The posted lessons are part of the Differential Equations course that I taught at Montgomery College in Germantown Maryland.

The lessons are written according to *Differential Equations* , Third Edition, by Blanchard, Devaney, and Hall , Brooks/Cole as the text book adopted for the class.

For any questions, comments or objections

You may write to me at

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Lesson for the week 1:

Corresponds to the sections 1.1 and 1.2 of the text book.

A phenomenon that involve changing quantities may be described by a differential e

Assume that all the dependent variables that we are discussing in the examples 1-5
are differentiable

Example 1:

The rate of change of the number of people (y) in a certain place depends directly on the number itself at any time (t)

This phenomenon may be described by the differential equation

$\frac{dy}{dt} = ky$, where k is a constant.

Example 2:

The rate of change in the population of a certain species depends directly on the number itself and at the same time on the carrying capacity of the environment.

If

y : The number present at time t
L : the carrying capacity

this phenomenon may be described by the differential equation

$\frac{dy}{dt} = ky\left(1 - \frac{y}{L}\right)$, where k is called the growth rate parameter, this stays constant in a given context

Example 3:

When an object is brought from a specific temperature to an environment in a different temperature, the temperature of the object changes.

The Newton's law of cooling states that the rate of change of the temperature y with respect to the time t, is proportional to the difference of y and the temperature of the surrounding medium.

For example, if an object is taken out of the oven at $450^\circ F$ and is placed in a room at a tempearture $70^\circ F$

$\frac{dy}{dt} = k(70 - y)$, where k is a constant.

Example 4:

Let us consider a Law of Free Market that was given by Adam Smith

For a certain commodity, let the supply equal s units, the demand equal d units,
and the price equal p units at a time t.

Express $u = d - s$ and call it excess demand.

If $u > 0$, then $\frac{dp}{dt} > 0$, and therefore $\frac{du}{dt} < 0$

That is, if the demand is greater than supply, the price increases, and therefore the excess demand decreases.

Example: We all experience the Newton's Law of graviation, the famous inverse square law

$$\frac{d^2y}{dt^2} = \frac{k}{y^2}$$

Briefly:

A differential equation is an equation that involves the derivatives of the function of interest.

Read the section 1.1 of the text and make sure to understand the following terminologies:

Initial conditions

General Solution

Particular Solution

Equilibrium Solutions

Solving Differential Equations:

Example 1:

An object is removed from an oven and it is placed in a room at $70^\circ F$

y: temperature of the object at time t

Newton's Law of cooling gives us the following differential equation

$\frac{dy}{dt} = k(y - 70)$

The above equation can be solved analytically by separation of variables

in the following manner

$\frac{dy}{y-70} = kdt$

$\rightarrow$

$\int \frac{dy}{y-70} = \int kdt$

$\rightarrow$

$\ln|y - 70| = kt + C$ (see the footnote 1 for computation of the integral)

If the oven is hotter than $70^{\circ}F$, we have $y > 70$ and therefore $|y - 70| = y - 70$

$\ln|y - 70| = kt + C$

$\rightarrow$

$y - 70 = e^{kt+C}$

$\rightarrow$

$y - 70 = e^{C}e^{kt}$

$\rightarrow$

$y - 70 = ce^{kt}$, where $c = e^{C}$

$\rightarrow$

$y = 70 + ce^{kt}$ Now we have a general solution of the differential equation.

If the temperature of the oven is $450^{\circ}F$, then $y = 450^{\circ}F$, when $t = 0$ units

the condition mentioned above is called an initial condition which can be used

to find a particular solution of this differential equation that corresponds to the initial condition $y(0) = 450$

Subsituion in the above equation yields

$450 = 70 + ce^{k(0)}$
$\rightarrow$
$450 - 70 = c$
$\rightarrow$
$c = 380$

and a particular solution is

$y = 70 + 380e^{kt}$

Let us work on some exercises in the text book

#4 on the Page 14:

Given a population model

$$\frac{dP}{dt} = 0.3\left(1 - \frac{P}{200}\right)\left(\frac{P}{50} - 1\right)P$$

P is the population at time t

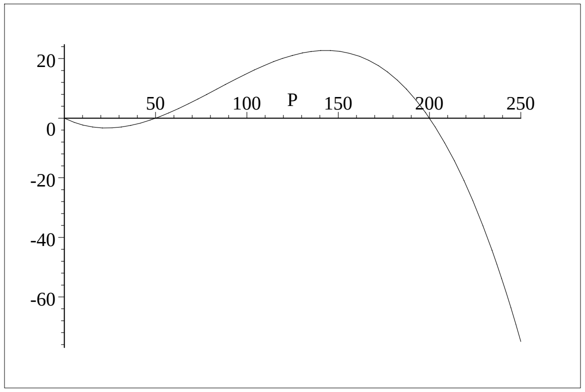
a)
For what values of P is the population in equilibrium

that is

the values of P for which $\frac{dP}{dt} = 0$
$0.3\left(1 - \frac{P}{200}\right)\left(\frac{P}{50} - 1\right)P = 0$
$P = 0, P = 50, P = 200$

b)
For what values of P is the population increasing

that is $\frac{dP}{dt} > 0$
$0.3\left(1 - \frac{P}{200}\right)\left(\frac{P}{50} - 1\right)P$



