Hypothesis Testing:

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In this part, **we shall discuss hypothesis tests about population mean**.

Using Examples such as

To test if the meantime μ **for all the passengers to reach the boarding area after checking the luggage is no more than 45 minutes**, **we may record** "**such timing**" **for a simple random sample of appropriate number of passengers and see if the mean** $\overline{x}$ **of this sample supports the value** $\mu \leq$ **45 minutes**.

If I would like to check whether or not my average blood sugar level is 90 mgDl, I shall take the blood sugar measurements several times and see if the mean of these sample supports $\mu = 90$ **mgDL for me**.

The mean weight of the mangoes of "**Chausa**" **variety is 220 gm**. **The owner of a big** "**Chausa**" **mango FARM claims that the mean weight of the mangoes from his farm is higher**. **A simple random sample of the mangoes from this farm will be taken to see if the sample is a good evidence in favor of claim of the Farm owner**.

The results of the variability of the sample means may be used to perform the mathematical calculations for such tests.

Let us start with an example of testing whether a milk supply does has added water in it.

Even though a large part of natural milk is already water, still **added water in the milk can have impacts on the taste and preservation of milk that can have**
business and health consequences.

Milk products are used widely by big restaurant chains like McDonald as well as an average road side vendor for beverages.

These days, **modern instruments like cryoscopes** (**the one shown here is supplied by**
Advanced Instrument Corporation , **www.aicompanies.com**)

ADVANCED 4C3
MULTI-SAMPLE CRYOSCOPE

Such an instrument can check the freezing points of samples of size 2ml to help determine whether milk has added water in it. Ordinarily milk already is 88% water, we are talking about added water contents here. Added water raises the freezing pointof milk.

Let us assume that in the region of our interest the freezing point of supply (measured for a 2ml volume) from pure milk shows approximately
a normal distribution with mean $\mu = -0.540°C$ and $\sigma = 0.010°C$. To obtain such information more accurately, refer to the CRC Handbook of Chemistry and Physics from the CRC press.

Example:

The manager of a large restaurant chain suspects that one of the suppliers is adding water in the milk supply. He decides to take 20 samples of volume 2 ml each (assume simple random) and determine the mean of the freezing point of these 20. He finds that the mean freezing point for these 40 is $\overline{x} = -.52683°C$ which of course is higher than $-0.540°C$

Let us compute the probability of a sample showing such an extreme or more extreme sample mean that is $\overline{x}$ if the milk supply is actually pure that is $\mu = -0.540°C$

Since the population here is normal, the distribution of $\overline{x}$ for the sample sizes of 20 will show approximately a normal distribution with

mean $\mu = -0.540°C$ and standard deviation $\frac{0.010}{\sqrt{20}} \approx 2.2361 \times 10^{-3} = 0.0022361°C$
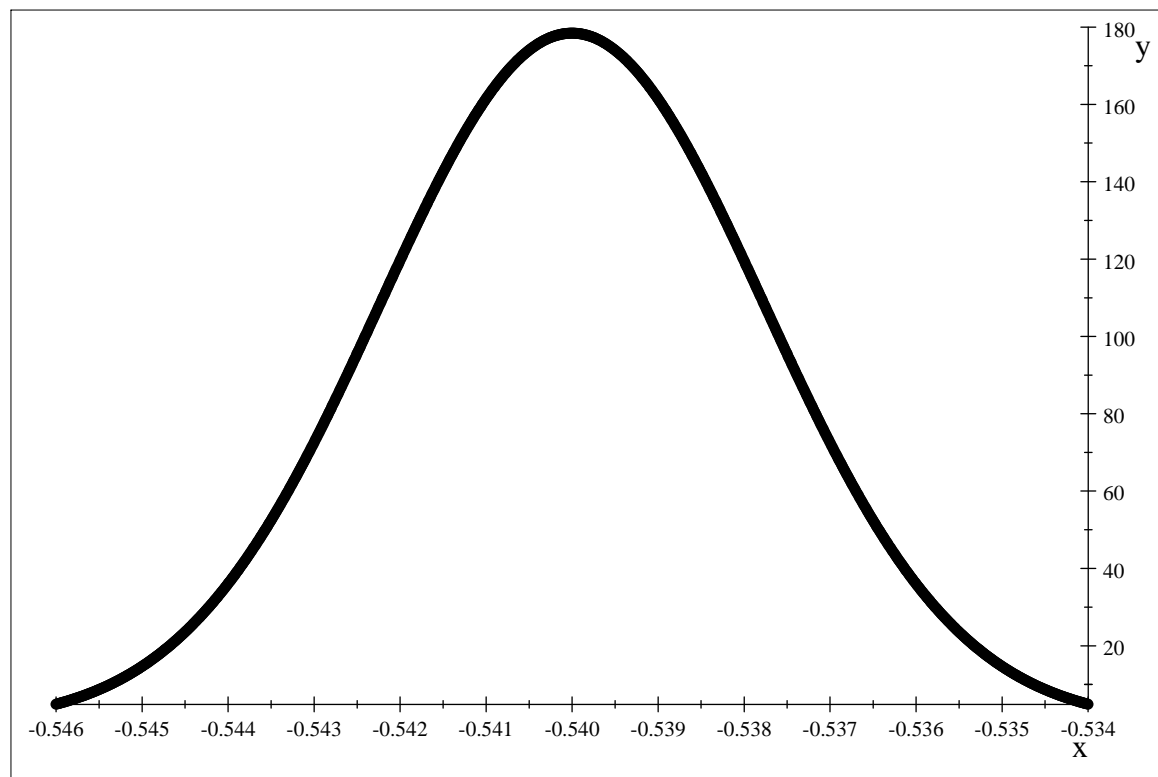
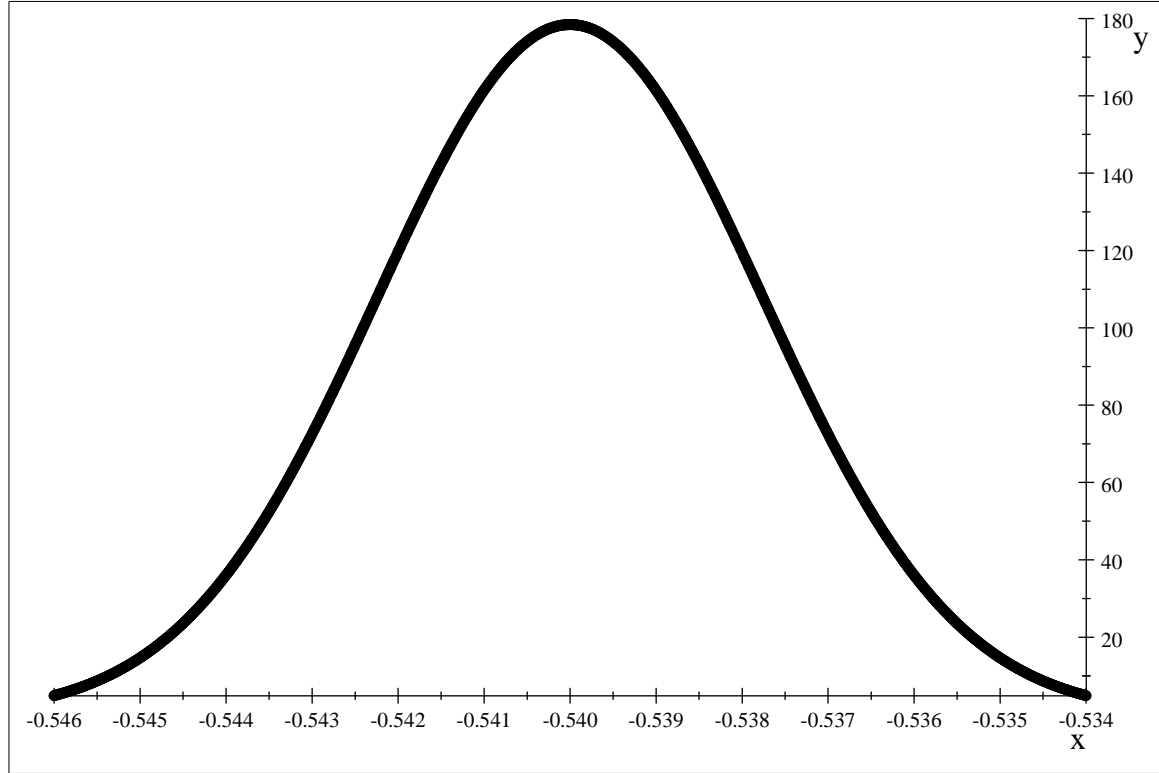The distribution of the sample means of such simple random samples will look like

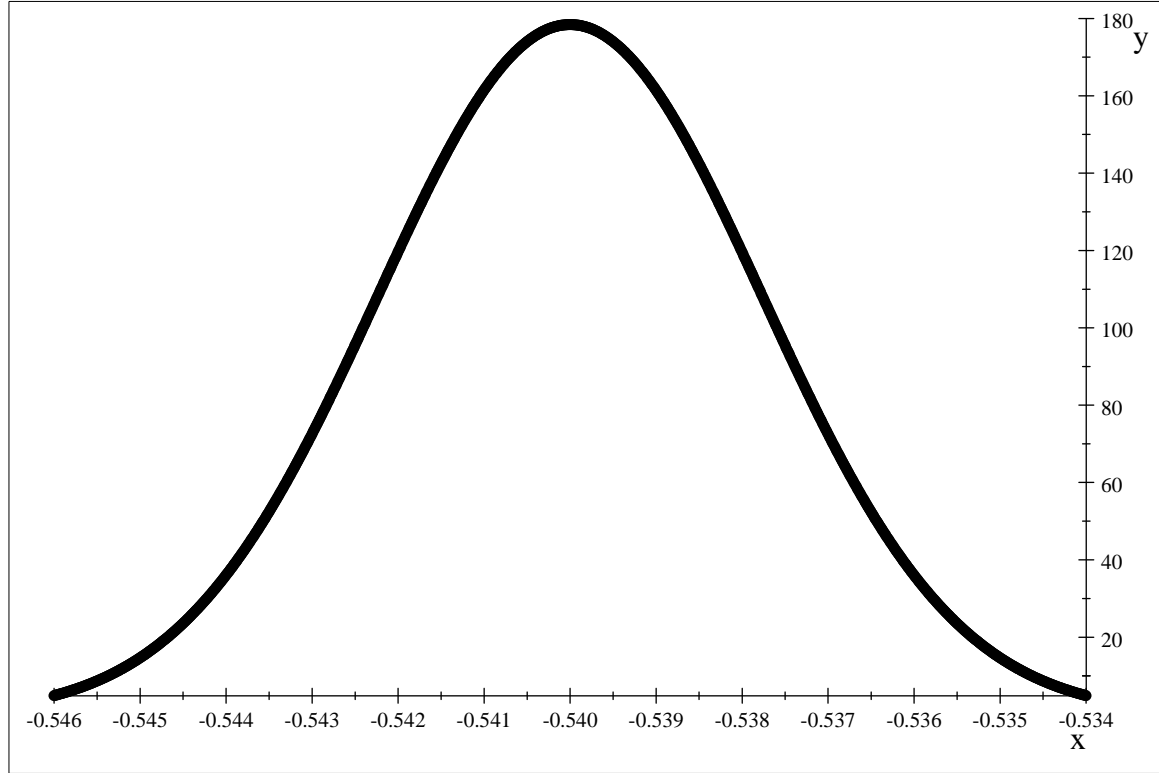$$\mathbf{f(u)} = \frac{1}{0.0022361\sqrt{2\pi}} \exp\left(-\left((u+.54)^2/\left(2 \times 0.0022361^2\right)\right)\right)$$
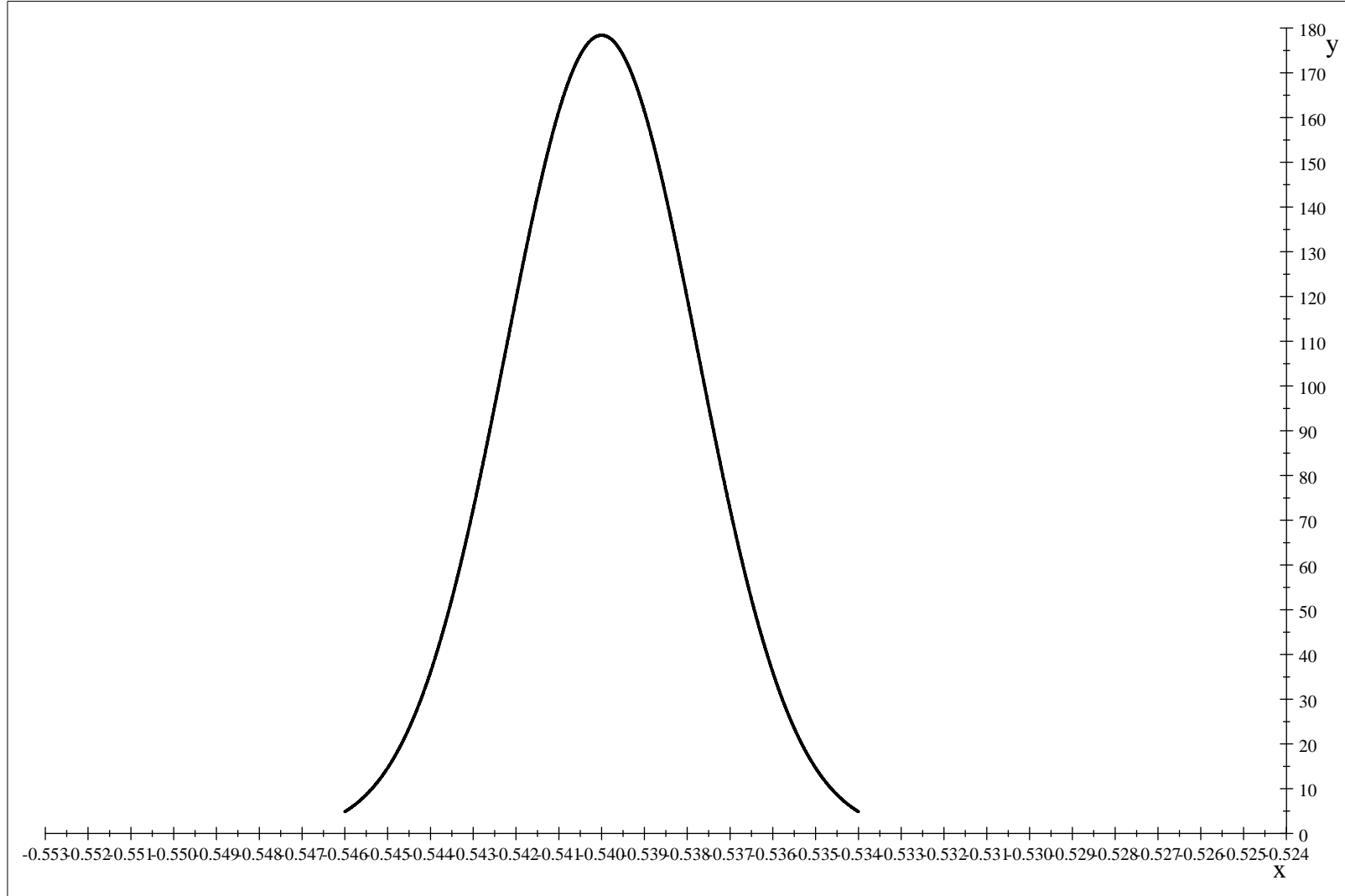
$$-.54 - 6 \times 0.0022361 = -0.5534166$$

