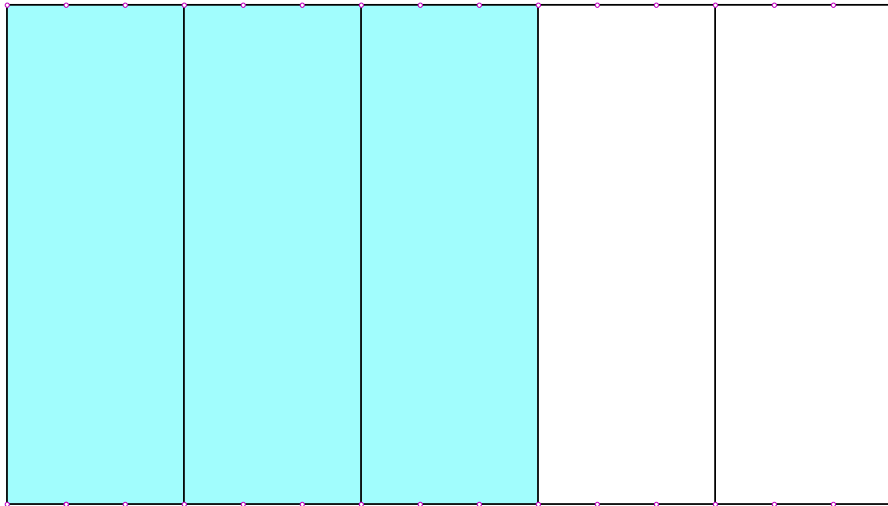


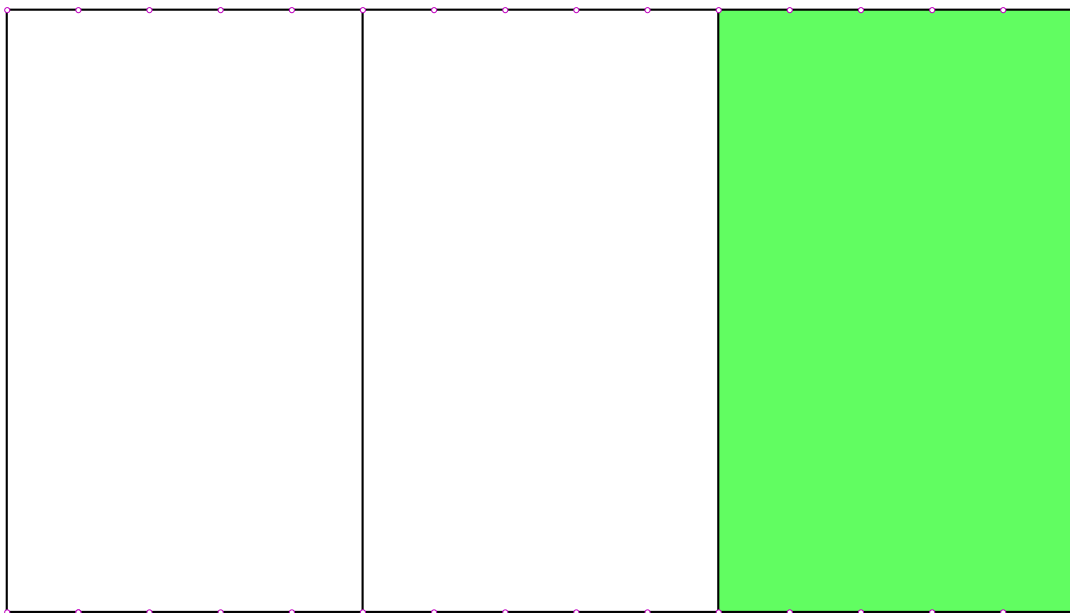
Fractions with illustrative examples

Relate the following with $\frac{3}{5}$



The colored portion is $\frac{3}{5}$, 3 out of 5 equal portions of the whole.

