A lesson for October 24

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The following lesson corresponds to the chapter 10 of the text book.

First Part of Our Study is

The distribution of the sample means.

First we are going to take up the the cases in which the population is at least 20 times larger than the sample.

For example: If we take a simple random sample of size 30 from the employees of a large corporation which has 1100 employees,
$\frac{1100}{30} = 36.66666667$**, the population is 36.67 times more than the sample size (meets the at least 20 times larger criterion.)**

On the other hand if we take a simple random sample of 50 students from a small college with 700 students
$\frac{700}{50} = 14$ **, the at least 20 times criterion is not met.**

When the population is at least 20 times larger than the sample, we have

If x (the population) has mean μ **and standard deviation** σ
then $\overline{x}$ **mean of simple random samples of size n, has mean** μ **and st dev** $\frac{\sigma}{\sqrt{n}}$

If x is normal, $\overline{x}$ **is also normal.**

Let us consider the following example for an illustration of the above result:

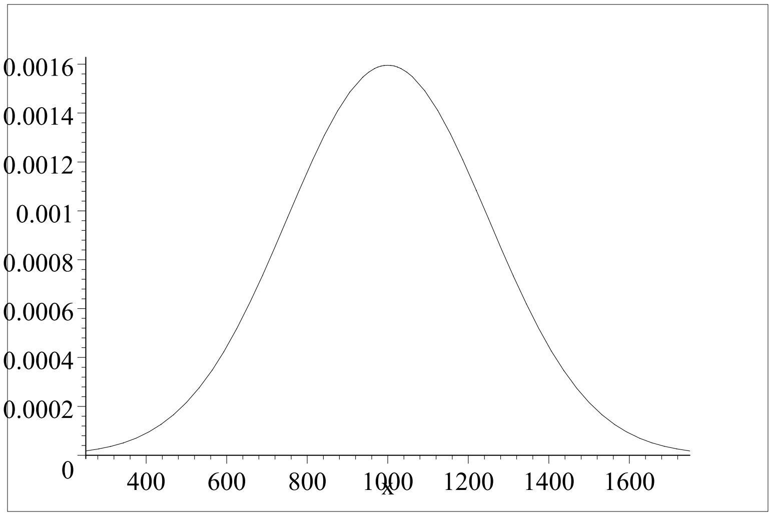
Example:
Given that the life of a certain make of light bulb shows a normal distribution with mean 1000 hours and st dev σ **of 250 hours.**

$\mu =$ **1000**
$\sigma =$ **250**

$b(x) = \frac{1}{250\sqrt{2\pi}} e^{-\left((x-1000)^2/(2\times 250^2)\right)}$ **(you may ignore this expression, used only to sketch the graph)**

The distribution of x is given by

$b(x)$



If we look at the distribution of the average life of 4-pack of bulbs from the above population,

that is simple random samples of size 4 from the above population, the mean of those $\overline{x}$ has

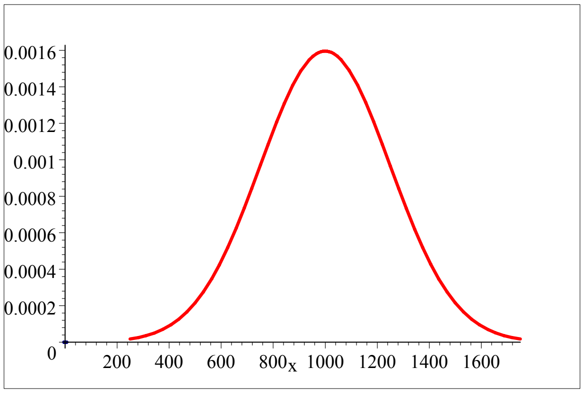
mean 1000 hours

st dev is $\frac{250}{\sqrt{4}} = 125$ **hours**

and the distribution is normal.

$a(x) = \frac{1}{125\sqrt{2\pi}} e^{-\left((x-1000)^2/(2\times 125^2)\right)}$ **(you may ignore this expression, used only to sketch the graph)**

The following picture shows that the distribution of $\overline{x}$ has less spread than x.



red: x

blue: $\overline{x}$

Let us compute the probability that the average life of 4 randomly selected bulbs from
the above popualtion is less than 720 hours.
that is
Probability that $\overline{x} < 720$
Since $\overline{x}$ **have a normal distribution, we can transform to z as shown below**
$$z < \frac{720 - 1000}{125} = -2.24$$

$$\begin{array}{lll} & & .04 \\ & & \downarrow \\ -2.2 & \rightarrow & .0125 \end{array}$$

z<-2.24
area
0.0125

Therefore the probability that average life of randomly selected 4 bulbs will be less than
720 hours is 0.0125.