$$-.54 + 6 \times 0.0022361 = -0.5265834$$

f(u)

You may note that the area to the right of $\overline{x} = -.525°C$ is extremely small, which shows that if the milk is from a supply for which $\mu = -.54°C$
then it is highly unlikely that $\overline{x} \geq -.525°C$ will occur for a simple random sample of size 20.

Or, we transform $\overline{x}$ to z and get

For $\overline{x} = -.52683$

$$z = \frac{-.52683 - (-0.540)}{\left(\dfrac{0.010}{\sqrt{20}}\right)} = 5.\,889\,803\,052\,734\,446\,060\,341\,763\,443\,438\,181\,604\,150\,588\,759\,216\,758\,757\,729\,54$$

$$\int_{5.89}^{\infty} \frac{1}{\sqrt{2\pi}}e^{-z^2/2}\mathbf{dz} = 1.\,930\,978\,094\,185\,306\,586\,459\,066\,787\,927\,523\,891\,511\,567\,773\,777\,044\,614\,717\,56 \times 10^{-9}$$

$$\int_{5.86}^{\infty} \frac{1}{\sqrt{2\pi}}e^{-z^2/2}\mathbf{dz}$$

so that the probability that

$$\overline{x} \geq -.52683°C$$

is approximately the same as

$$z \geq \frac{-.52683 - (-0.540)}{\left(\dfrac{0.010}{\sqrt{20}}\right)} \cong 5.\,89$$

$z \geq 5.89$

**This value is out of range from our z-table
and we can say that the probability of a sample
showing values in this range (if the supply is pure)
is almost 0**

We can notice that the sample mean is improbably far from the mean that we should have seen for milk with no **ADDED** water in it.

Therefore the manager can be confident that the milk supplier is adding water in the milk supply.

We can formalize the above steps in terms of hypothesis testing in the following steps

The null hypothesis is the statement "milk has no added water in it" or in terms of the population mean μ, it is $\mu = -0.540°C$

The alternative hypothesis is that the "milk does have added water in it"
or in terms of the population mean μ, it is $\mu > -0.540$
Symbolically:

We have

$H_O : \mu = -0.540°C$ **a formulation of the statement "milk has no added water in it"**
$H_A : \mu > -0.540°C$ **a formulation of the statement "milk has added water in it"**

The sample value is $\overline{x} = -0.52683°C$

The test statistic for the test of one population mean (for known population standard deviation)

is given by

$$z = \frac{\overline{x} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$$

in this case, **as you saw above**,

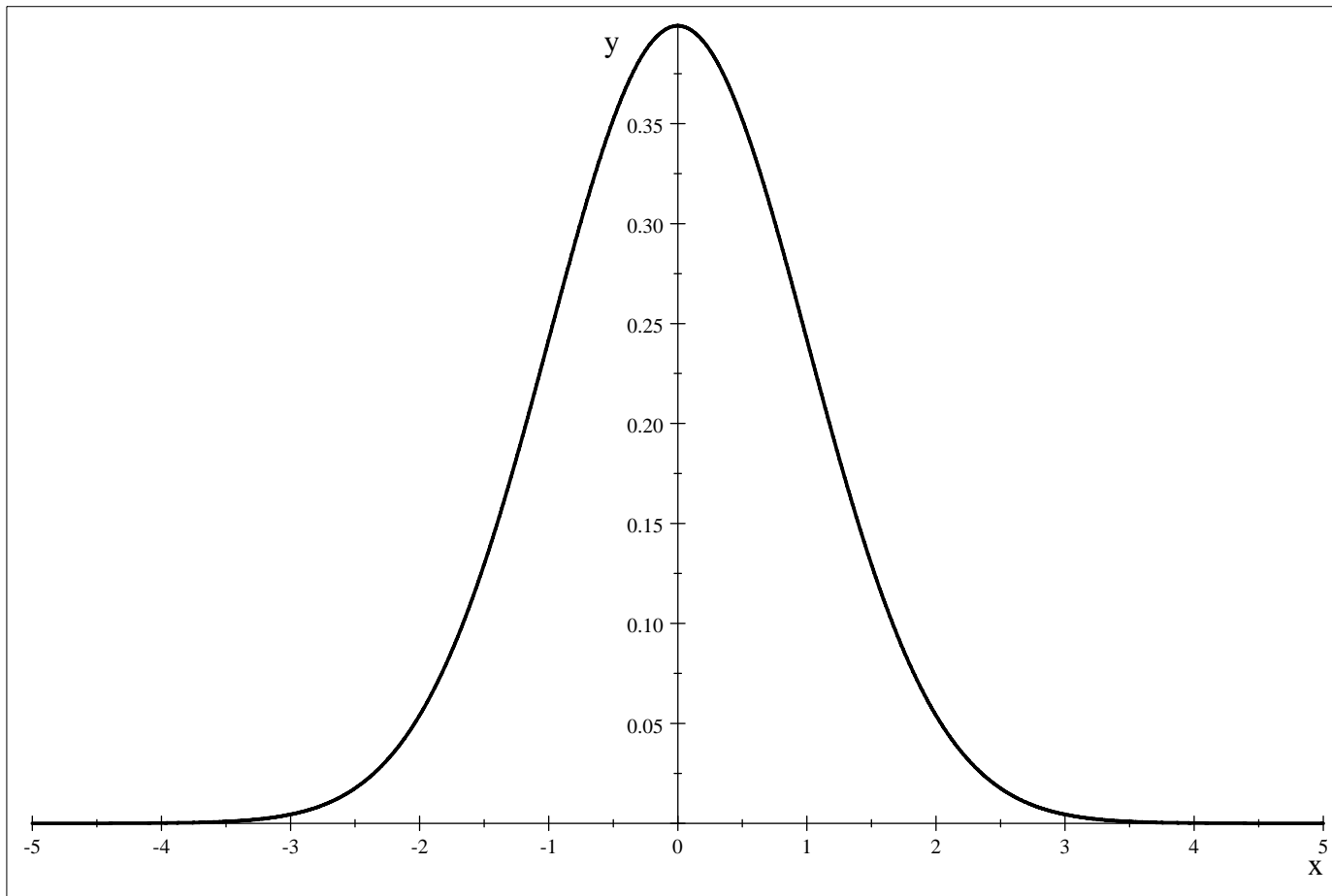$$\mathbf{z} = \frac{-.52683 - (-0.540)}{\left(\dfrac{0.010}{\sqrt{20}}\right)} = 5.89$$

The P_value is the probability that

$z \geq 5.89$ **(remember that our alternative is ">" that is "greater than")**

Remember that our $z-$**table does not go upto this value, beacuse there is very little tail area beyond** $z = 6.71$

$$\frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

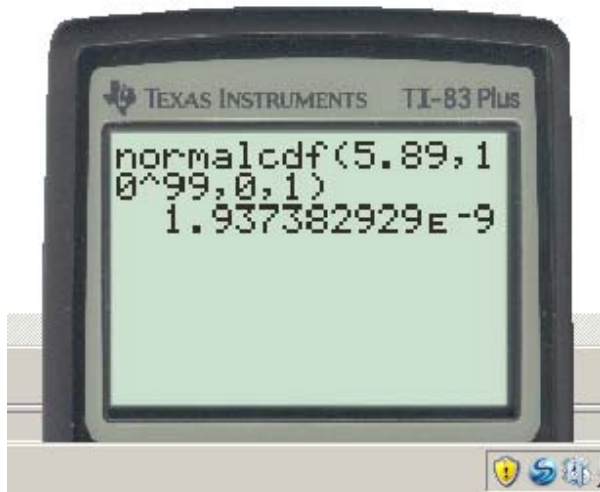Therefore the probability that $z \geq 5.89$ is approximately zero.

We may be able to find this probability by using technology as

$$\int_{5.89}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz := 1.930\,978\,094\,185\,306\,586\,459\,066\,787\,927\,523\,891\,511\,567\,773\,777\,044\,614\,717\,56 \times 10^{-9}$$

that is approximately 0.00000000193

OR may use a TI83plus calculator

to obtain

A small P_value goes against the null hypothesis,

therefore in this example, we reject the null hypothesis that

$H_O : \mu = -0.540°C$ **a formulation of the statement "milk has no added water in it"**

and decide in favor of the alternative that

$H_A : \mu > -0.540°C$ **a formulation of the statement "milk has added water in it"**

Of course, we may be making a wrong decision (that is rejecting a null hypothesis that is true)

but the probability of us making such an error is only

which is small

So small that the probability of getting six dots facing up on each die
in a roll of TEN fair dice
$$\frac{1}{6^{10}} = 1.\,653\,817\,168\,792\,020\,186\,6 \times 10^{-8}$$
is more than 1.93×10^{-9} **the probability of rejecting the true**
null hypothesis in this case.

Sometimes a level is set to indicate how small a P_value will be treated as
small for a test. Such a level is called the level of significance of a test.

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Summary:

To test a null hypothesis for the population mean:

Formulate the null : H_O
Formulate the alternative: H_A

select a simple random sample of the desired size (say n)
(**May have to apply the Central Limit Theorem**)

Compute the test satistic

$$z = \frac{\overline{x} - \mu}{\left(\dfrac{\sigma}{\sqrt{n}} \right)}$$

$\overline{x}$: **the sample mean**
μ: **the mean suggested by the null hnypothesis**

Then compute the P_value, **if the P_value is very small**,
then reject the null hypothesis.

If a level (**level of significance**) **is set for the test**,
then reject the null if the P_value is smaller
than the level of significance.

Example 2:

The owner of a 1000s of soda/snack vending machines in a big metropolitan area submits in

his tax documents that he gets a revenue of $80 per machine per day from these machines. Suspecting that the vendor is understating his income, an inspector decides to take a simple random sample of 50 machines on randomly selected days, and test

the null hypothesis $H_0 : \mu = 80$
against the alternative $H_A : \mu > 80$

at 1% level of significance. The inspector is able to assume that $\sigma = \$38.00$ (a GIVEN value.)