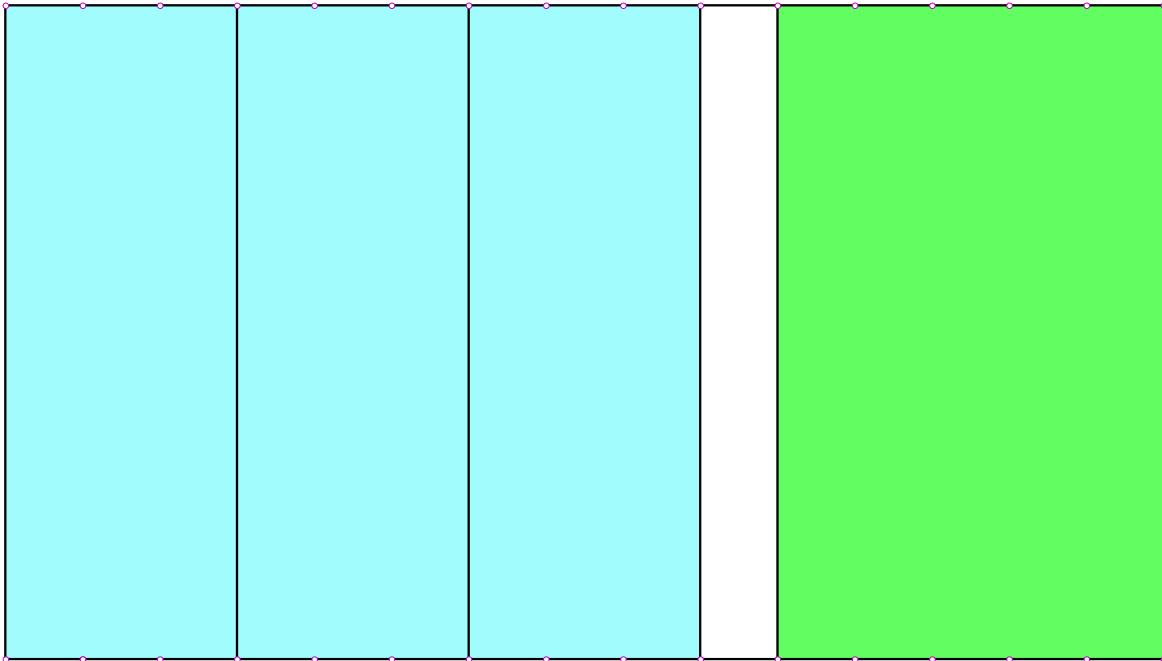
The colored portion in the following is $\frac{1}{3}$ that is 1 out 3 equal portions of a whole



If we have to determine the value of $\frac{3}{5} + \frac{1}{3}$

that is

the blue and the green area together in the following picture:

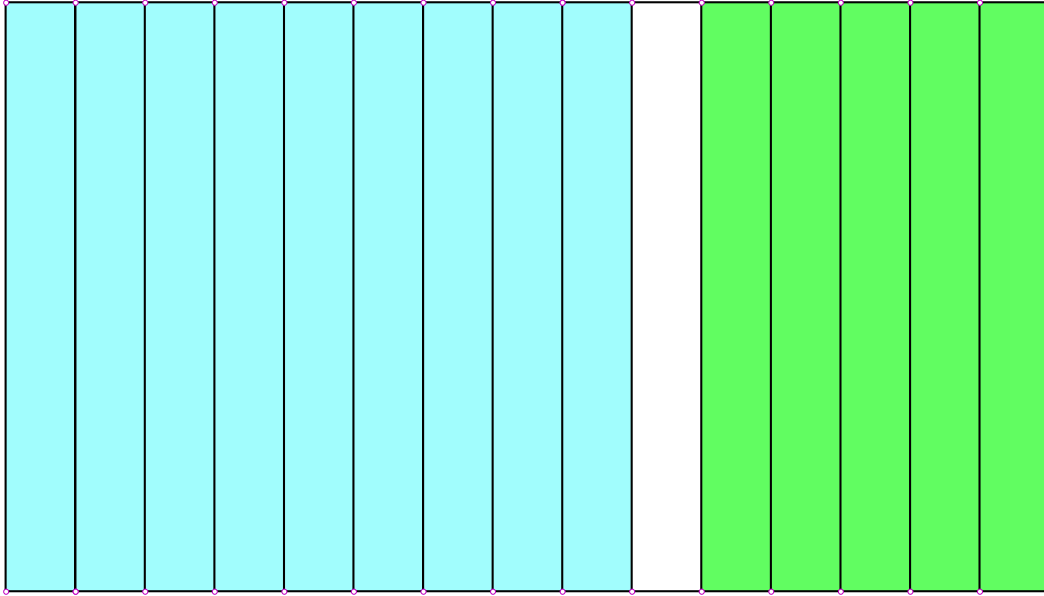


We can do so by

first taking the lowest common multiple of the two denominators that is 3 and 5:

The least common multiple of 3 and 5 is $3 \times 5 = 15$

Then divide the rectangle into 15 equal segments



Note that the two regions together are $\frac{14}{15}$

We could do it arithmetically by using the following process

$$\begin{aligned}
 & \frac{3}{5} + \frac{1}{3} \\
 & \text{denominator} \\
 & = \frac{3 \times 3}{5 \times 3} + \frac{1 \times 5}{3 \times 5} \\
 & = \frac{9}{15} + \frac{5}{15} \\
 & = \frac{14}{15}
 \end{aligned}$$

note that 15 is a common

Example 2:

Adding rational numbers

To add

$\frac{1}{12} + \frac{3}{8}$: Note that $12 \times 8 = 96$ is a common denominator and will still give us the correct answer, but a good idea is to find the lowest common denominator

One way to find a common denominator is to look at prime factorizations of the denominators,

here:

$$\begin{aligned}8 &= 2 \times 2 \times 2 \\12 &= 2 \times 2 \times 3\end{aligned}$$

This suggests that THREE factors of TWOs and one factor of THREE are enough to cover both 8 and 12

That is $2 \times 2 \times 2 \times 3 = 24$

and note that

$$\begin{aligned}24 &= 3 \times 8 \\24 &= 2 \times 12\end{aligned}$$

and we are relieved of the unnecessary excess baggage that we would have to carry by using 96.

NOW LET US ADD

$$\begin{aligned}\frac{1}{12} + \frac{3}{8} &\text{ using 24 as the common denominator (which is the lowest common denominator)} \\&= \frac{2}{24} + \frac{9}{24} \\&= \frac{11}{24}\end{aligned}$$

Example 3:

Add

$$\frac{5}{18} + \frac{7}{15}$$

For the lowest common denominator,
look at

$$18 = 2 \times 3 \times 3$$
$$15 = 3 \times 5$$

To cover both of these, we need ONE factor of TWO, TWO factors of THREE, and ONE factor of FIVE

$2 \times 3 \times 3 \times 5 = 90$ the lowest common multiple of the 8 and 15, or the lowest common denominator in this case

Note that

$$18 \times 5 = 90$$

and

$$15 \times 6 = 90$$

using these

we have

$$\frac{5}{18} + \frac{7}{15}$$
$$= \frac{25}{90} + \frac{42}{90}$$
$$= \frac{67}{90}$$

4. Subtraction:

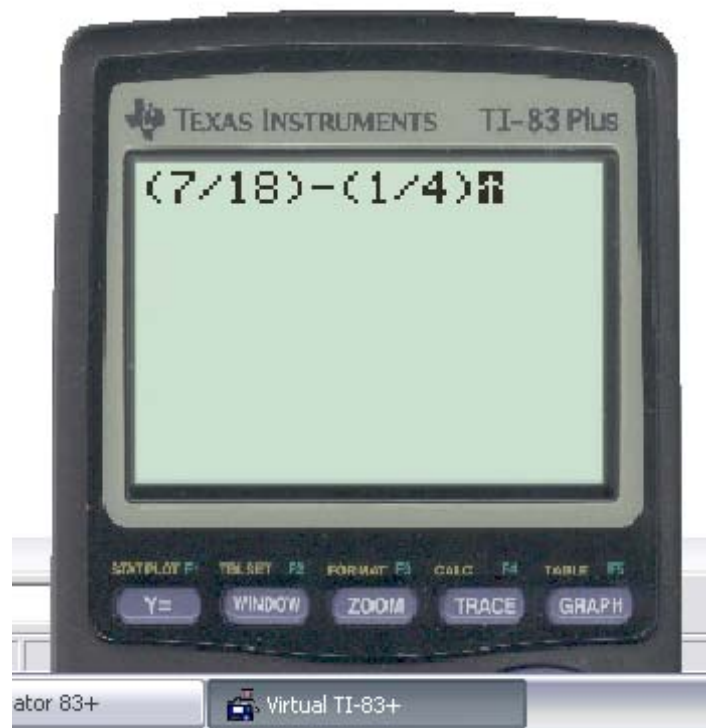
$$\frac{7}{18} - \frac{1}{4}$$
 Note that the lcd here is 36 and $36 = 18 \times 2$, also $36 = 4 \times 9$