CENTRAL LIMIT THEOREM:

If x has mean μ **and standard deviation** σ**,**
then for simple random samples of size n from this population,
$\overline{x}$ **shows appromately a normal distribution if the sample size large. Many practioners treat 30 as a large enough sample size to meet this criterion.**
Again, the mean of $\overline{x}$ **is still** μ **and standard deviation is** $\frac{\sigma}{\sqrt{n}}$

Let us look at an illustration through the following example.

Suppose a population has mean $\mu = 27100$ **and standard deviation** $\sigma = 5200$ **and we take a simple random sample of size 30 from this population. We would like to find the probability that the sample mean** $\overline{x}$ **is going to be within 1000 of the population mean 27,100.**
this means $-1000 < \overline{x} - 27100 < 1000$
Since, $\overline{x}$ **has approximately a normal distribution**

Note that $z = \dfrac{\overline{x} - 27100}{\left(\dfrac{5200}{\sqrt{30}}\right)}$

Therefore

$-1000 < \overline{x} - 27100 < 1000$

$\Leftrightarrow$

$$\frac{-1000}{\left(\frac{5200}{\sqrt{30}}\right)} < \frac{\overline{x} - 27100}{\left(\frac{5200}{\sqrt{30}}\right)} < \frac{1000}{\frac{5200}{\sqrt{30}}}$$

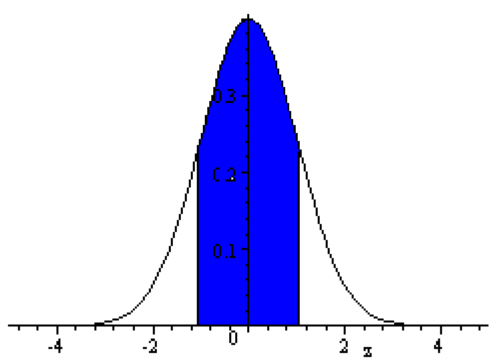
$\Leftrightarrow$

$$\frac{-1000}{\left(\frac{5200}{\sqrt{30}}\right)} < \mathbf{z} < \frac{1000}{\frac{5200}{\sqrt{30}}}$$

$\Leftrightarrow$

$\mathbf{-1.05 < z < 1.05}$

Therefore the probability of $-1000 < \overline{x} - 27100 < 1000$
is the same as the probability of $-1.05 < z < 1.05$
which is the asme as the shaded area in the two tails in the following picture.
$s(z) = \dfrac{1}{\sqrt{2\pi}} e^{-z^2/2}$ **(you may ignore this expression, used only to sketch the graph)**
s(z)



The area to the left of z=–1.05 according to the table is

	.05
	$\downarrow$
–1.0 $\rightarrow$	**.1469**

The area to the left of z=1.05 according to the table is

$$.05$$
$$\downarrow$$

1.0 $\rightarrow$ **.8531**

Therefore, the area between z=-1.05 **and z=1.05 is**
$.8531 - .1469 = 0.7062$
Therefore the probability of $-1000 < \overline{x} - 27100 < 1000$ **is 0.7062.**

Consider another illustration of this idea with the following

Example:

Suppose that the insurance claims for a particular feature, shows a mean of $\$987.00$ **and standard deviation \$781.00. If the same data holds, find the probability that the average claim on a random sample of 100 cars will be more than \$1100.00.**

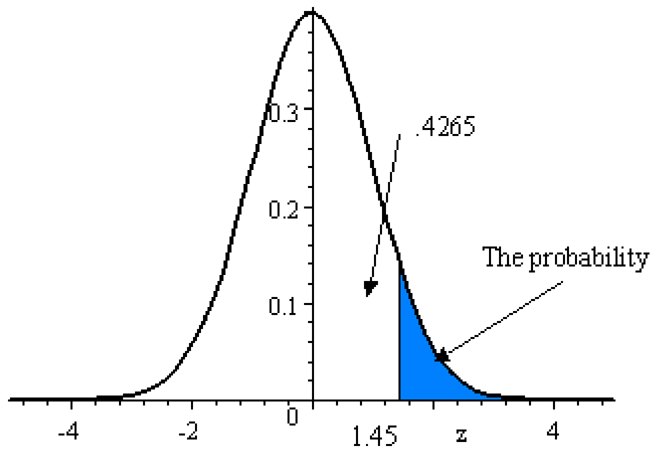
To find the
Probability that
$\overline{x} > 1100$
or
$$\rightarrow z > \frac{1100 - 987}{\left(\frac{781}{\sqrt{100}}\right)} = 1.4469$$
$\rightarrow z > 1.45$
From the z-table