Let us see what the inspector will conclude if the sample mean turns out to be $\overline{x} = \$86.00$

Note that the test statistic here is $z = \dfrac{86-80}{\left(\frac{38}{\sqrt{50}}\right)} = 1.1164843913471803017$

The P_value is the probability that
z > 1.1164843913471803017

From the z-table

$.02$

$\downarrow$

1.1 $\rightarrow$ $.8686$

the P_value is $1-.8686 = 0.1314$

Since, the P_value>0.01 the level of significance

we do not reject the null hypothesis,

in plain language, **the sample does not prov**

An analogy of hypothesis test

(**warning**:
Do not think that hypothesis testing is always in an accusation setting)

Jury Trial:

Null: **Not Guilty**
Alternative: **Guilty**

Not Guilty unless proven so beyond a reasonable doubt.

Example 3:

Al is manager of the cheese packaging company. One kind of package is meant to contain 32 oz of cheese.
To see if the filling process is working right (there is no underfilling to avoid liability and there is no overfilling to avoid loss of profit) Al takes a simple random sample of 36 packages and finds that $\overline{x} = 32.7$ oz for these 36 packages. The packaging process has $\sigma = 2$ oz..
Formualte the appropriate hypotheses and state your conclusion at 5% level of significance.
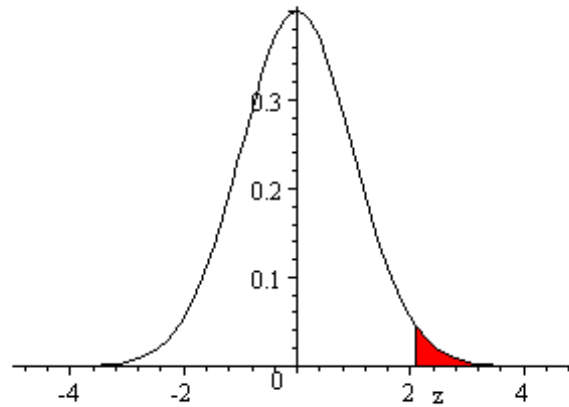
$\mathbf{H}_0$: μ = **32**
$\mathbf{H}_A$: μ ≠ **32**
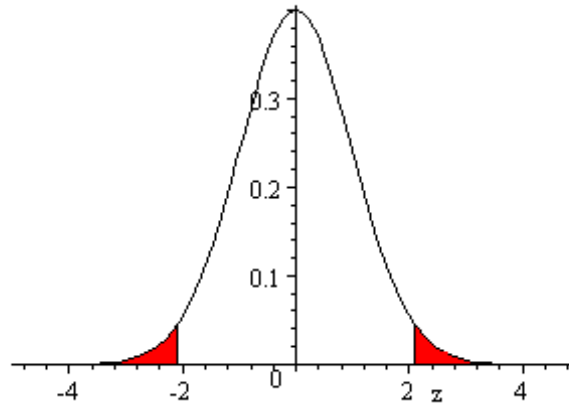
Test statistic:

$$z = \frac{32.7 - 32}{\left(\dfrac{2}{\sqrt{36}}\right)} = 2.1$$

s(z)

The tail value is $.5 - .4821 = 0.0179$

Two sided, **Two talied**

P_value is $2 \times .0179 = 0.0358$

0.0358<.05

Reject the null.

When the null is rejected, **then the sample results are called significant**.

Example 4:

Julie would to like to determine if the average time that people spend talking on the phone to a certain overseas location has changed from $\mu = 30$ minutes. She takes a simple random sample of 40 such calls and finds that $\overline{x} = 28.2$ minutes. Use $\sigma = 8$ minutes and determine if Julie has good evidence to suggest a change from the believed average of $\mu = 30$ minutes. Use a .05 level of significance.

H$_O$: μ = 30
H$_A$: μ ≠ 30

$$z = \frac{28.2 - 30}{\left(\dfrac{8}{\sqrt{40}}\right)} = -1.423$$

Area under the z-curve to the left of z=-1.423
is $.5 - .4222 = 0.0778$

P_value is $2 \times .0778 = 0.1556$

0.1556 **is not less than .05**, **the null is not rejected**, **the sample does not show a good evidence of change**.

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Recall:

$z_{.05}$ **is the value of z such that the area to the right of this value is .05**.

We had found that $z_{.05} = 1.645$
$z_{.025} = 1.96$

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If the alternative is < **(less than)** , **and the level of significance is** α, **then we reject the null hypothesis when the test statistic is less than** $-z\alpha$

If the alternative is $>$ (greater than) , and the level of significance is α, then we reject the null hypothesis when the test statistic is greater than z_α.

If the alternative is $\neq$ (greater than) , and the level of significance is α, then we reject the null hypothesis when the test statistic
is less than $-z_{\alpha/2}$ or is greater than $z_{\alpha/2}$

Example 5: **A university uses a web interface to enhance its face to face classes.**
The mean time that the participants spend in handing the tecnical aspect
(accessing the course page, downloading, uploading etc) of the course is 4 hours each semester.
Another interface claims that if the university uses their services, the above time will go down.
The interface is used for a simple random sample of 100 participants, and they find
that $\overline{x} = 3.5$ hours for the semester. Take $\sigma = 1.5$ hours.
State the appropriate null and alternative.
Find the P_value.
State, whether you will accept or reject the null hypothesis at 1% level of significance.

H_O: μ = 4 $\left(\text{may also say: } \mu \geq 4\right)$
H_A: μ < 4
Test statistic:
$$z = \frac{3.5 - 4}{\left(\frac{1.5}{\sqrt{100}}\right)} = -3.3333$$
P_value is
Probability that $z < -3.33$
$.5 - .4996 = 0.0004$

The P_value is very small, **reject the null**.
Accept the alternative.

Example 6:

The employees of a contractor (a very large company) make 17 mistakes per
document, **while processing the documents of a government department**.
Another contractor (C) , claims that his employees show a lower average.
In a simple random sample of 35 employees from the contractor C, **we find that**
$\overline{x} = 14$. **Take** $\sigma = 8$ **and**
Formulate the null and alternative hypothesis
Compute the test statistic
Compute the P_value
State your conclusion, **whether you shall accept or reject the null at 1% level of significance**.

$\mathbf{H}_O$: **μ = 17**
$\mathbf{H}_A$: **μ < 17**

$$\mathbf{z} = \frac{14 - 17}{\left(\dfrac{8}{\sqrt{35}}\right)} = -2.2185$$

P_value is 0.0132

0.0132>0.01

do not reject the null hypothesis at 1% level of significance.

Example 7: **To illustrate the role of a confidence interval in hypothesis testing**.

The mean yearly expenses on the professional development of the professionals in a certain speciality has been $2000. To see if this mean has "changed" in view of the on line professional development offerings* (could change in either directions, less because it would cost less to do activities on line, more because people may like to explore more opportunities), a simple random sample of 36 professionals is asked about the expenses that they incurred on professional development. Assume $\sigma = \$800$

If the mean for these 36 professionals is $\overline{x} = \$2151$

a) **To formulate the null and the alternate hypotheses**
b) **To compute the test statistic**
c) **To find the P_value**
d) **To report our conclusion at a 5% level of significance**
e) **To compute a 95% confidence interval**
f) **To check whether the confidence interval contains the value of $2000**
g) **To relate the answers in the parts d) and f) written above.**

a)

Formulate the hypotheses
H_O: μ = **2000**
H_A: μ ≠ **2000**

b)
the test statistic
$$z = \frac{\overline{x} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$$
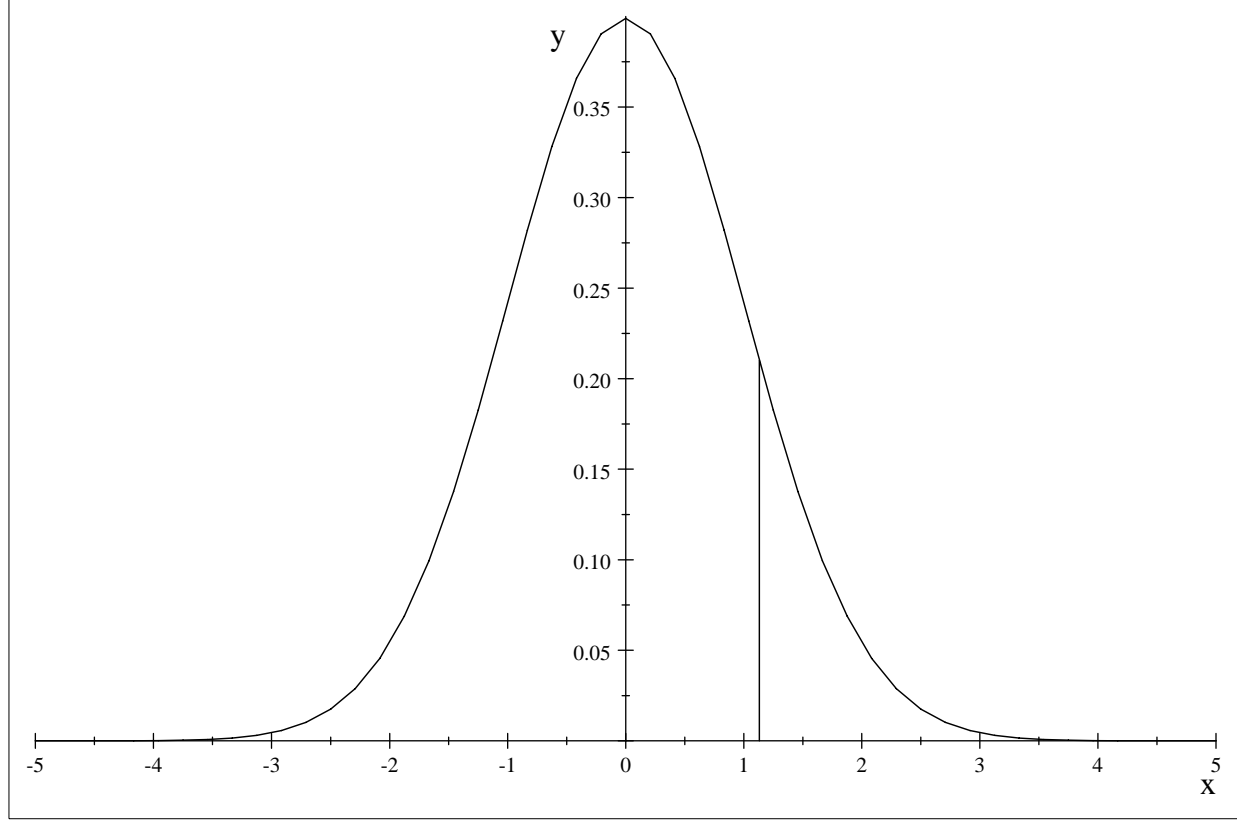
in this case, it is