therefore we may write

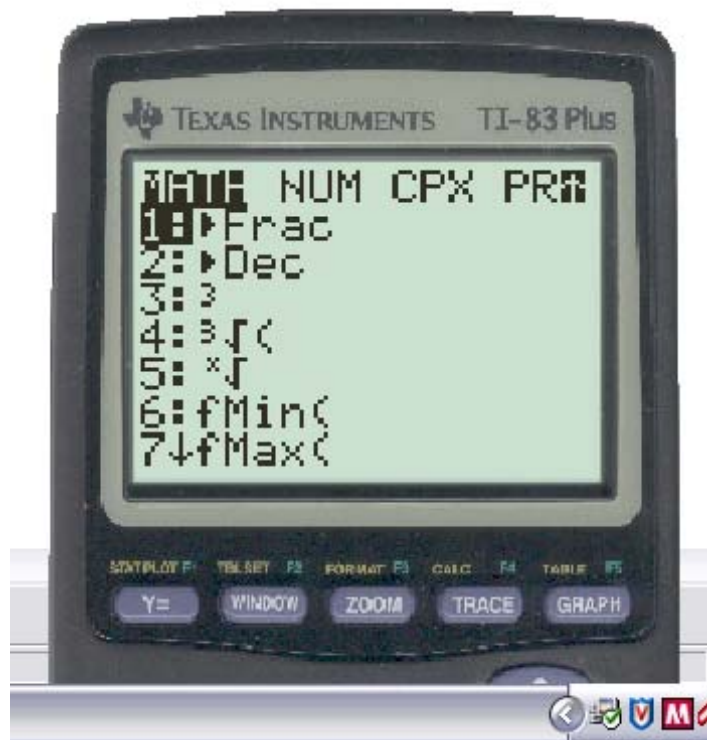
$$\begin{aligned} & \frac{7}{18} - \frac{1}{4} \\ &= \frac{7 \times 2}{18 \times 2} - \frac{1 \times 9}{4 \times 9} \\ &= \frac{14}{36} - \frac{9}{36} \\ &= \frac{14-9}{36} \\ &= \frac{5}{36} \end{aligned}$$