$\frac{dP}{dt} > 0$
$(50, 200)$
c)
$\frac{dP}{dt} < 0$

For
$(0,50) \cup (200,\infty)$

#10 on the page 17

It is given in the situation that the rate at which the quantity of a radioactive isotope decays is proportional to the amount of isotope present and we are assuming that $\lambda > 0$

a)

$\frac{dr}{dt} = -\lambda r$

b)
$\frac{dr}{dt} = -\lambda r$
$r = r_0$ when $t = 0$

#11
$\frac{dr}{dt} = -\lambda r$
$r = r_0$ when $t = 0$

$\frac{dr}{dt} = -\lambda r$
$\rightarrow$
$dr = -\lambda r dt$
$\rightarrow$
$\frac{dr}{r} = -\lambda dt$
$\rightarrow$

$\int \frac{dr}{r} = \int -\lambda dt, \quad r > 0$

$\rightarrow$

$\ln r = -\lambda t + C$

given that

$t = 0, r = r_0$

$\rightarrow$

$\ln r_0 = -\lambda(0) + C$

$\rightarrow$

$C = \ln r_0$

$\ln r = -\lambda t + \ln r_0$

$\rightarrow$

$r = e^{(-\lambda t + \ln r_0)}$

$\rightarrow$

$r = e^{-\lambda t} e^{\ln r_0}$

$\rightarrow$

$r = e^{-\lambda t} r_0$

$\rightarrow$

$r = r_0 e^{-\lambda t}$

a)

Half life is 5230 years

When $t = 5230$

$r = \frac{r_0}{2}$

$\frac{1}{2} r_0 = r_0 e^{-5230\lambda}$

$\rightarrow$

$e^{-5230\lambda} = \frac{1}{2}$

$\rightarrow$

$\ln e^{-5230\lambda} = \ln\left(\frac{1}{2}\right)$

$\rightarrow$

$-5230\lambda = \ln\left(\frac{1}{2}\right)$

$\rightarrow$

$\lambda = -\frac{1}{5230}\ln\left(\frac{1}{2}\right) = 1.325329217 \times 10^{-4}$

Please finish the other parts on your own.

Post a question in the discussion area if you have difficulty.

Page 18:
#16:

Growth rate is $\frac{dy}{dt}$

Given that the carrying capacity is 2500

and the growth parameter is 0.3

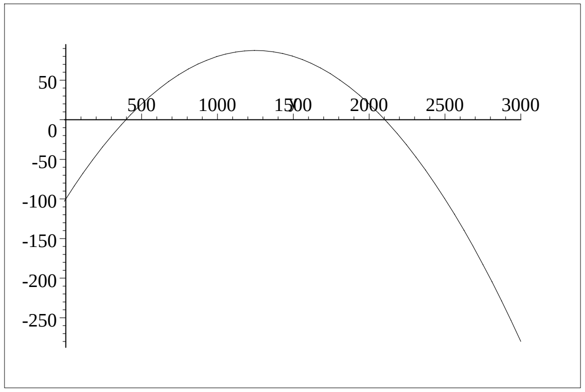
The logistic model is

$\frac{dy}{dt} = .3y\left(1 - \frac{y}{2500}\right)$

a)

Since we are harvesting 100 fish each year

$\frac{dy}{dt} = .3y\left(1 - \frac{y}{2500}\right) - 100$
$\frac{dy}{dt} > 0$
for
$.3y\left(1 - \frac{y}{2500}\right) - 100 > 0$
$.3y\left(1 - \frac{y}{2500}\right) - 100$



x-axis has the y-values
y-axis has $\frac{dy}{dt}$ values
$\frac{dy}{dt} > 0$

on the interval whose end points are given by the solutions of the quadratic equation

$.3y\left(1 - \frac{y}{2500}\right) - 100 = 0$, Solution is: $\{y = 396.08743617003346806\}, \{y = 2103.9125638299665319\}$ (see the footnote 2)

Since the fish population will expressed in terms of whole numbers,

we can round these numbers to 396 and 2104

$\frac{dy}{dt} < 0$
for
y in
$(0, 396) \cup (2104, \infty)$

APOLOGY: I used y for P, sorry

For the population starting with the initial value of 2500>2104

$\frac{dy}{dt} < 0$

The growth rate decreases

and eventually the population should decrease towards the equilibrium 2104

b)