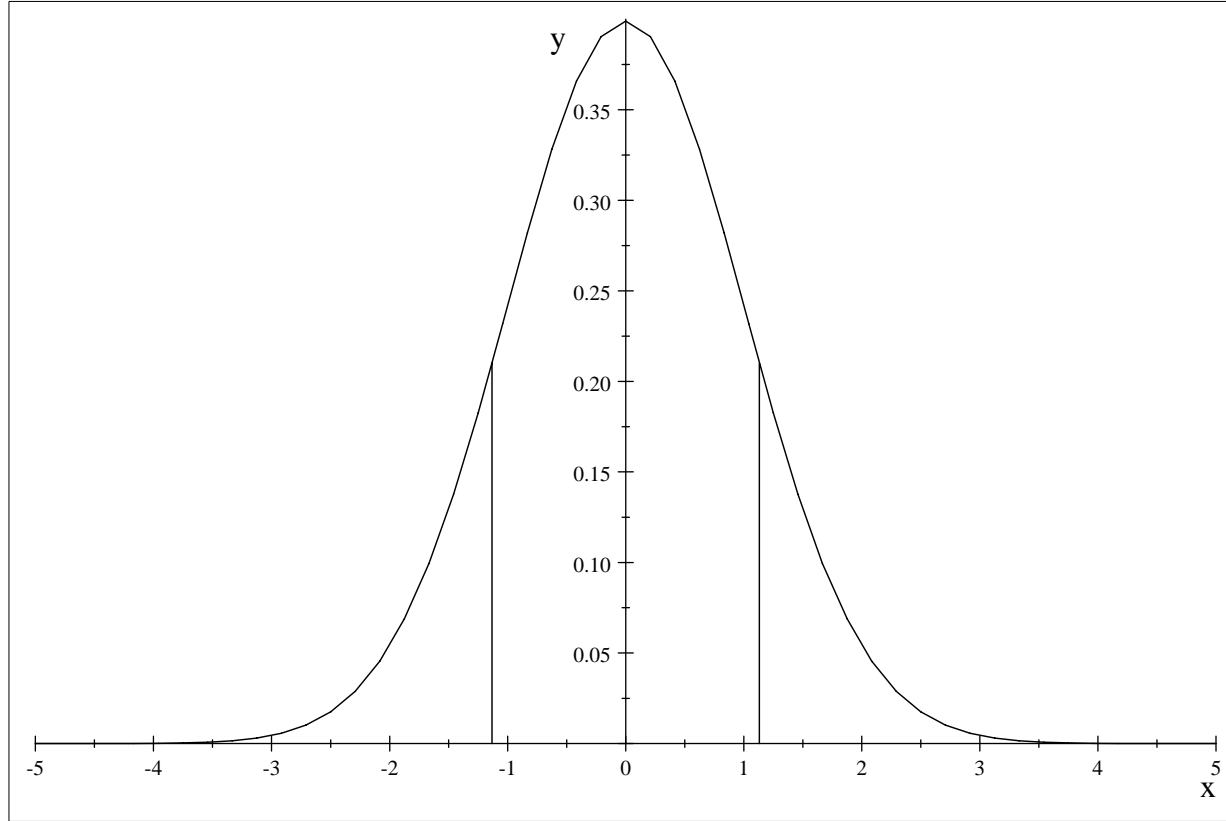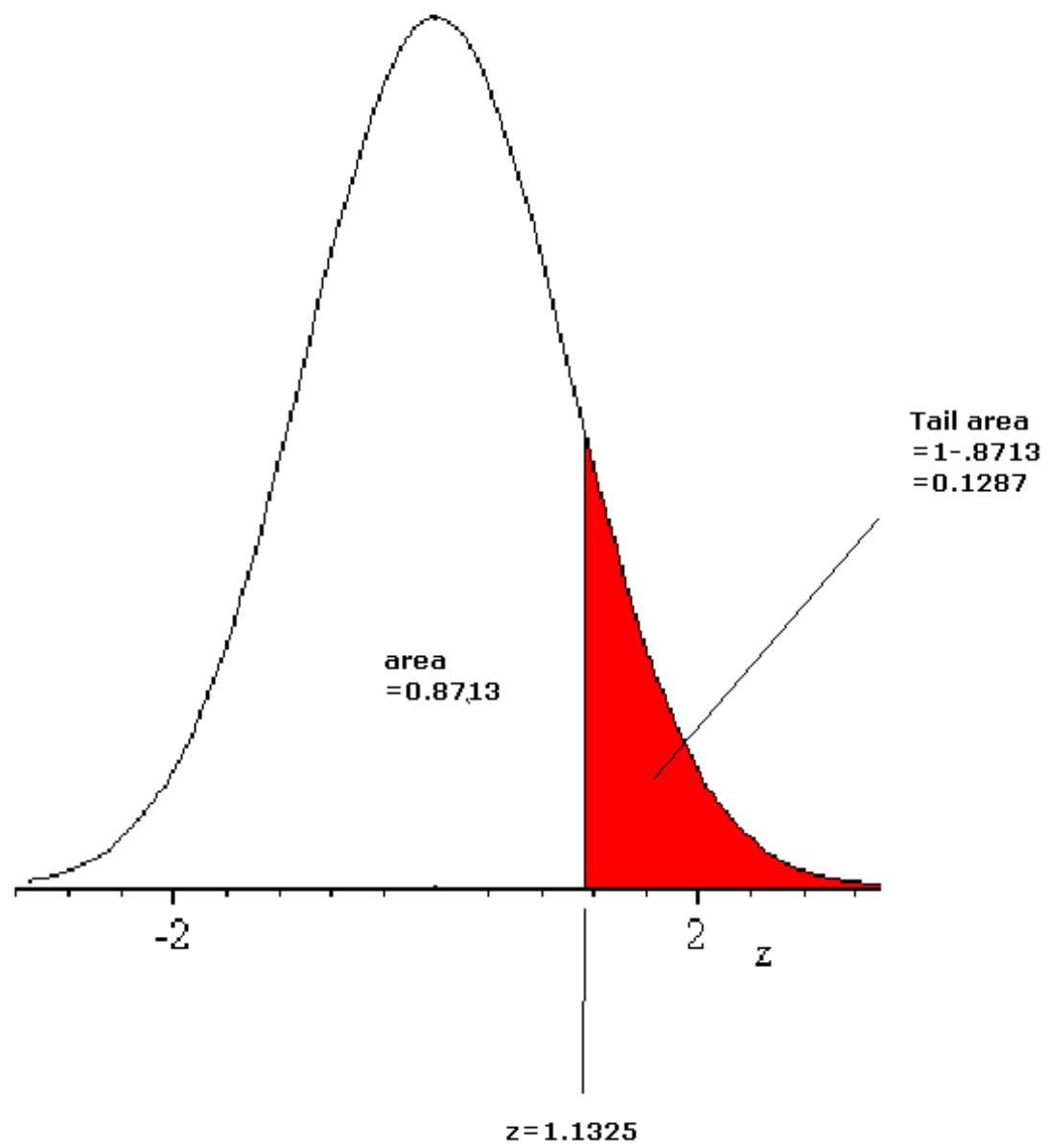$$z = \frac{2151 - 2000}{\left(\frac{800}{\sqrt{36}}\right)} = 1.1325$$

c)

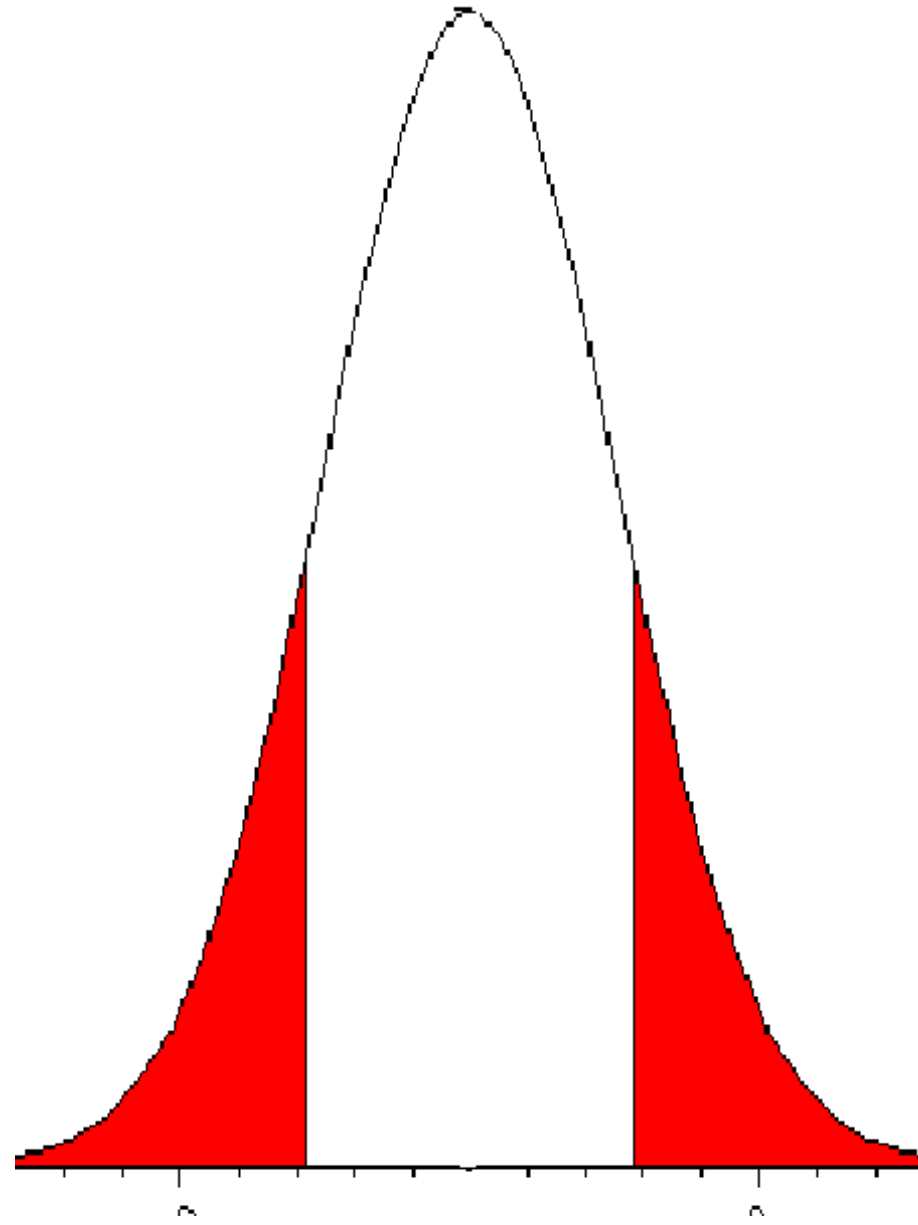**Look at the tail area**

$$\frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

y

0.35

0.30

0.25

0.20

0.15

0.10

0.05

-5   -4   -3   -2   -1   0   1   2   3   4   5

x

Tail area
=1-.8713
=0.1287

area
=0.8713

-2

2

z

z=1.1325

Remember that it is a two tailed test (alternative is ≠)

Therefore the P_value is made up of both the tails

The P_value is $2 \times .1287 = 0.2574$

d)
Since $0.2574 > 0.05$

we do not reject the null hypothsis

we do not have a good evidence at 5% level of significance that the mean under consideration is now different from $2000.00

e)

To construct a 95% confidence interval based on this sample mean of $\$2151$

the margin of error for 95% confidence level is

$$1.96\frac{800}{\sqrt{36}} = \$261.33$$

therefore a 95% confidence interval is

$(2151 - 261.33, 2151 + 261.33) = (1889.67, 2412.33)$

f)

This interval does include the value $2000

g)

For a two tailed test:

When the null is NOT rejected at 5% level of significance, **a 95% confidence interval includes the value of the population mean that the null states** (**which is $2000.00 in this example.**)

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Here is an alternate method to accept or reject the null hypothesis.

It is called the critical value approach:

Works as shown below, **we shall use the critical value approach**

mostly in

Recall:

$z_{.05}$ **is the value of z such that the area to the right of this value is .05**.

We had found that

$z_{.05}$= **1.645**
$z_{.025} = 1.96$

If the alternative is < **(less than)** , **and the level of significance is** α, **then we reject the null hypothesis when the test statistic is less than** $-z\alpha$

If the alternative is $>$ (greater than) , and the level of significance is α, then we reject the null hypothesis when the test statistic is greater than z_α.

If the alternative is $\neq$ (Not equal to) , and the level of significance is α, then we reject the null hypothesis when the test statistic
is less than $-z_{\alpha/2}$ or is greater than $z_{\alpha/2}$

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Hypothesis test for population mean when σ is not known

In fact this is the case that we shall see much more often than the case when σ is known

If we take simple random samples of size n from a normal population with mean
μ then $\dfrac{\overline{x} - \mu}{\left(\frac{s}{\sqrt{n}} \right)}$ has a t-distribution with $n - 1$ degrees of freedom.

Formulate the null and the alternative
Compute the test statistic
Find or estimate the P_value
If the P_value is very small by your standards, reject the null

OR if a level of significance α is given, reject the null hypothesis if the
P_value is less than α
If the null is rejected, we say that the results are statistically significant

Critical value approach

at $n - 1$ **degrees of feedom**

If the alternative is $<$ **(less than) , and the level of significance is** α, **then we reject the null hypothesis when the test statistic is less than** $-t_\alpha$

If the alternative is $>$ **(greater than) , and the level of significance is** α, **then we reject the null hypothesis when the test statistic is greater than** t_α.

If the alternative is $\neq$ **(Not equal to) , and the level of significance is** α, **then we reject the null hypothesis when the test statistic**
is less than $-t_{\alpha/2}$ **or is greater than** $t_{\alpha/2}$

....

Example 1:

The label on a cough medication bootle says that the alcohol contents are
20 ml. A doctor suspects this claim because she thinks that they contain more alcohol.
Assume a normal population for the alcohol contents and that In 20 randomly selected
bottles she finds the mean content of alcohol as 24.19 ml with standard deviation 3.19 ml.

Note that σ **is not known in this case**, **3.19 ml is the standard deviation of the sample of size 20.**

a) State the null hypothesis
$\mathbf{H}_O$: μ = 20
b) State the alternative hypothesis

$\mathbf{H}_A$: μ > **20**

c) **Compute the test statistic**

$$t = \frac{\overline{x} - \mu}{\left(\dfrac{s}{\sqrt{n}}\right)}$$

$$t = \frac{24.19 - 20}{\left(\dfrac{3.19}{\sqrt{20}}\right)} = 5.87405945 \qquad\qquad \textbf{df is 19}$$

P_value is the area to the right of t=5.87405945

d) **Report your conclusion at 5% level of significance**.

$$.05$$

$$\downarrow$$

19 → **1.729**

5.8741 > 1.729 **the value of** $t_{.05}$ **at 19 degrees of freedom**
area to the right of t=1.729 is 0.05.
Area to the right of t=5.87405945 **is less than .05**
Reject the null
Have a good evidence that the alcohol contents are
more than 20 ml on an average.