If you are using a TI83plus calculator,

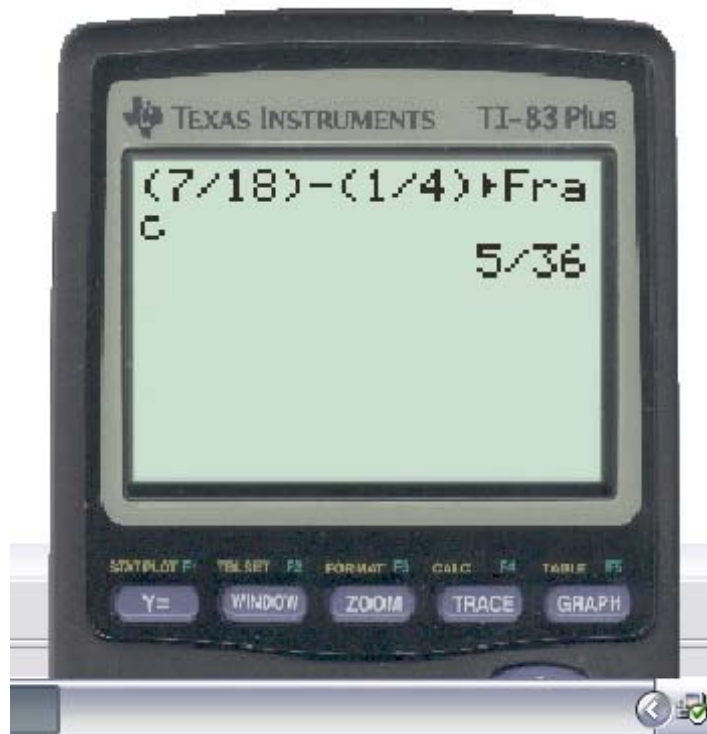
First enter the fractions



Then Press Math Key and SElect Frac

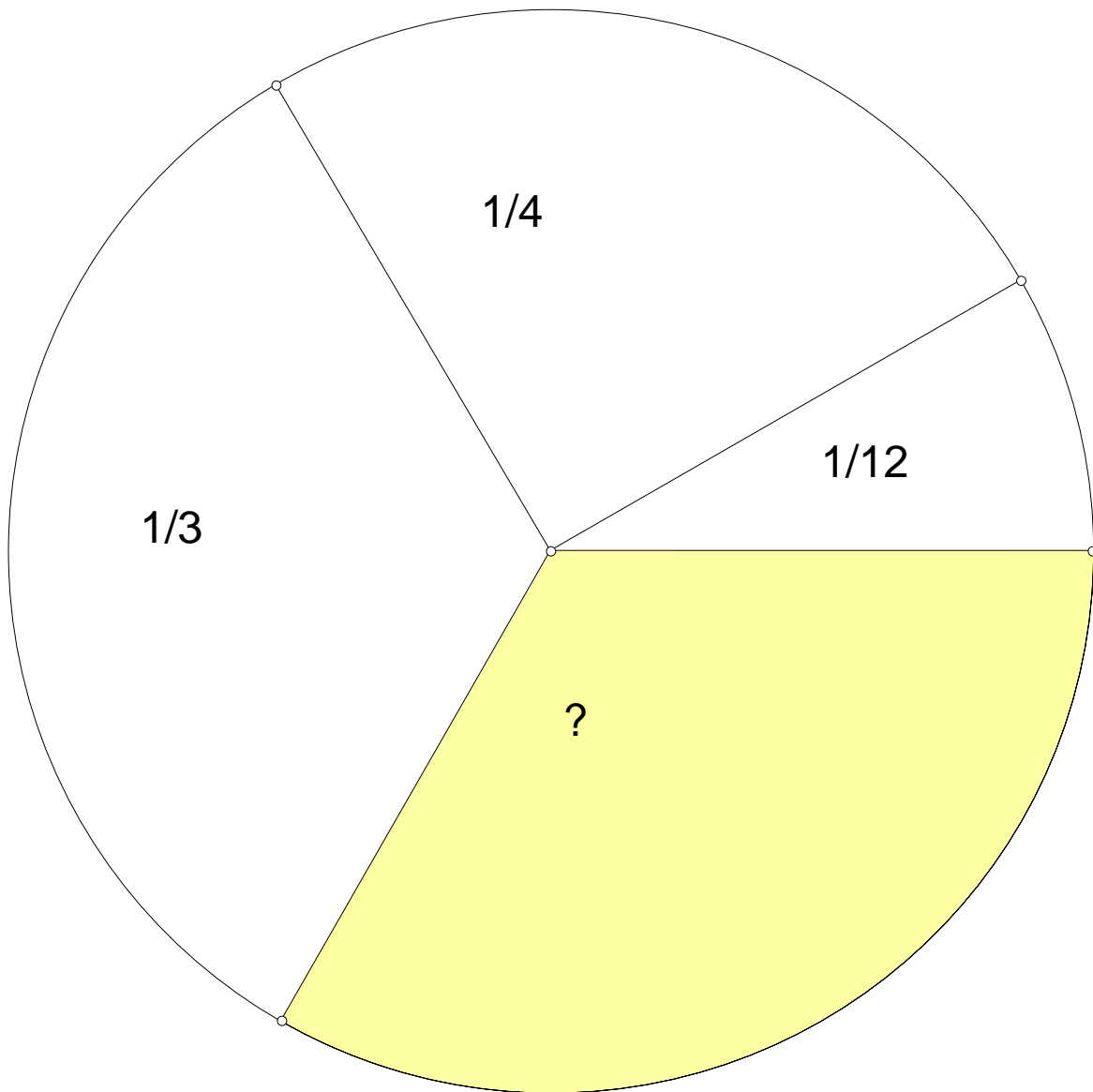


Press Enter to see the answer



Using Pictures:

Example 5:



To find the value of the colored portion:

