These are the lessons taught with the use of the text book Linear Algebra by David Lay

Greetings:

I am trying to keep these lessons as close to actual class room settings as possible.

They do not intend to replace the text book actually they will involve the text book.

An advantage of a distance learning course is that you may be able to go ahead in case you have time now,
to attend to something later.

For that reason, I am posting all the lessons from the Last semester with slight modifications.

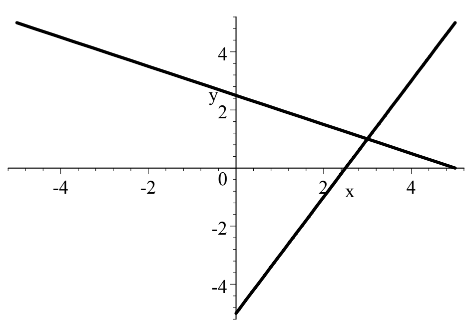
I shall make modifications further as we go on.

Please forgive me if I missed to modify a lesson for the current semester.

Let us get to the work now.

Recall a very simple technique of solving linear equations.
Called the process of elimination.
First, I may look unneccessarily detailed to you.
But watching these details would make not only solving such equations but also many other complicated things simple.

Example 1.1: Suppose that we want to solve the system
$$2x - y = 5$$
$$x + 2y = 5$$
which geometrically is the point of intersection

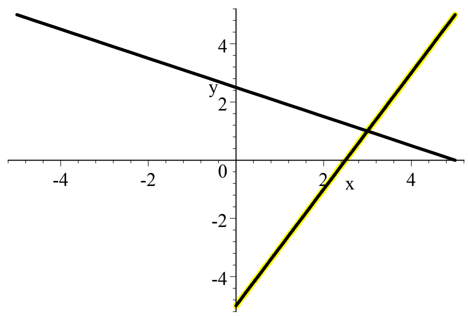


If we muliply the top equation by 2
we get

$4x - 2y = 10$

$x + 2y = 5$

note that the new equation still is equivalent to the old equation.



If add the two equations

$4x - 2y = 10$

$x + 2y = 5$

We get

$5x = 15$

$\to x = 3$

substitute this value in any of the above two equations,
you get

$3 + 2y = 5 \to 2y = 5 - 3 \to 2y = 2 \to y = 1$

Or

$x=3$

$y=1$

is the solution of the above system.

Again, note that when solving such a linear system, having even more than two equations, the system remained equivalent even if

We interchange two equations
We multiply an equation by a non zero number
We add an equation to a multiple of another equation

Those of you, who used a graphing calculator to solve a linear system like,

$2x - y = 5$

$x + 2y = 5$

First wrote the following matrix, called an augmented matrix, for the system, which is

$$\begin{bmatrix} 2 & -1 & 5 \\ 1 & 2 & 5 \end{bmatrix}$$

Then an operation known as rref (row reduced echelon form) in many calculators, will transform the matrix to

$\begin{bmatrix} 2 & -1 & 5 \\ 1 & 2 & 5 \end{bmatrix}$**, row echelon form:** $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \end{bmatrix}$

and you read the solution as x=3, y=1 or simply (3,1)

To transform a matrix to its Echelon Form (to be formally defined later) we use one or a combination of the following steps called row reduction process by using row operations on a matrix.

The row operations on a matrix are

1. Interchanging any two rows
2. Multiplying a row by a non zero number
3. Adding a row with a multiple of another row

Example 1.2
To solve the system
$x - y + 2z = 1$
$x - 2y + z = -2$
$3x - 2y + 2z = 1$

First, write the augmented matrix

$$\begin{bmatrix} 1 & -1 & 2 & 1 \\ 1 & -2 & 1 & -2 \\ 3 & -2 & 2 & 1 \end{bmatrix}$$

Our first target is to transform the first column to look like

1
0
0

To do that, we may replace
the row number 2 by row number 2 plus negative 1 times the row 1,
or symbolically