$$.05$$
$$\downarrow$$

1.4 $\rightarrow$ **.9265**

this means that the area between z=0 and z=1.45 is 0.4265

But we want the area to the right of z=1.45.



The area is $0.5 - 0.4265 = 0.0735$

or the probability that the average will exceed $1100 is 0.0735.

Example 4.

The following example is taken from Introduction to Business Statistics by Ronald Weiers, fourth edition, page 290.

From the past experience, an airline has found the luggage weight for individual air travelers on their trans-Atlantic route to have a mean of 80 pounds and standard deviation 20 pounds. The plane is consistently fully booked and holds 100 passengers. The pilot insists on having extra fuel if the TOTAL luggage weight exceeds 8300 pounds. On what percentage of the flights will she end up having extra fuel.

For the weight of the luggage (x) of an individual passenger, we have $\mu = 80$**lbs and** $\sigma = 20$**lbs.**
Note that extra fuel is needed when

total $>$ **8300**
or
average $> \frac{8300}{100} = 83.0$ **(8300 is the total for 100 passengers)**
or
$\bar{x} >$ **83**
or

z $> \dfrac{83-80}{\left(\dfrac{20}{\sqrt{100}}\right)} =$ **1.5**

We have to find the area to the right of z=1.5 under the standard normal curve. From the Normal Curve Areas Table , the area to the left of z=1.5 is 0.9332 therefore the area to the right of z=1.5 is $1-.9332 = 0.0668$**.**

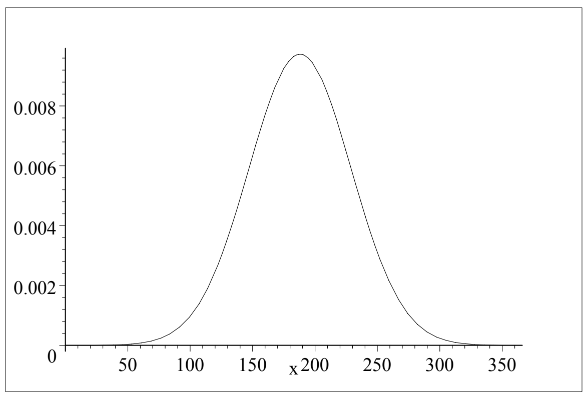
About 6.68% of the flights will need extra fuelling.

Some worked out problems from the text book:

10.8 on the page 258:

c(x) $= \dfrac{1}{41\sqrt{2\pi}}$ **e**$^{-\left((x-188)^2/(2\times 41^2)\right)}$ (You may ignore the expression.)

$c(x)$



Population

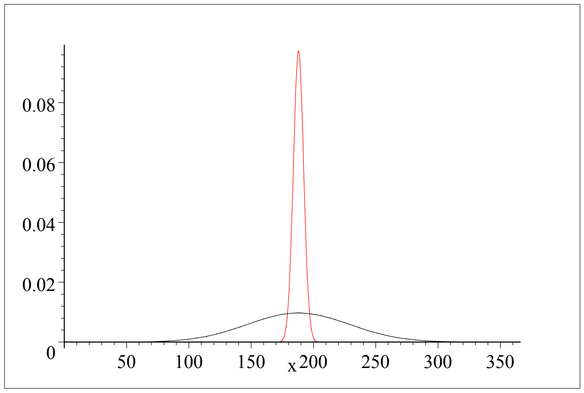
a)

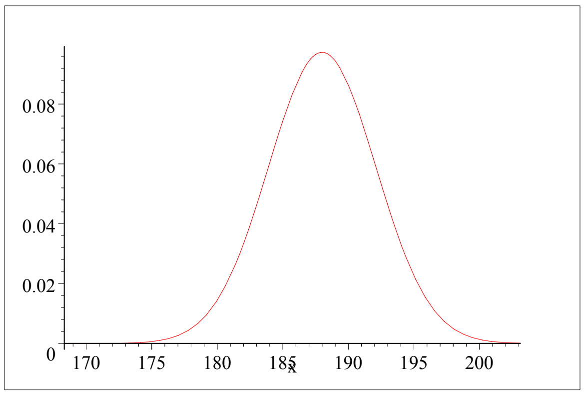
n=100, to state the distribution of $\overline{x}$

