\ v = 93 \quad m/\sec$

$$93 = -3.72(0) + C$$

$$C = 93$$

$v = -3.72t + 93$

$v = 0$ when $-3.72t + 93 = 0 \rightarrow t = \dfrac{93}{3.72}$ sec

The max height is the value of s when $t = \dfrac{93}{3.72}$ sec

$v = -3.72t + 93$

$\dfrac{ds}{dt} = -3.72t + 93$

$s = -3.72\left(\dfrac{t^2}{2}\right) + 93t + D$

using that $s = 0 \quad t = 0$

$0 = -3.72\left(\dfrac{0^2}{2}\right) + 93(0) + D$

$D = 0$

$s = -3.72\left(\dfrac{t^2}{2}\right) + 93t$

max s when $t = \dfrac{93}{3.72}$ sec

$$-3.72\left(\frac{(93/3.72)^2}{2}\right) + 93(93/3.72) = 1162.5 \text{ meters}$$

Let $f(x) = x^2 + \dfrac{a}{x}$.

(a) What value of a makes f have a local minimum at $x = 2$?

(b) What value of a makes f have a point of inflection at $x = 1$?

(c) Is there any value of a for which f can have a local maximum? Why or why not?

26.

$f(x) = x^2 + \frac{a}{x}$

a) Local MIN at $x = \alpha$

$f'(\alpha) = 0 \qquad f''(\alpha) < 0$

$f'(x) = 2x - \frac{a}{x^2}$

If have a local min at $x = 2$

must have

$2(2) - \frac{a}{2^2} = 0 \rightarrow a = 16$

$f''(x) = 2 + \frac{2a}{x^3}$

$x = 2$

$2 + \frac{2a}{2^3} = \frac{8 + a}{4} > 0$ when $a = 16$

therefore $a = 16$ will give a local min at $x = 2$

13 (b)

$\int x^n dx = \frac{x^{n+1}}{n+1} + C \qquad n \neq -1$

$\int \frac{2x^2 + 5x}{x^3} dx = \int \left(\frac{2x^2}{x^3} + \frac{5x}{x^3} \right) dx = \int \left(\frac{2}{x} + \frac{5}{x^2} \right) dx$

$\int \left(\frac{2}{x} + \frac{5}{x^2} \right) dx = 2 \int \frac{dx}{x} + 5 \int \frac{1}{x^2} dx$

$\int \frac{dx}{x} = \ln|x|$

$\int \frac{1}{x^2} dx = \int x^{-2} dx = \frac{x^{-2+1}}{-2+1} = -\frac{1}{x}$

$\int \left(\frac{2}{x} + \frac{5}{x^2} \right) dx = 2 \ln|x| - \frac{5}{x} + C$

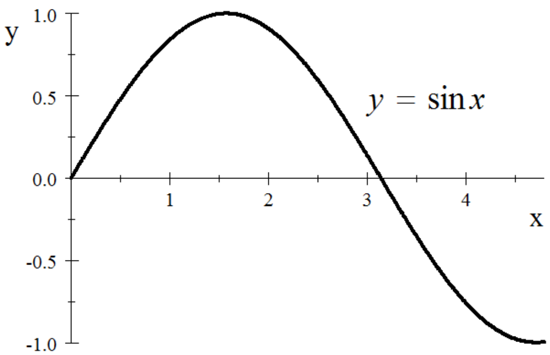
$$\int \frac{\sin(2x)}{\sin x}dx = \int \frac{2\sin x\cos x}{\sin x}dx = 2\int \cos x dx = 2\sin x + C$$

