Lesson 21:

In this section, we shall look at the procedure to transform the coordinate vectors relative to a non standard basis to a coordinate vector relative to another non standard basis.

Let us look at the procedure though an example.

Example.

We know that $\left\{\begin{bmatrix}1\\0\\0\end{bmatrix},\begin{bmatrix}0\\1\\0\end{bmatrix},\begin{bmatrix}0\\0\\1\end{bmatrix}\right\}$ is a basis of R^3 called the standard basis.

Take $\begin{bmatrix}3\\5\\7\end{bmatrix} \in R^3$

$$\begin{bmatrix}3\\5\\7\end{bmatrix}= 3\begin{bmatrix}1\\0\\0\end{bmatrix}+5\begin{bmatrix}0\\1\\0\end{bmatrix}+7\begin{bmatrix}0\\0\\1\end{bmatrix}$$

call 3,5,7 the coordiantes of the vector $\begin{bmatrix}3\\5\\7\end{bmatrix}$ with respect to the standard basis.

May express by $\begin{bmatrix}3\\5\\7\end{bmatrix}_{\text{standard basis}} = \begin{bmatrix}3\\5\\7\end{bmatrix}$

Know B=$\left\{\begin{bmatrix}1\\1\\1\end{bmatrix},\begin{bmatrix}1\\-1\\1\end{bmatrix},\begin{bmatrix}1\\1\\-1\end{bmatrix}\right\}$ is another basis of R^3.

To find the coordinates of $\begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix}$ relative to B,

look at $\begin{bmatrix} 1 & 1 & 1 & 3 \\ 1 & -1 & 1 & 5 \\ 1 & 1 & -1 & 7 \end{bmatrix}$, row echelon form: $\begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$

6,-1,-2 are the coordiantes of $[x] = \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix}$ relative to B, denoted

by $\begin{bmatrix} 6 \\ -1 \\ -2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix}_B = [x]_B$

May note $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 6 \\ -1 \\ -2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix}$

recall the notations

$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} = P_B, \begin{bmatrix} 6 \\ -1 \\ -2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix}_B$

$P_B[x]_B = [x]$

shall extend this to start 4.7 tomorrow.

Let C= $\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \right\}$ be another basis of R^3

Let us make sure that this is a basis of R^3

$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$, row echelon form: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Note that $P_C = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$

Say we need $[x]_C$

Know that $P_C[x]_C = [x]_{\text{standard}} = P_B[x]_B$

Note

$P_C[x]_C = P_B[x]_B$

Therefore

$\boxed{[x]_C = P_C^{-1} P_B [x]_B \text{ and } [x]_B = P_B^{-1} P_C [x]_C}$

the change of basis formulas

$P_C^{-1} P_B$ is called the change of coordinates matrix from B to C, denoted by $P_{C \leftarrow B}$

$P_B^{-1} P_C$ is called the change of coordinates matrix from C to B, denoted by $P_{B \leftarrow C}$

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8. on page 277 in the third edition

Given that $B = \left\{ \begin{bmatrix} -1 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ -5 \end{bmatrix} \right\}$ and $C = \left\{ \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$ two bases of R^2. We have to find the change of coordinates matrix from B to C and the change of coordinates matrix from C to B.

$P_B = \begin{bmatrix} -1 & 1 \\ 8 & -5 \end{bmatrix}$, $P_C = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$

$P_{C \leftarrow B} = P_B^{-1} P_C = \begin{bmatrix} -1 & 1 \\ 8 & -5 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$

$P_{B \leftarrow C} = P_C^{-1} P_B = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}^{-1} \begin{bmatrix} -1 & 1 \\ 8 & -5 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix}$

10. on page 277 in the third edition

Given that B= $\left\{\begin{bmatrix} 7 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix}\right\}$ and C= $\left\{\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \end{bmatrix}\right\}$ two bases of R^2. We have to find the change of coordinates matrix from B to C and the change of coordinates matrix from C to B.

$P_B = \begin{bmatrix} 7 & 2 \\ -2 & -1 \end{bmatrix}, P_C = \begin{bmatrix} 4 & 5 \\ 1 & 2 \end{bmatrix}$

$P_{C \hookrightarrow B} = P_B^{-1} P_C = \begin{bmatrix} 7 & 2 \\ -2 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 4 & 5 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ -5 & -8 \end{bmatrix}$

$P_{B \hookrightarrow C} = P_C^{-1} P_B = \begin{bmatrix} 4 & 5 \\ 1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 7 & 2 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} 8 & 3 \\ -5 & -2 \end{bmatrix}$

Please finish section 4.7 and post the difficulties in the discussion area.