Or may use a TI83plus calculator as shown below

Press the STAT key

Select TESTS

ttest

```
TEXAS INSTRUMENTS    TI-83 Plus

T-Test
 µ>20
 t=5.874059452
 P=5.87141E⁻6
 x̄=24.19
 Sx=3.19
 n=20
```
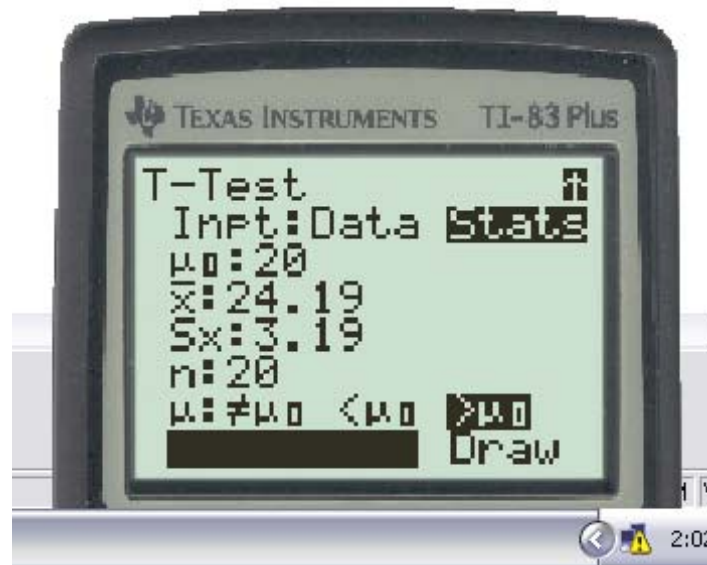
Note that the P_value is 0.00000587141 which is much smaller than 0.05. therefore the null hypothesis is rejected.

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Example 2:

**Contents in the bottles of an expensive medication show that the average amount
in the bottles is 500 ml. A pharmacy suspects that the bottles are being underfilled**.

Null: $\mu = 500$ **ml**

Alternative: $\mu < 500$ **ml**

A simple random sample of 25 bottles shows a mean of 497 ml and standard deviation 4.18 ml.

c) Find the test statistic $\quad t = \dfrac{\overline{x} - \mu}{\left(\dfrac{s}{\sqrt{n}}\right)}$

$t = \dfrac{497 - 500}{\left(\dfrac{4.18}{\sqrt{25}}\right)} = -3.58851675$

P_value is the area to the left of t$= -3.58851675$
df $\quad 25 - 1 = 24$
d) Report your conclusion at 1% level of significance.

$$.01$$

$$\downarrow$$

$$24 \qquad \rightarrow \qquad 2.492$$

area to the left of $t = -2.492$ **is .01**
Since $-3.58851675 < -2.492,$
the P_value<.01
Reject the null

Or may use a TI83plus calculator as shown below

Press the STAT key

Select TESTS

ttest
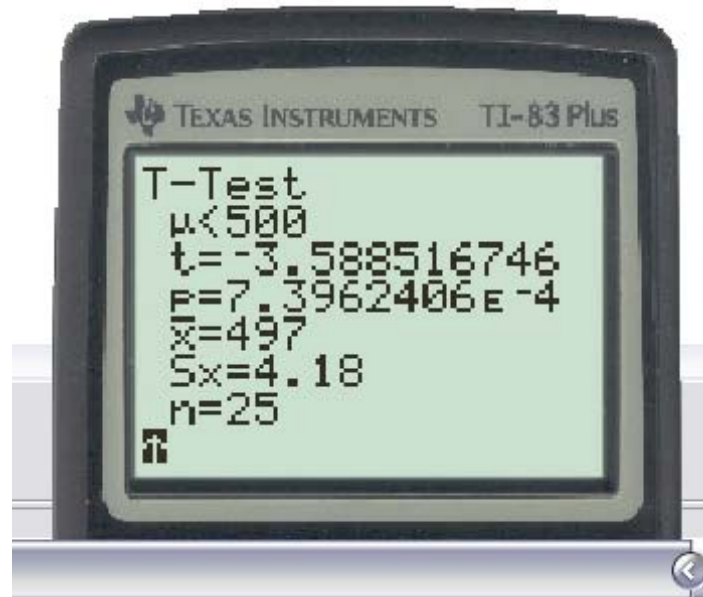
P_value is 0.00074<0.01

Example 3:

The union of the inflight service of a certain customer friendly airline claims that the diastolic blood pressure of the inflight service workers is higher than 80 mmHg. To test this claim, the diastolic blood pressure of 35 workers is measured and for these 35 workers, $\bar{x} = 83.5$ mmHG and $s = 3.7$ mmHg. At 5% level of significance, does this sample show a good evidence that the diastolic blood pressure of the inflight service workers is higher than 80 mmHg?

Note here that

H_O: μ = 80

H$_A$: μ > 80

$$t = \frac{\overline{x} - \mu}{\left(\frac{s}{\sqrt{n}}\right)}$$

or

$$t = \frac{83.5 - 80}{\left(\frac{3.7}{\sqrt{35}}\right)} = 5.596291686715853013\,3$$

the degrees of freedom is

35 − 1 = 34

Note from the t-table that

.05

↓

34   →   1.6909

which means that for 34 degrees of freedom, $t_{.05} = 1.6909$

Since, the test statistic $t = 5.596291686715853013\,3 > t_{.05}$
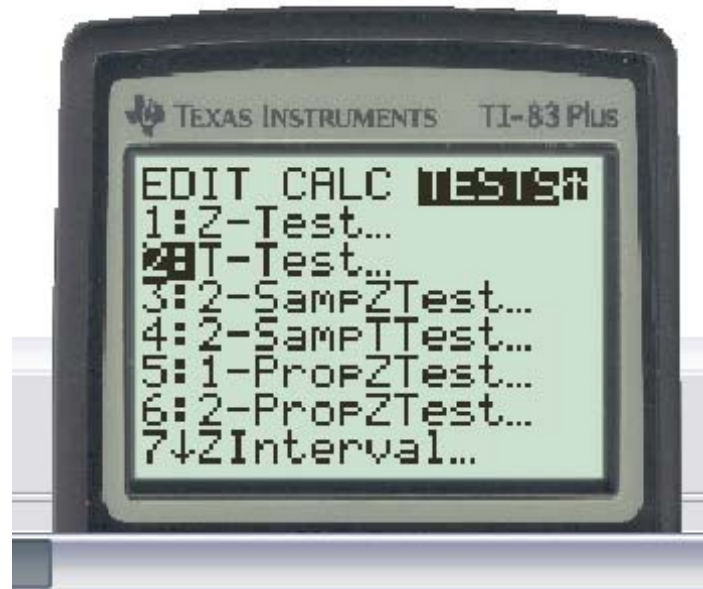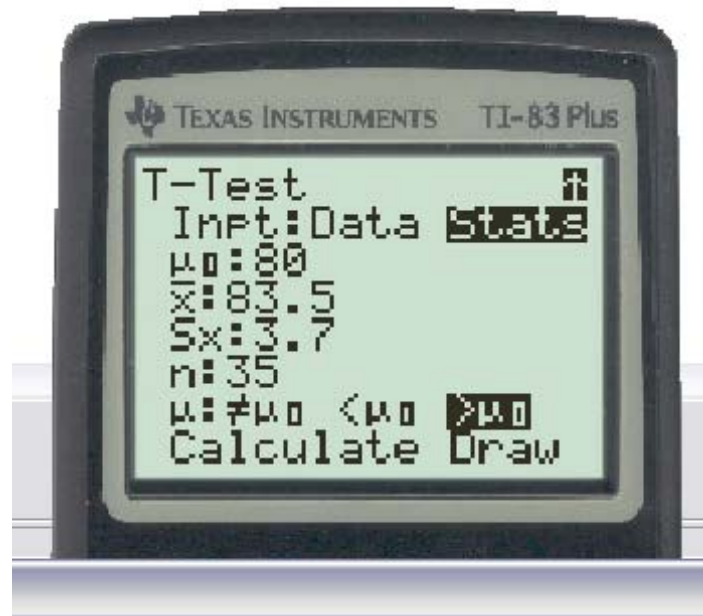
reject the null hypothesis, the sample provides a good evidence

that the diastolic
blood pressure of the inflight service workers is higher than 80 mmHg.
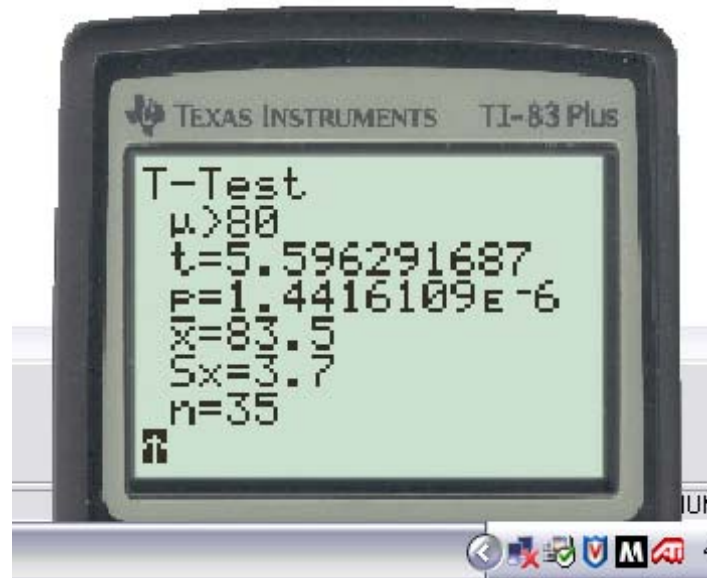
Or may use a TI83plus calculator as shown below

Press the STAT key

Select TESTS

ttest

The P_value is 0.0000014416

_____

Hypothesis Test for Population Proportion:

Formulate the null : H_O
Formulate the alternative: H_A

select a simple random sample of the desired size (say n)

Compute the test satistic

$$z = \frac{\hat{p} - p}{\sqrt{\dfrac{p(1-p)}{n}}}$$

$\hat{p}$: **the sample proportion**
p: **the population proportion that is suggested by the null hypothesis**
n: **the sample size**

Then compute the P_value, **if the P_value is very small**,
then reject the null hypothesis.

If a level (**level of significance**) **is set for the test**,
then reject the null if the P_value is smaller
than the level of significance.

Example 1:
The manufacturer of an inexpensive "**walkman**" **CD player claims that no more than 10% of the product**
are below standard. **Find the probability that more than 29 of 200 randomly selected products of this type**
will be substandard.

That is
if we take $p =.10$ (**here we mean that the population proportion is 0**.**10**)
then to find the probability that
$\hat{p} > \dfrac{29}{200} = 0.145$

or in other words, **we have**

Null Hypothesis: $H_O : p =.10$
Alternative Hypothesis: $H_O : p >.10$

Test Statistic: $z = \dfrac{\hat{p} - p}{\sqrt{\dfrac{p(1-p)}{n}}}$

Test statistic: $z = \dfrac{.145 - .10}{\sqrt{\dfrac{.10(1 - .10)}{200}}} = 2.12132034$

The P_value is the probability that z>2.12

$1 - .983 = 0.017$

If the P_value is very small, reject the null hypothesis.

Reject the claim that p=0.10

If a level of significance α is set for a test, reject the null if the P_value is smaller than α

Example2: A popular medication to treat depression is believed to give 15% of the patients "insomnia."
A variation in that medication is supposed to reduce such a proportion.
To check this, the new medication is given to 250 randomly selected patients
and it is found that 23 of them had insomnia.
Carry out a hypothesis test to see if you can reject the
null hypothesis $(H_O : p = .15)$ at the level $\alpha = .01$

$\mathbf{H}_O$**: p** $= 0.15$
$\mathbf{H}_A$**: p** < 0.15

Test Statistic:
$\hat{p} = \dfrac{23}{250} = 0.092$

$$z = \dfrac{.092 - .15}{\sqrt{\dfrac{.15(1 - .15)}{250}}} = -2.56828317$$

P_value is the probability that z<-2.56828317
From the z-table, **this probability is 0.0051**
Since 0.0051$<$**.01**
Reject the null that $(H_O : p = .15)$

We are saying that we have a good evidence that the proportion of people that get insomnia by using the new medication is less than 0.15.