Note that the non colored portion adds to $\frac{1}{3} + \frac{1}{4} + \frac{1}{12}$: 12 itself
is the lowest common denominator

$$\frac{1}{3} + \frac{1}{4} + \frac{1}{12}$$

$$= \frac{4}{12} + \frac{3}{12} + \frac{1}{12}$$
$$= \frac{8}{12}$$

$$= \frac{2}{3}$$

The leftover from 1 is

$$1 - \frac{2}{3} = \frac{3}{3} - \frac{2}{3} = \frac{1}{3}$$

This may be thought of as a practical problem like

$\frac{1}{12}$ of a circular plot is given to Alex

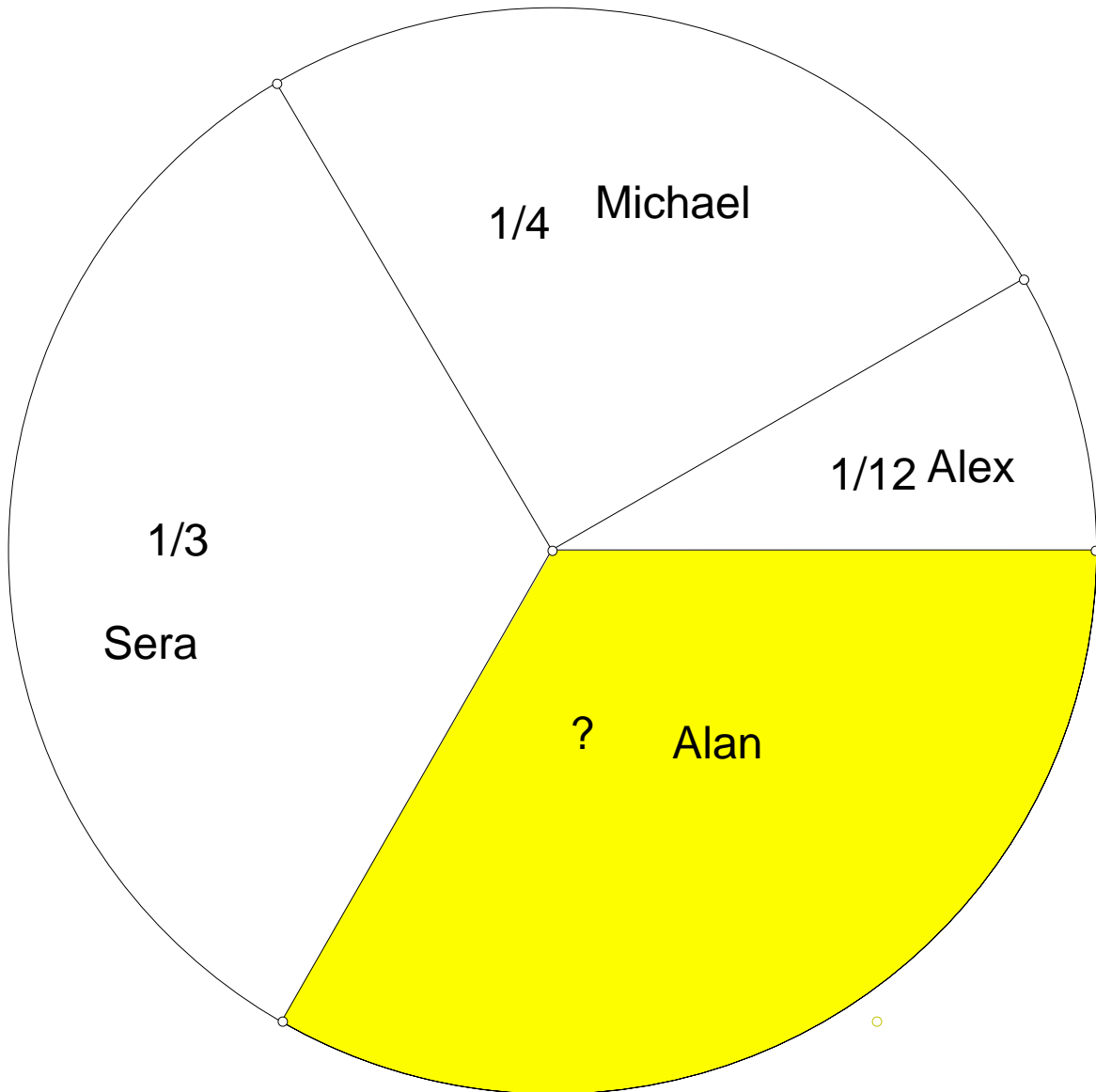
$\frac{1}{4}$ to Michael

$\frac{1}{3}$ to Sera

The rest to Alan

How much is left for Alan?