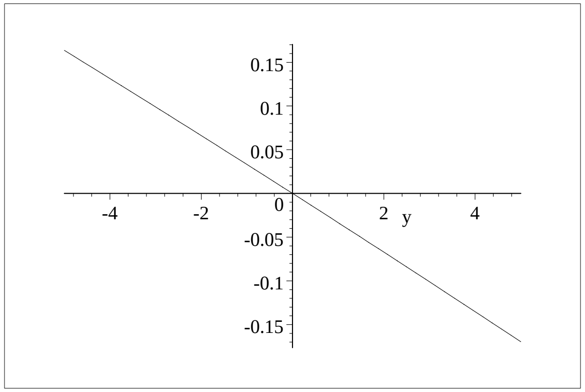
In this case, we are harvesting $\frac{y}{3}$ of the fish each year

therefore the model is

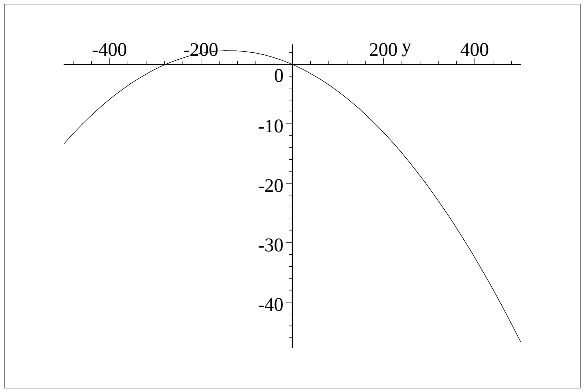
$$\frac{dy}{dt} = .3y\left(1 - \frac{y}{2500}\right) - \frac{y}{3}$$

A plot of y on the horizontal axis and $\frac{dy}{dt}$ on the vertical axis gives

$$.3y\left(1 - \frac{y}{2500}\right) - \frac{y}{3}$$



Adjusting the window



If a third of the population is harvested each year, note that the population with decay to 0 for any positive initial value.

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Sections 1.1 and 1.2

Pick an exercise from the section 1.2 please

Page 33:
#5

$\frac{dy}{dt} = ty$

$\frac{dy}{y} = tdt$

$\rightarrow$

$\int \frac{dy}{y} = \int tdt$

$\rightarrow$

$\ln y = \frac{t^2}{2} + C$

$\rightarrow$

$y = e^{\left(\frac{t^2}{2}+C\right)}$

$\rightarrow$

$y = e^C e^{(t^2/2)}$

$\boxed{e^C = D}$

$y = De^{(t^2/2)}$

32.

$\frac{dy}{dt} = 2ty^2 + 3t^2y^2, \quad y(1) = -1$

$\rightarrow$

$\frac{dy}{dt} = (2t + 3t^2)y^2$
$\rightarrow$
$\frac{dy}{y^2} = (2t + 3t^2)dt$
$\rightarrow$
$\int \frac{dy}{y^2} = \int (2t + 3t^2)dt$
$\rightarrow$
$-\frac{1}{y} = t^2 + 3\frac{t^3}{3} + C$
$\rightarrow$
$-\frac{1}{y} = t^2 + t^3 + C$
$t = 1, y = -1$
$-\left(\frac{1}{-1}\right) = 1^2 + 1^3 + C$
$1 = 2 + C$
$C = -1$

$-\frac{1}{y} = t^2 + t^3 - 1$
$y = -\frac{1}{t^2+t^3-1}$

Work on the following problems in the sections 1.1 and 1.2

1.1: 1, 3, 5, 11, 15, 19

1.2: 3, 5, 11, 15, 19, 29, 31

Footnotes

1. Evaluating $\int \frac{dy}{y-70}$

Let $y - 70 = u \rightarrow dy = du$

$$\int \frac{dy}{y-70} = \int \frac{du}{u} = \ln|u| = \ln|y - 70|$$

2.

To solve

$$.3y\left(1 - \frac{y}{2500}\right) - 100 = 0$$

multiply by 2500

$$.3 \times 2500y\left(1 - \frac{y}{2500}\right) - 2500 \times 100 = 0$$
$\rightarrow$
$$750y - .3y^2 - 250000 = 0$$
$\rightarrow$
$$.3y^2 - 750y + 250000 = 0$$

Solutions are

$$\frac{750-\sqrt{750^2-4\times.3\times250000}}{2\times.3} = 396.087\,436\,170\,033\,468\,05$$
$$\frac{750+\sqrt{750^2-4\times.3\times250000}}{2\times.3} = 2103.912\,563\,829\,966\,532$$