$\mathbf{r}_2 \rightarrow \mathbf{r}_2 + (-1)\mathbf{r}_1$

or

$\mathbf{r}_2 \rightarrow \mathbf{r}_2 - \mathbf{r}_1$

which is performed as

$\mathbf{r}_2$	**1**	**–2**	**1**	**–2**
$-\mathbf{r}_1$	**–1**	**1**	**–2**	**–1**
$\mathbf{r}_2-\mathbf{r}_1$	**0**	**–1**	**–1**	**–3**

Therefore

$$\begin{bmatrix} 1 & -1 & 2 & 1 \\ 1 & -2 & 1 & -2 \\ 3 & -2 & 2 & 1 \end{bmatrix}$$

$\downarrow$ $\mathbf{r}_2 \rightarrow \mathbf{r}_2 - \mathbf{r}_1$

$$\begin{bmatrix} 1 & -1 & 2 & 1 \\ 0 & -1 & -1 & -3 \\ 3 & -2 & 2 & 1 \end{bmatrix}$$ **continue in this manner**

$\downarrow$ $\mathbf{r}_3 \rightarrow \mathbf{r}_3 - 3\mathbf{r}_1$

$$\begin{bmatrix} 1 & -1 & 2 & 1 \\ 0 & -1 & -1 & -3 \\ 0 & 1 & -4 & -2 \end{bmatrix}$$

Now we would like to transform the second column

$$\begin{matrix} -1 \\ -1 \\ 1 \end{matrix} \quad \textbf{to} \quad \begin{matrix} 0 \\ 1 \\ 0 \end{matrix}$$

The usual strategy is to obtain 1 in the second entry of the column.
In this case, it may be done by

$r_2 \rightarrow (-1)r_2$ **which, we shall denote informally as just** $-r_2$

Therefore

$$\begin{bmatrix} 1 & -1 & 2 & 1 \\ 0 & -1 & -1 & -3 \\ 0 & 1 & -4 & -2 \end{bmatrix}$$

$\downarrow\ -\mathbf{r}_2$

$$\begin{bmatrix} 1 & -1 & 2 & 1 \\ 0 & 1 & 1 & 3 \\ 0 & 1 & -4 & -2 \end{bmatrix}$$

$\mathbf{r}_1+\mathbf{r}_2\ \downarrow\ \mathbf{r}_3-\mathbf{r}_2$

$$\begin{bmatrix} 1 & 0 & 3 & 4 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 5 & 5 \end{bmatrix}$$ **Now we can transform the third column to** $\begin{matrix} 0 \\ 0 \\ 1 \end{matrix}$ **by**

$\downarrow\ \frac{1}{5}\mathbf{r}_3$

$$\begin{bmatrix} 1 & 0 & 3 & 4 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$\mathbf{r}_1-\mathbf{3r}_1\ \downarrow\ \mathbf{r}_2-\mathbf{r}_3$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Therefore the solution is x=1,y=2,z=1
or (1,2,1.)

Note: When solving a system of linear equations by using the method of row opeartins, we may read the solution at any stage when it is convenient.

For example, let us look at the exercise number 16 in the text.
First note that the augmented matrix is

$$\begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ -2 & 3 & 2 & 1 & 5 \end{bmatrix}$$ **To transform the first column,do the next step**

$\downarrow\ \mathbf{r}_4+\mathbf{2r}_1$

$$\begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 3 & 2 & -3 & -1 \end{bmatrix}$$

$$\downarrow \ \frac{1}{2}\mathbf{r}_2$$

$$\begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 3 & 2 & -3 & -1 \end{bmatrix}$$

$$\downarrow \ \mathbf{r}_4 - \mathbf{3r}_2$$

$$\begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & -1 & -3 & -1 \end{bmatrix}$$

$$\downarrow \ \mathbf{r}_4 + \mathbf{r}_3$$

$$\begin{bmatrix} 1 & 0 & 0 & -2 & -3 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

We shall continue with this concept in the lessons 2 and 3.
that is
i) to see if a system is consistent.
ii) to see whenther a consistent system has a unique solution or infinitely many solutions.