Example3:

It is believed that 58% of the of high school senios in a cettain metro area would like to attend big schools for their degree if they can manage the "money" for it.
To see if this proportion has changed, a random sample of 371 high school seniors is taken and 191 of them said that they would ike to attend big schools for their degree if they can manage the "money" for it. Carry out a hypothesis test at 5% level to see that if the sample is a good evidence for the change in the aforesaid proportion.

$\mathbf{H}_O$: $\mathbf{p = 0.58}$
$\mathbf{H}_A$: $\mathbf{p \neq 0.58}$

$\hat{p} = \dfrac{191}{371} = \mathbf{0.514824798}$

Test Statistic $z = \dfrac{0.514824798 - .58}{\sqrt{\dfrac{.58(1 - .58)}{371}}} = -2.54349389$

Here the P_value consists of both the tails

Probability that z<-2.54 **is .0055** (from the table)
P_value is $2 \times .0055 = 0.011$

0.011$<$**.05**

Reject the null

Do see a good evidence at 5% level that the aforesaid proportion has changed.

Example:

The manufacturer of an inexpensive walkman claims that no more than 10% of their product is substandard. To check this claim a simple random sample of size 300 is taken. In this simple random sample of size 300 of these products,47 are found to be substandard. Carry out a hypothesis test at 5% level of significance to see if the sample is a good evidence that more than 10% are defective.

Testing for population proportion:

$H_O : p = .10$ (**or** $p \leq .10$)
$\mathbf{H}_A: \mathbf{p} > .\mathbf{10}$

In the sample $\overline{p} = \frac{47}{300} = 0.15667$

Test statistic:

$$\mathbf{z} = \frac{\overline{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

$$z = \frac{.15667 - .10}{\sqrt{\frac{.10(1-.10)}{300}}} = 3.2718$$
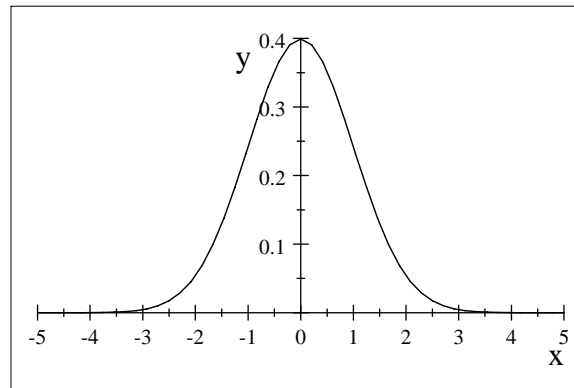
Critical value approach:

Reject the null if the computed z$>z_{.05}$, $z_{.05} = 1.645$

Here, **you have** $3.2718 > 1.645$, **therefore reject the null hypothesis at 5% level**.

P_value approach:

P_value is the area under the z-curve to the right of z=3.2718

$$s(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

The P_value is approximately 0

or

$$\int_{3.2718}^{\infty} s(z)dz = 5.3433 \times 10^{-4}$$

The P_value$<$.05

Reject the null at 5% level.

The sample shows a good evidence that the proportion of substandards is higher than 0.10

_____

review of calculation of $z_{.01}$

$.5 - .01 = 0.49$

Look for the value .49 in the area portion of the table

$$.03$$

$$\uparrow$$

$$2.3 \qquad \leftarrow \quad .4901$$

$z_{.01} = 2.33$

_____

Example 2:

A brand of cough medication claims that no more than 10 ml of a 500 ml bottle has alchohol in it. To see if this claim is correct a simple random sample of 15 bottles is taken and the alchohol in it is measured. For this sample $\overline{x} = 16.7$ **ml and** $s = 3.41$ **ml. At 1% level of significance, does the sample provide a good evidence that the alchohol contents in 500 ml bottles is higher than 10 ml on an average?**
(assume that the alchohol contents are normally distributed)

H_O: μ = 10
H_A: μ > 10

Do not know σ, and the sample size is small,
use t as the test statistic

$$t = \frac{\overline{x} - \mu}{\left(\frac{s}{\sqrt{n}}\right)} \qquad \text{degrees of freedom } n - 1$$

$$\mathbf{t} = \frac{16.7 - 10}{\left(\frac{3.41}{\sqrt{15}}\right)} = \mathbf{7.6097}$$

Reject the null if $t > t_{.01}$ **df** $15 - 1 = 14.0$
t-table

$$.01$$

$$\downarrow$$

14 $\qquad \rightarrow \qquad$ **$t_{.01} = 2.624$**

$7.6097 > 2.624$, **reject the null hypothesis**

Example 3:

It is believed that the mean of the finish time for a standard run for high school students is 18.45 minutes. A shoe supplier claims that if the students put their specific make on, they will run faster. They even give 10 pairs of shoes to a school district free. School district selects 10 students randomly and finds that $\overline{x} = 17.9$ minutes and $s = 3.86$ minutes. Carry out a hypothesis test to determine if this sample is a good evidence to show that the students who put these kind on run faster. Use 1% level of significance.

$H_O : \mu = 18.45 \qquad\qquad (\mu \geq 18.45)$
$\mathbf{H}_A: \mathbf{\mu < 18.45}$

assuming normal distribution for the population

$$t = \frac{17.9 - 18.45}{\left(\dfrac{3.86}{\sqrt{10}}\right)} = -0.45058 \quad \textbf{df is } 10 - 1 = 9.0$$

reject the null if $t < -t_{.01}$ **at 9 df**

$$.01$$

$$\downarrow$$

9 $\qquad \rightarrow \qquad$ $\mathbf{t}_{.01} = \mathbf{2.821}$

$-\mathbf{t}_{.01} = -\mathbf{2.821}$

-0.45058 **is NOT less than** -2.821
do not reject the null

we do not see a good evidence for improvement in time taken to finish.

_____

When we make a conclusion about the null hypothesis based on a level of significance,
we may make mistake of

either rejecting a true null hypothesis

OR

not rejecting a false null hyopthsis

The conclusions may be of different forms as indicated in the following table

	Null is true	Null is not true
Null Rejected	Very bad , type I error	OK
Null not Rejected	Very good	bad, type II error

Note that the level of significance is the maximum probability of making type I error.

Example 4:

The owner of sncak/soda vending machines (1000s of them) in a large region submits that he gets a revenue of $75 per day from each machine. The tax authorities suspect this claim and take a simple random sample of 50 machines , record revenues on randomly selected days and find that $\overline{x} = \$81.36$ from each machine. Use $\sigma = \$39.16$

and carry out a hypothesis test to determine if this sample shows a good evidence that the vendor is making more money than he is stating. Use 1% level of significance.

Ayaz:

$\mathbf{H}_O\text{: }\mu = 75 \qquad\qquad (\mu \leq 75)$

$\mathbf{H}_A\text{: }\mu > 75$

$$z = \frac{81.36 - 75}{\left(\frac{39.16}{\sqrt{50}}\right)} = 1.1484$$

$z_{.01} = 2.33$, **and** 1.1484 **IS NOT GREATER THAN 2.33**
Do not reject the null hypothesis.
The p_value is $.5 - .3749 = 0.1251$ **not less than .01**
do not reject the null

_____

We would always like to keep the probability of making a type I error small,

**but at the same time we should see that the probability of making a type II error is small
for nujll hypotheses that are very far from the true value of** μ

To see how the probability of making a type II error depends on the true value of μ,
let us look at the following example

Example 5:

The owner of sncak/soda vending machines (1000s of them) in a large region submits that he gets a revenue of $70 per day from each machine. The tax authorities suspect this claim and take a simple random sample of 50 machines to record revenues on randomly selected days . They shall use $\sigma = \$39.16$ **and carry out a hypothesis test to39 determine if this sample shows a good evidence that the vendor is making more money than he is stating. They are using a 1% level of significance**.

Question:

Find the probability of a type II error if in fact $\mu = \$75$

or find the probability that the null is not rejected when in fact $\mu = \$75$

Null is not rejected if $z < z_{.01}$

That is when

$$\frac{\overline{x} - 70}{\left(\frac{39.16}{\sqrt{50}}\right)} < \mathbf{2.33}$$

or

$$\overline{x} - \mathbf{70} < \mathbf{2.33}\frac{39.16}{\sqrt{50}}$$

or

$$\overline{x} < \mathbf{70} + \mathbf{2.33}\frac{39.16}{\sqrt{50}}$$

or

$$\overline{x} < \mathbf{82.904}$$

$$\mathbf{t(u)} = \frac{1}{\left(39.16/\sqrt{50}\right)\sqrt{2\pi}}\mathbf{e}^{-\left((u-70)^2/\left(2\times\left(39.16/\sqrt{50}\right)^2\right)\right)}$$

$$t(u)$$
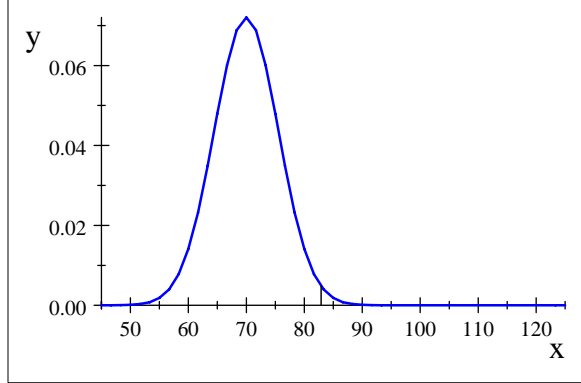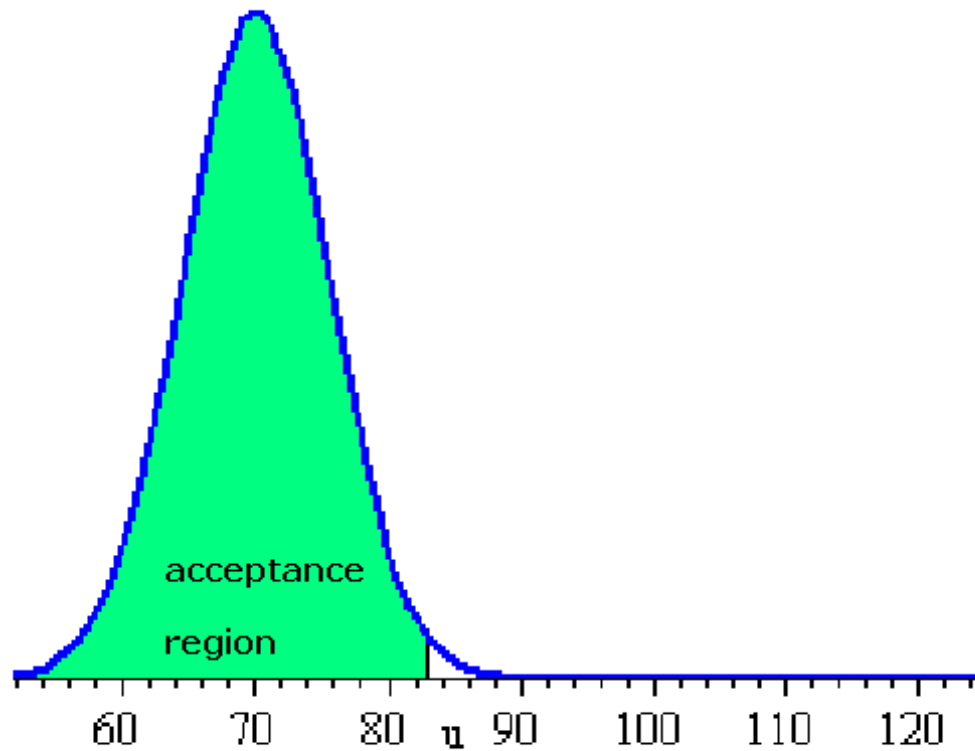
The probability of the type II is the probability that
$\bar{x} < 82.904$ **when actually** $\mu = 75$

or the probability that
$$z < \frac{82.904 - 75}{\left(\frac{39.16}{\sqrt{50}}\right)} = 1.43$$

The area under the standard normal curve below 1.43 is 0.9236

therefore the probbaility of making a type II error when $\mu = 75$ **is 0.9236**