For $\overline{x}$**, the mean 188 units, st dev** $\dfrac{41}{\sqrt{100}} = 4.1$ **units**

Since the population is almost normal, the distribution of $\overline{x}$ **is also approximately normal.**

b)





mean of $\overline{x}$ is 188 units and st dev 4.1 units
Probability that
$\overline{x} <$ **180**
z $< \frac{180 - 188}{4.1} = -1.95121951$

$$\begin{array}{lcc} & & .05 \\ & & \downarrow \\ -1.9 & \rightarrow & .0256 \end{array}$$

The probability is 0.0256

Page 262:

10.12

x : $\mu = 1.6 \qquad \sigma = 1.2$

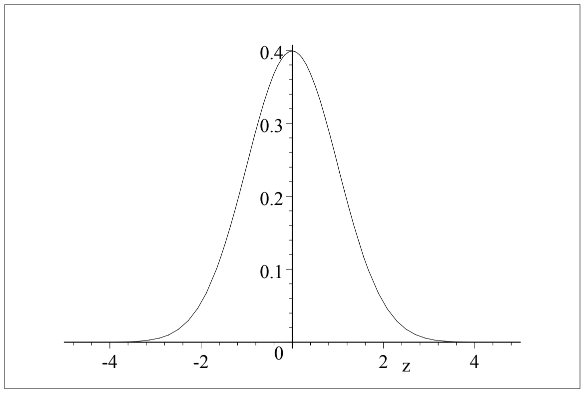
n=200,
$\overline{x}$ **has mean 1.6, st dev is** $\frac{1.2}{\sqrt{200}} = 8.48528137 \times 10^{-2} \cong 0.084853$
Probability using the Central Limit theorem:
$\overline{x}$ **is approximately normal**

$\overline{x}$ **exceeds 2**

$\overline{x} >$ **2**

z $> \frac{2-1.6}{0.084853} =$ **4. 714 034 86**

$\frac{1}{\sqrt{2\pi}} e^{-z^2/2}$



the probability is approximately 0.

$\int_{4.71403486}^{\infty} \frac{1}{\sqrt{2\pi}} \mathbf{e}^{-z^2/2} \mathbf{dz} = \mathbf{1.2142954 \times 10^{-6}}$

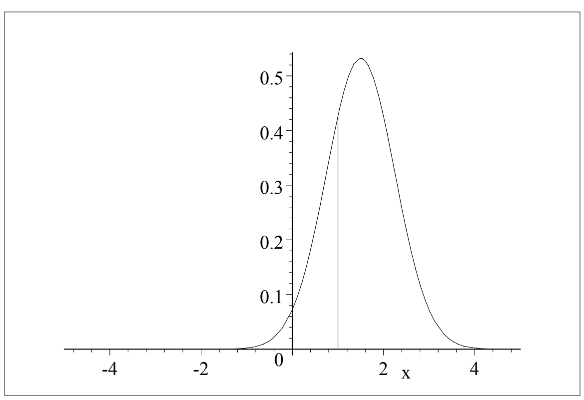
10.28 on the page 273

a)

One reason: counts are whole numbers

Another reason:
A normal distribution with mean 1.5, st dev .75

$r(x) = \frac{1}{.75\sqrt{2\pi}} e^{-\left((x-1.5)^2/\left(2\times.75^2\right)\right)}$



It looks very dangerous if the indicated proportion in the graph equals the proportion of cars with less tahn one person travelling

b)
n=700
$\overline{x}$ **has approximately a normal distribution with mean 1.5**
and standard deviation $\frac{.75}{\sqrt{700}} = 2.83473355 \times 10^{-2} \cong 0.028347$

c)
Probability

Total number of people in a random sample of 700 is more than 1075 people
that is $\overline{x} > \frac{1075}{700} = 1.53571429$

$\overline{x} > 1.53571429$

z $> \frac{1.53571429 - 1.5}{0.028347}$

z $> \mathbf{1.25989664}$

$$\begin{array}{ccc} & & \mathbf{.06} \\ & & \downarrow \\ \mathbf{1.2} & \rightarrow & \mathbf{.8962} \end{array}$$

Probability is $1 - .8962 = 0.1038$

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An example to review the procedure:

Given that the weights of packages of an expensive compound shows approximately a normal distribution with mean 32 oz and standard deviation 2 oz.

