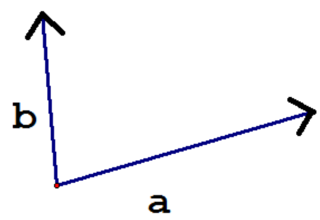
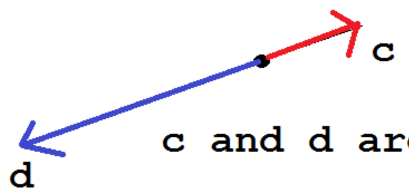


a and b are linearly independent
or a and b are not linearly dependent



c and d are linearly dependent

1.

Is the set $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \end{bmatrix} \right\}$ linearly dependent?

NO

they can not expressed as a multiple of each other

2.

Is the set $\left\{ \begin{bmatrix} 2 \\ 5 \end{bmatrix}, \begin{bmatrix} \sqrt{2} \\ \sqrt{5} \end{bmatrix} \right\}$ linearly dependent?

NO

3.

Is the set $\left\{ \begin{bmatrix} 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 4 \\ 10 \end{bmatrix} \right\}$ linearly dependent?

YES because

$$\begin{bmatrix} 4 \\ 10 \end{bmatrix} = 2\begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

4.

Is the set $\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 10 \end{bmatrix} \right\}$ linearly dependent?

YES

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0\begin{bmatrix} 4 \\ 10 \end{bmatrix}$$

5.

Is the set $\left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix} \right\}$ linearly dependent?

YES

$$\begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

6.

$$\left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} \right\}$$ are linearly dependent

because

$$\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} = -1 \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + 2 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Formally

If

$$\alpha_1 \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} + \alpha_2 \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + \alpha_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

with at least one of the α_s non zero

say $\alpha_3 \neq 0$

Then I can solve $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ in terms of $\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

i.e.

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

for some real numbers x_1 and x_2

i.e.

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$ should be an augmented matrix

of a consistent system

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$, row echelon form: $$\begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

We noted that subsetset of R^n with a non zero vector in it, should be linearly dependent

If we have two non zero vectors in R^2 , then they are linearly dependent if and only if they are collinear

If we have three non zero vectors in R^3 , then they are linearly dependent if and only if they are coplanar

sec 1.7 #14 If the set is linearly dependent

$$\begin{bmatrix} 1 & -3 & 2 \\ -2 & 7 & 1 \\ -4 & 6 & h \end{bmatrix}$$ should an aug matrix of consistent system

$$\begin{bmatrix} 1 & -3 & 2 \\ -2 & 7 & 1 \\ -4 & 6 & h \end{bmatrix} \begin{matrix} \rightarrow \\ r_2 + 2r_1 \\ r_3 + 4r_1 \end{matrix} \begin{bmatrix} 1 & -3 & 2 \\ 0 & 1 & 5 \\ 0 & -6 & h+8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & 2 \\ 0 & 1 & 5 \\ 0 & -6 & h+8 \end{bmatrix} \begin{matrix} \rightarrow \\ \square \\ r_3 + 6r_2 \end{matrix} \begin{bmatrix} 1 & -3 & 2 \\ 0 & 1 & 5 \\ 0 & 0 & h+38 \end{bmatrix}$$

For consistency

$h + 38 = 0$

or $h = -38$

$$\begin{bmatrix} 1 & -3 & 2 \\ -2 & 7 & 1 \\ -4 & 6 & -38 \end{bmatrix}$$, row echelon form: $$\begin{bmatrix} 1 & 0 & 17 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & 2 \\ 0 & 1 & 5 \\ 0 & -6 & h+8 \end{bmatrix} \underset{r_3 + 6r_2}{\rightarrow} \begin{bmatrix} 1 & -3 & 2 \\ 0 & 1 & 5 \\ 0 & 0 & h+38 \end{bmatrix}$$

For consistency

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$$\begin{bmatrix} 1 & -3 & 2 \\ -2 & 7 & 1 \\ -4 & 6 & -38 \end{bmatrix}$$, row echelon form: $$\begin{bmatrix} 1 & 0 & 17 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

#20 linearly dependent because the set formed contains the zero vector

Crow's question

We can not span R^4 with less than 4 vectors

TRUE

Another of Crow's comments

Another of Crow's comments

Can we have a linearly independent set of three vectors in R^4?

YES

Can we have a linearly independent subset of more than 4 vectors in R^4?
NO

#28
S=$\{v_1, v_2, v_3, v_4\}$ is linearly independent in R^4
$\Rightarrow$
R=$\{v_1, v_2, v_3\}$ is linearly independent in R^4
YES

Annette
If a set is linearly independent,
there is no vector in the set that is the linear combination of the others
"If there is no vectors in S that is a linearly combination of the others, then it can not be true for R either"

For Avril
#28 continued

if R=$\{v_1, v_2, v_3\}$ is linearly dependent
one of these say v_3 can be written as a linear combination of v_1, v_2
$v_3 = \alpha_1 v_1 + \alpha_2 v_2$
which makes
$v_3 = \alpha_1 v_1 + \alpha_2 v_2 + 0v_4$

i.e. v_3 is a linear combination of v_1, v_2, v_4
i.e.
S=$\{v_1, v_2, v_3, v_4\}$ is linearly dependent $\rightarrow\leftarrow$
because $\{v_1, v_2, v_3, v_4\}$ is linearly independent

Avril's another question

Can a subset of a linearly dependent set of vectors in R^n be linearly independent?

YES

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$$ is linearly dependent

but

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$$ is linearly independent

Linear Transformations

Example 1

Consider

$$f : R^2 \to R^2$$

$$f\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} 5x_1 \\ 3x_2 \end{bmatrix}$$

$$f\left(\begin{bmatrix} 2 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} 10 \\ 3 \end{bmatrix}$$

Linear Transformations
Example 1

Consider

$$f : R^2 \to R^2$$

$$f\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 5x_1 \\ 3x_2 \end{bmatrix}$$

$$f\left(\begin{bmatrix} 2 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 10 \\ 3 \end{bmatrix}$$

informally
A linear transformation $T : R^n \to R^m$ is a map that preserves linear combinations

$$f : R^2 \to R^2$$

$$f\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 5x_1 \\ 3x_2 \end{bmatrix}$$

is a linear transformation

$$f : R^2 \to R^2$$

$$f\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 5x_1 + 1 \\ 3x_2 - 2 \end{bmatrix}$$

is not a linear transformation

Start formally