0.9236

Null

actual population
mean
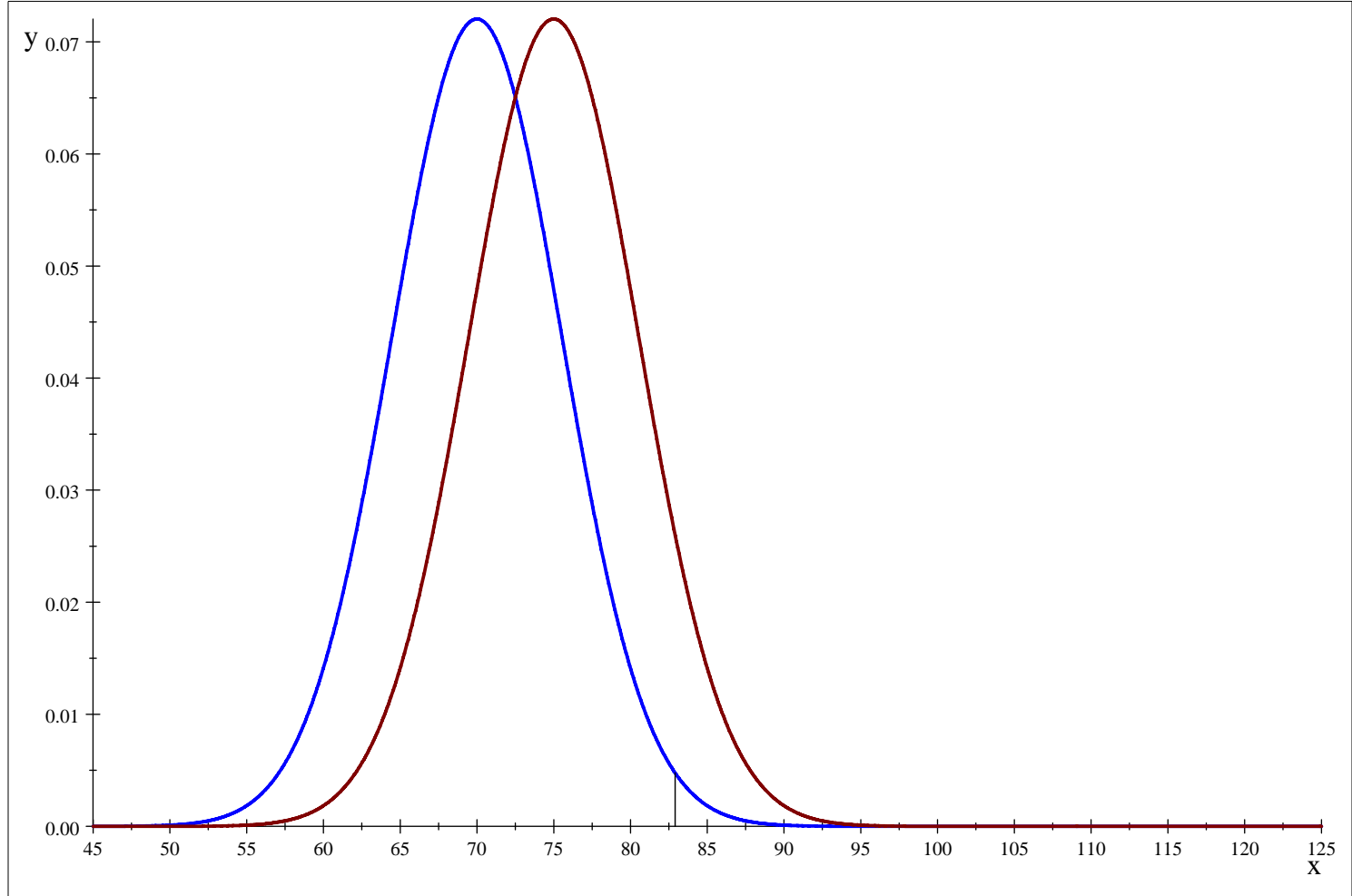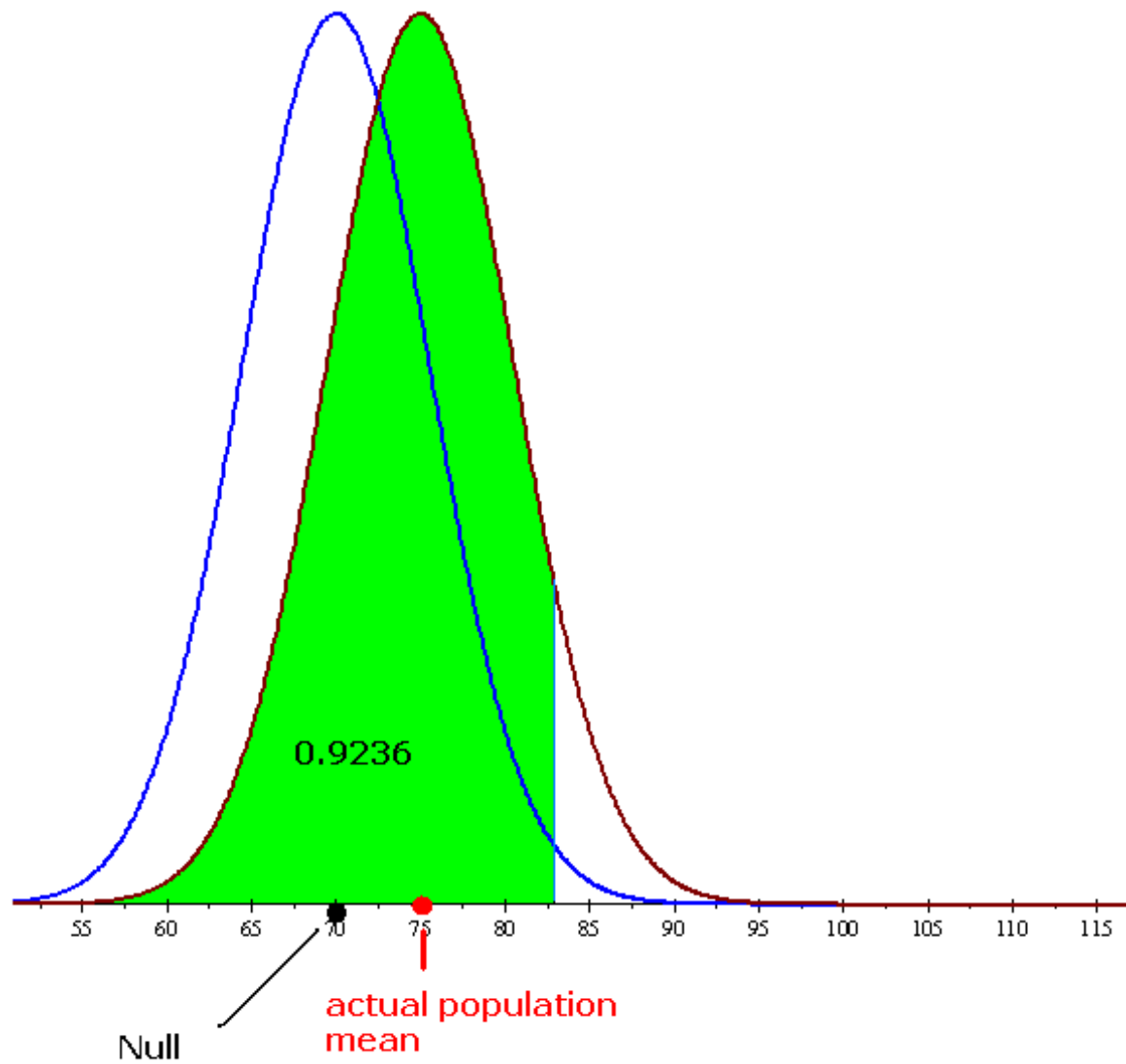
The probability of the type II eroor when in fact $\mu = 80$

is that of

$$z < \frac{82.904 - 80}{\left(\frac{39.16}{\sqrt{50}} \right)} = 0.52$$

the area under the standard normal curve to the left of z=0.52 is 0.6985

therefore the probability of making a type II error in such a situation is 0.6985

0.6985

Null

actual population mean
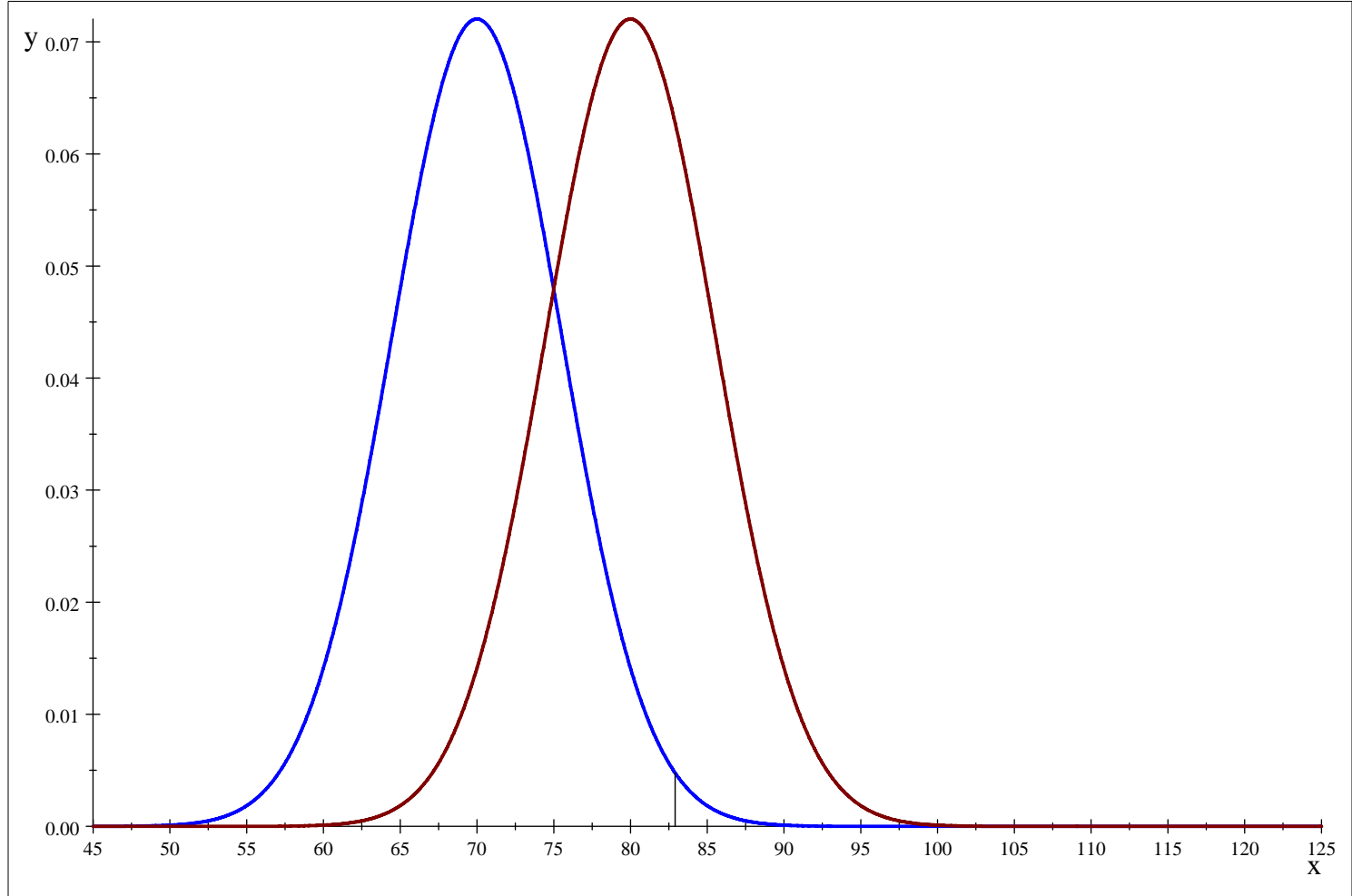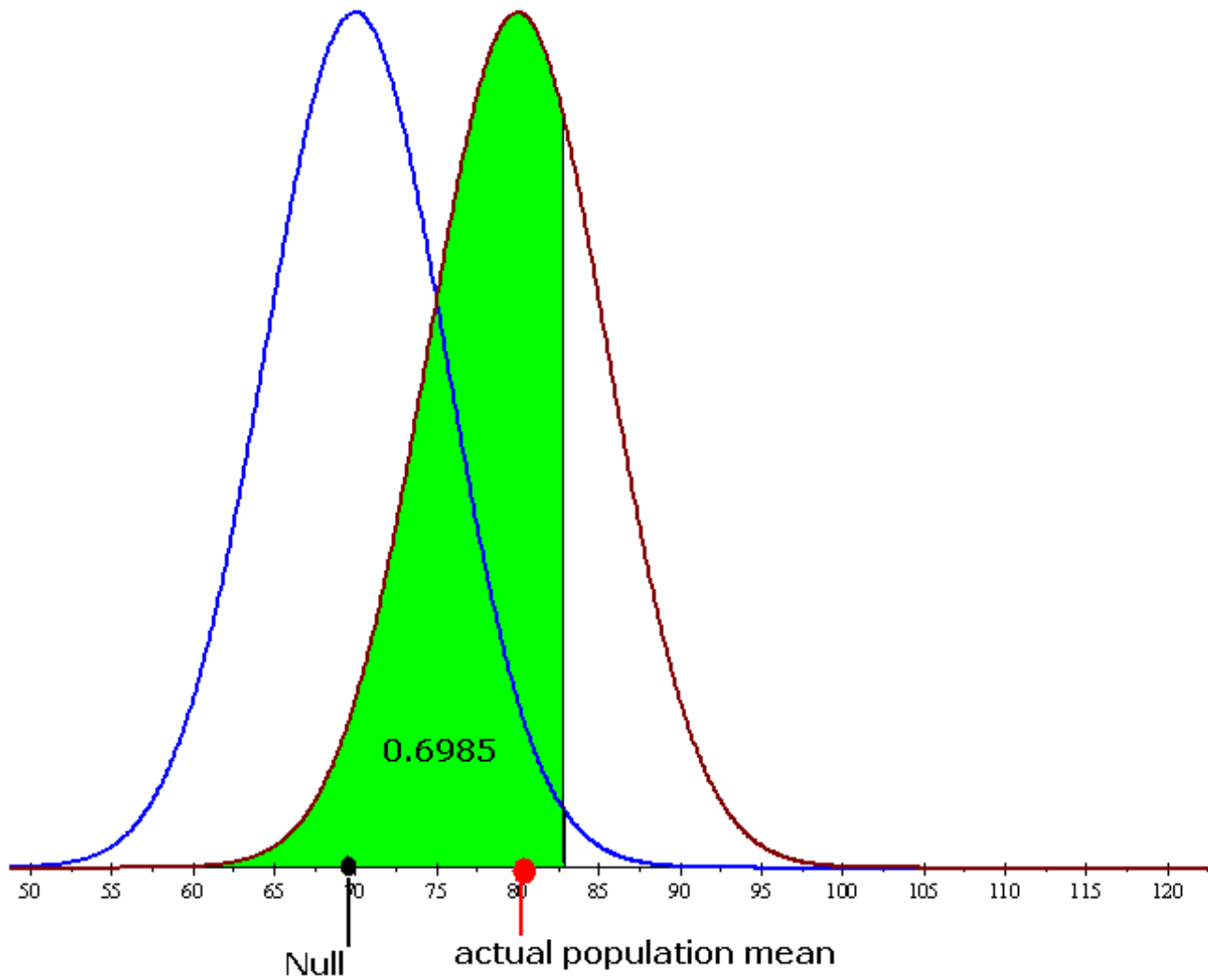
In case, we have the actual value of $\mu = 100,$
the probability of a type II error is