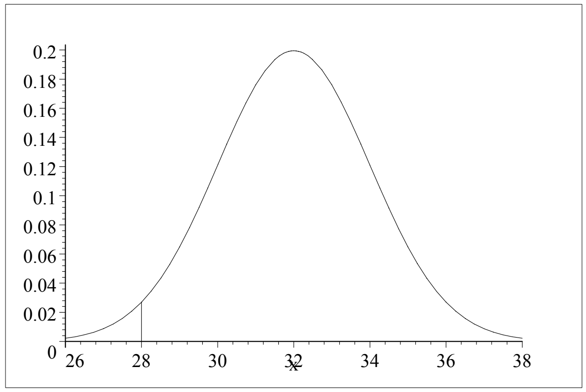
a) Find the probability that the weight of an individual package will be less than 28 oz.
b) Find the probability that the mean weight $\overline{x}$ **of randomly selected 10 packeges will be less than 28 oz.**
c) Find the level L such that the probability that the mean weight $\overline{x}$ **of randomly selected 10 packeges will be less than L is only 0.01.**

$\mathbf{p}(\mathbf{x}) = \frac{1}{2\sqrt{2\pi}} \mathbf{e}^{-\left((x-32)^2/\left(2\times 2^2\right)\right)}$

$\mathbf{a}(\mathbf{x}) = \frac{1}{\left(2/\sqrt{10}\right)\sqrt{2\pi}} \mathbf{e}^{-\left((x-32)^2/\left(2\times\left(2/\sqrt{10}\right)^2\right)\right)}$

a)

$p(x)$

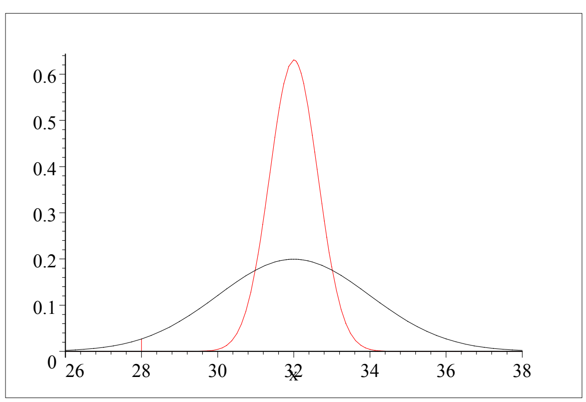


The probability that
x $<$ **28**
z $< \frac{28-32}{2} = -2.0$

$.00$
$\downarrow$
$-2.0 \quad .0228$

The probability is 0.0228.
chances are 2.28% that an individual package will weigh less than 28 oz.
b)
Since x is normal,
$\overline{x}$ **has a normal distribution with mean 32 oz and st dev** $\frac{2}{\sqrt{10}} = 0.632455532$ **oz**

Recall that $\frac{2}{\sqrt{10}}$ **is the standard deviation of sample means of simple random samples of size 10**



looks like the probability that $\overline{x} < 28$ **oz is approximately 0.**
Still, let us see

$\overline{x} < \mathbf{28}$

$\mathbf{z} < \frac{28-32}{0.632455532} = -\mathbf{6.32455532}$

FYI

$$\int_{-\infty}^{-6.32455532} \frac{1}{\sqrt{2\pi}} \mathbf{e}^{-z^2/2} \mathbf{dz} = \mathbf{1.27} \times \mathbf{10}^{-10}$$

c)

recall:

Find the level L such that the probability that the mean weight $\overline{x}$ of randomly selected 10 packeges will be less than L is only 0.01.

To find z such that the area under the z-curve to the left of z is 0.01

$$\begin{array}{ccc} & & \mathbf{.03} \\ & & \uparrow \\ -\mathbf{2.3} & \leftarrow & \mathbf{.0099} \end{array}$$

$\mathbf{z} = -\mathbf{2.33}$

$\frac{L-32}{0.632455532} = -\mathbf{2.33}$

$\rightarrow \mathbf{L} - \mathbf{32} = -\mathbf{2.33} \times \mathbf{0.632455532}$

$\rightarrow L = 32 - 2.33 \times 0.632455532 = 30.5263786$ **oz**