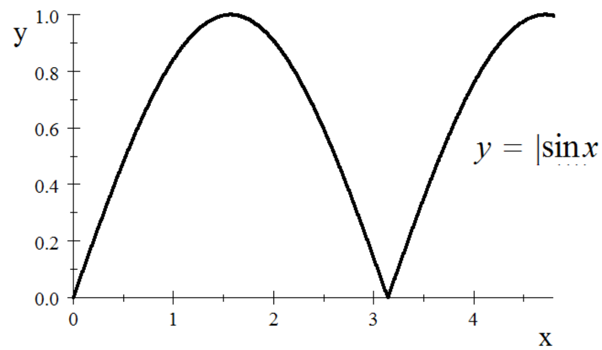
$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\sin(2x) = \sin(x+x) = \sin x\cos x + \cos x\sin x = 2\sin x\cos x$$

$$\sin(2x) = 2\sin x\cos x$$

$$\int_0^{3\pi/2} |\sin x| dx$$





$$|\sin x| = \sin x \qquad [0, \pi]$$

$$|\sin x| = -\sin x \qquad \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$$

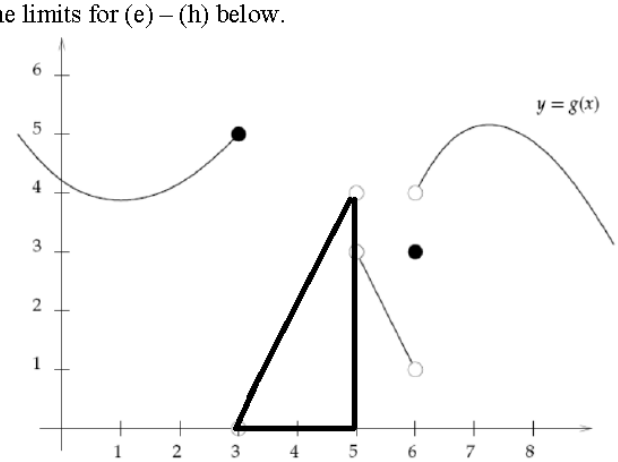
$$\int_0^{3\pi/2} |\sin x| dx = \int_0^{\pi} \sin x dx - \int_{\pi}^{3\pi/2} \sin x dx$$

$$\int_0^{\pi} \sin x dx = -\cos x|_0^{\pi} = -\cos(\pi) - (-\cos 0) = 2$$

$$\int_{\pi}^{3\pi/2} \sin x dx = -\cos x|_0^{3\pi/2} = -\cos\left(\frac{3\pi}{2}\right) - (-\cos 0) = 0 + 1 = 1$$

$$\int_0^{3\pi/2} |\sin x| dx = 3$$

Use the graph of the function g to evaluate the limits for (e) – (h) below.



(e) $\lim_{x \to 5^-} g(x)$ **4**

(f) $\lim_{h \to 0} \frac{g(4+h) - g(4)}{h}$

derivative of g at 4
slope of the tangent line at 4
slope is 4/2 or 2

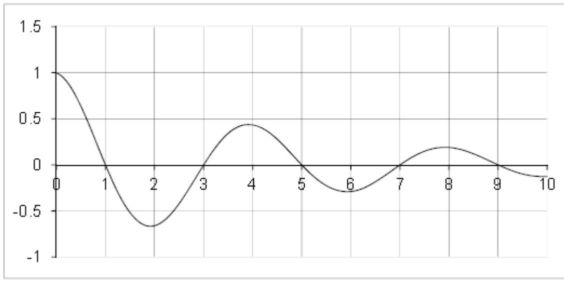
(g) $\lim_{x \to 6^+} \frac{x-5}{g(x)}$ $\frac{\textbf{6-5}}{\textbf{4*}}$

(h) $\lim_{x \to 6} g(x)$

g(x) approaches 4, as x approaches 6 from the right

DNE because the limit from the left is not equal to limit from the right

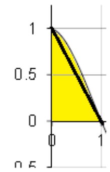
29. Let $g(x)$ be defined by $g(x)=\int_0^x f(t)\,dt$. The graph of f is shown below.



(d) Approximate the values of $g(1)$ and $g(3)$.

g(x) is the area under the graph from 0 to x

g(1)



.55 or may be .6

(c) On which subinterval(s) of $[0,10]$, if any, is the graph of $g(x)$ concave up? Justify your answer. **g is concave up when g'(x)=f''(x)>0 (2,4) U (6,8)**