the probability of

$$z < \frac{82.904 - 100}{\left(\frac{39.16}{\sqrt{50}}\right)} = -3.087$$

which is smaller than 0.001

True Population
Mean
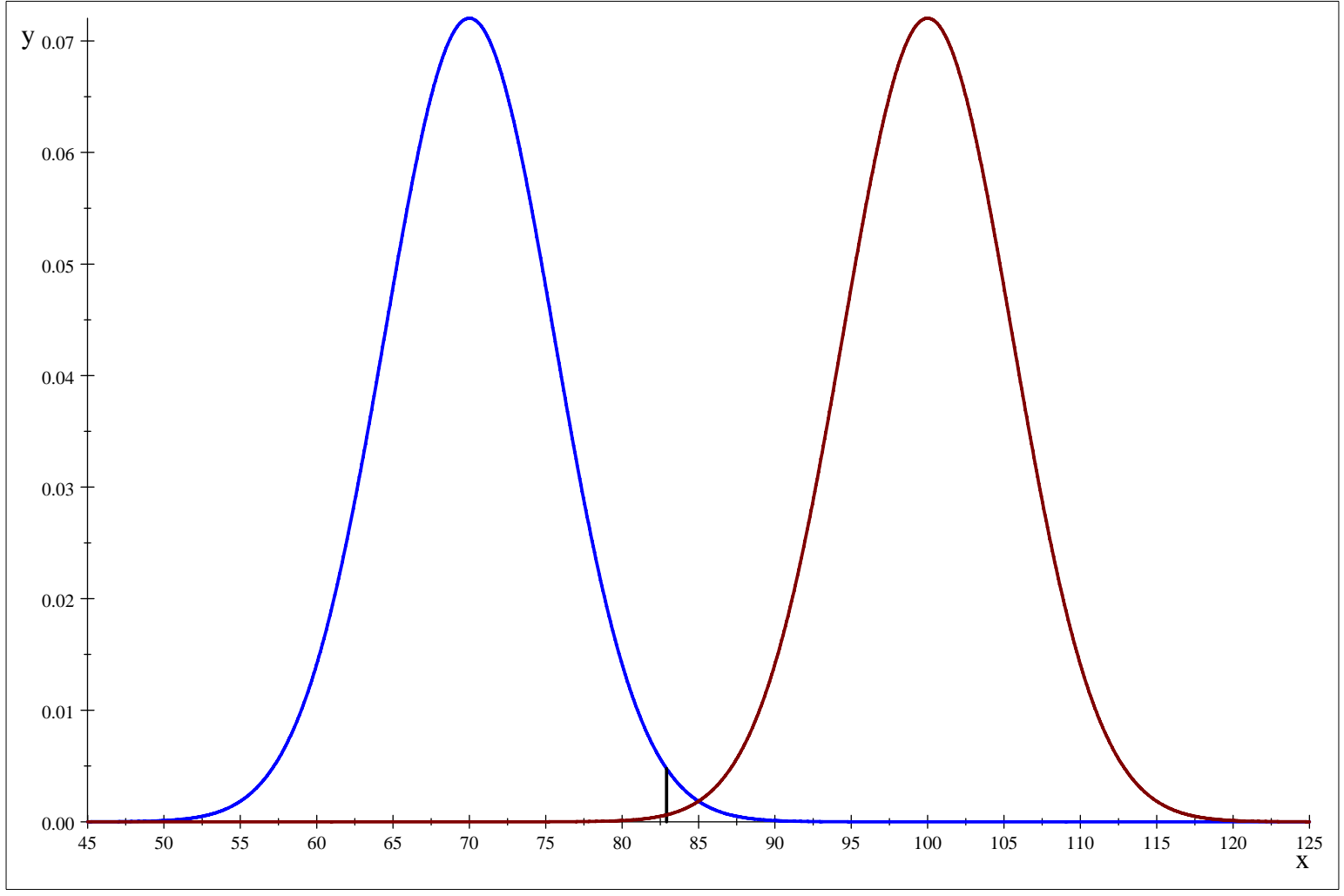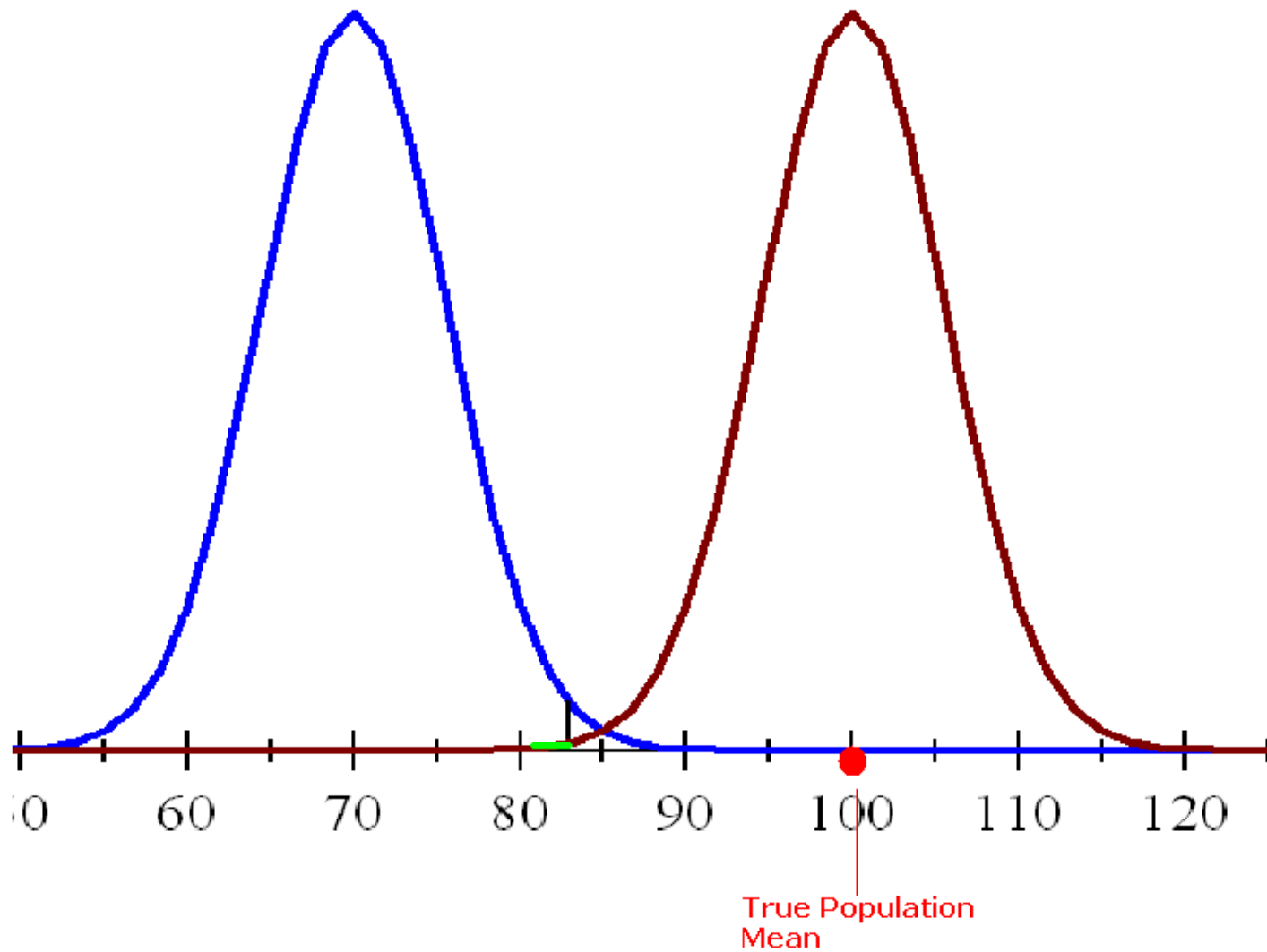
or we can use Excel to obtain a list of values of the type II error

depending on the various values of μ

actual mu	Beta
75	0.923240962
80	0.69998985
85	0.352540046
90	0.100041515
95	0.014475312
100	0.001010933
105	3.30583E-05

Power of a test $= 1-$ The probability of the type II error

We can plot the power curve to see that
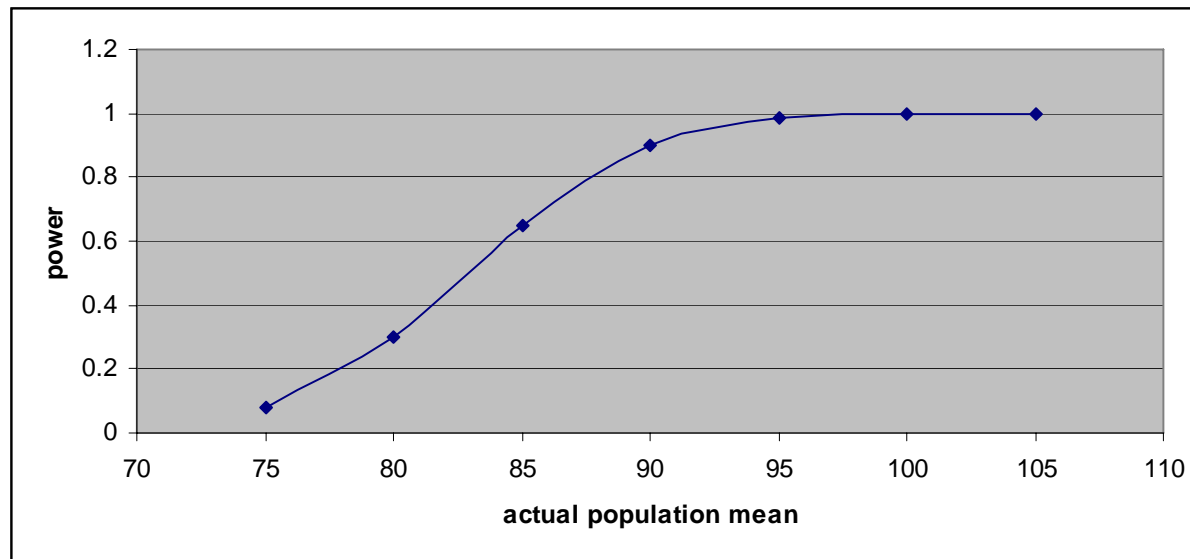
the power of this test higher for the value of μ that is much higher from the value stated in thenull hypothesis.