Lesson 5:
I am taking the exercises from the third edition but even if you have the second updated edition, you should be able to follow.
work continued on the section 1.3:

16 on page 38

To write 5 vectors in the span $\left\{ \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix} \right\}$

i) $2\begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix} + 5\begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} -4 \\ 0 \\ 19 \end{bmatrix}$

ii) $3\begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix} + \left(-\frac{1}{2}\right)\begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ \frac{9}{2} \end{bmatrix}$

iii) $\sqrt{2}\begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix} + 1\begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 3\sqrt{2} - 2 \\ 0 \\ 2\sqrt{2} + 3 \end{bmatrix}$

iv) $3\begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix} + (-2)\begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 13 \\ 0 \\ 0 \end{bmatrix}$

v) $\pi\begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix} + 2\begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 3\pi - 4 \\ 0 \\ 2\pi + 6 \end{bmatrix}$

18.

We have to determine the values of h for which $\begin{bmatrix} h \\ -5 \\ -3 \end{bmatrix}$ **is in the plane**

spanned by

$$\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} \text{ and } \begin{bmatrix} -3 \\ 1 \\ 8 \end{bmatrix}.$$

i.e. we have to determine the values of h for which $\begin{bmatrix} h \\ -5 \\ -3 \end{bmatrix}$ **is a linear combination of**

$$\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} \text{ and } \begin{bmatrix} -3 \\ 1 \\ 8 \end{bmatrix}.$$

i.e. (...make sure that you understand why...)
for which values of h does the augmented matrix

$\begin{bmatrix} 1 & -3 & h \\ 0 & 1 & -5 \\ -2 & 8 & -3 \end{bmatrix}$ **correspond to a consistent system.**

$$\begin{bmatrix} 1 & -3 & h \\ 0 & 1 & -5 \\ -2 & 8 & -3 \end{bmatrix}$$

$r_3 + 2r_1$ ↓

$$\begin{bmatrix} 1 & -3 & h \\ 0 & 1 & -5 \\ 0 & 2 & -3 + 2h \end{bmatrix}$$

$r_1 + 3r_2$ ↓ $r_3 - 2r_2$

$$\begin{bmatrix} 1 & 0 & h - 15 \\ 0 & 1 & -5 \\ 0 & 0 & 7 + 2h \end{bmatrix}$$

For a consistent system, must have the last entry in the third row as 0.

$7 + 2h = 0 \to h = -\frac{7}{2}$.

26.

Given that A= $\begin{bmatrix} 2 & 0 & 6 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix}$ **, b=** $\begin{bmatrix} 10 \\ 3 \\ 3 \end{bmatrix}$

a) To check if b is in the span of the columns of A i.e. b is a linear combination of the columns of A.

enough to check if the augmented matrix

$\begin{bmatrix} 2 & 0 & 6 & 10 \\ -1 & 8 & 5 & 3 \\ 1 & -2 & 1 & 3 \end{bmatrix}$ **correspond to a consistent system.**

$\begin{bmatrix} 2 & 0 & 6 & 10 \\ -1 & 8 & 5 & 3 \\ 1 & -2 & 1 & 3 \end{bmatrix}$**, row echelon form:** $\begin{bmatrix} 1 & 0 & 3 & 5 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

shows that b can be expressed as a linear combination of the columns.

b) To show that $\begin{bmatrix} 6 \\ 5 \\ 1 \end{bmatrix}$ **is a linear combination of the columns of A.**

In this case may be done in more than one way

one way

$$\begin{bmatrix} 6 \\ 5 \\ 1 \end{bmatrix} = \mathbf{0}\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} + \mathbf{0}\begin{bmatrix} 0 \\ 8 \\ -2 \end{bmatrix} + \mathbf{1}\begin{bmatrix} 6 \\ 5 \\ 1 \end{bmatrix}$$

or

$$\begin{bmatrix} 6 \\ 5 \\ 1 \end{bmatrix} = \mathbf{3}\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 8 \\ -2 \end{bmatrix} + \mathbf{0}\begin{bmatrix} 6 \\ 5 \\ 1 \end{bmatrix}$$

Section 1.4:

Recall the definition on page 41 in the third edition and on page 39 in the second edition.

The following examples illustrates the definition again:

1. $\begin{bmatrix} 2 & -1 \\ 1 & 4 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ 7 \end{bmatrix} = 5\begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix} + 7\begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 33 \\ -3 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 2 & 4 \\ -2 & 5 & -3 \end{bmatrix}\begin{bmatrix} 7 \\ 11 \\ -9 \end{bmatrix} = 7\begin{bmatrix} 1 \\ -2 \end{bmatrix} + 11\begin{bmatrix} 2 \\ 5 \end{bmatrix} - 9\begin{bmatrix} 4 \\ -3 \end{bmatrix} = \begin{bmatrix} -7 \\ 68 \end{bmatrix}$

Note that we can represent a system of linear equation in the following different ways.

$2x_1 - x_2 + 5x_3 = 1$
$x_1 - x_2 + 2x_3 = 4$
$2x_1 - 3x_2 + x_3 = 9$

may be written as an augmented matrix

$$\begin{bmatrix} 2 & -1 & 5 & 1 \\ 1 & -1 & 2 & 4 \\ 2 & -3 & 1 & 9 \end{bmatrix}$$

or as a vector equation

$$x_1\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} + x_2\begin{bmatrix} -1 \\ -1 \\ -3 \end{bmatrix} + x_3\begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 9 \end{bmatrix}$$

or as a matrix equation

$$\begin{bmatrix} 2 & -1 & 5 \\ 1 & -1 & 2 \\ 2 & -3 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 9 \end{bmatrix}$$ **or Ax=b**

with $A=\begin{bmatrix} 2 & -1 & 5 \\ 1 & -1 & 2 \\ 2 & -3 & 1 \end{bmatrix}$, $x=\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $b=\begin{bmatrix} 1 \\ 4 \\ 9 \end{bmatrix}$

Very important:

Read pages 41-43 to get used to all these different ways of expressing the same system.