Answer is $\frac{1}{3}$



Example 6:

Multiplication

$$\begin{aligned}
 & \frac{8}{15} \times \frac{9}{16} \\
 &= \frac{8 \times 9}{15 \times 16} \\
 &= \frac{8 \times 3 \times 3}{3 \times 5 \times 8 \times 2} \\
 &= \frac{3}{10}
 \end{aligned}$$

Example 7:

Division:

$$\begin{aligned} & \frac{6}{35} \div \frac{3}{7} \\ &= \frac{6}{35} \times \frac{7}{3} \quad \text{Change } \div \text{ to } \times \text{ and reverse the denominator} \\ &= \frac{6 \times 7}{35 \times 3} \\ &= \frac{2 \times 3 \times 7}{5 \times 7 \times 3} \\ &= \frac{2}{5} \end{aligned}$$

MIXED FRACTIONS:

May be seen at many places



here of course, $239\frac{9}{10}$ is meant to express 2.399

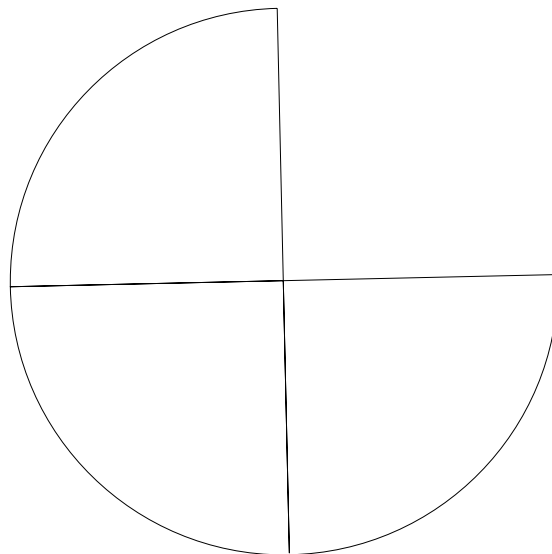
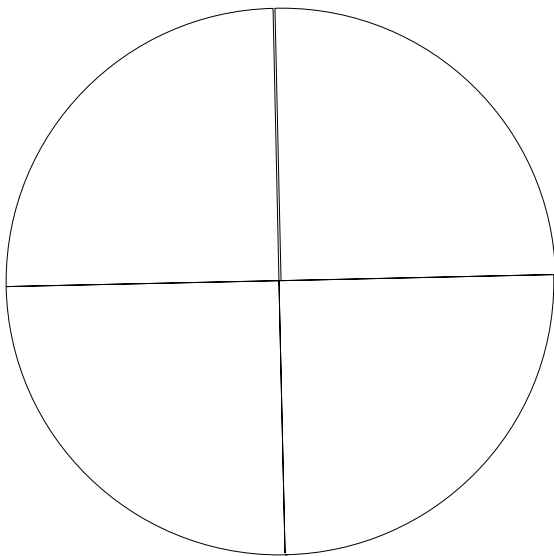
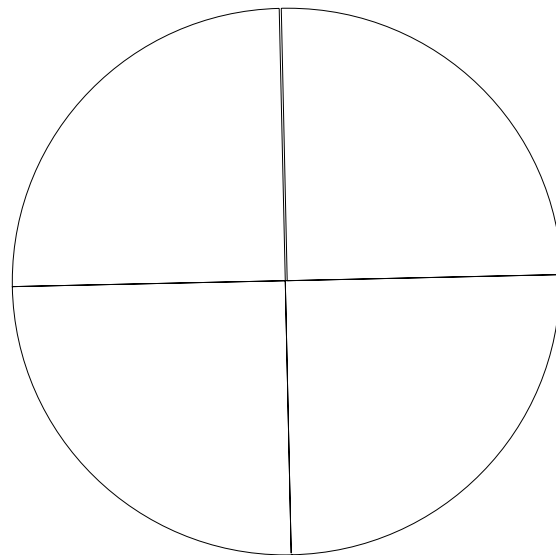
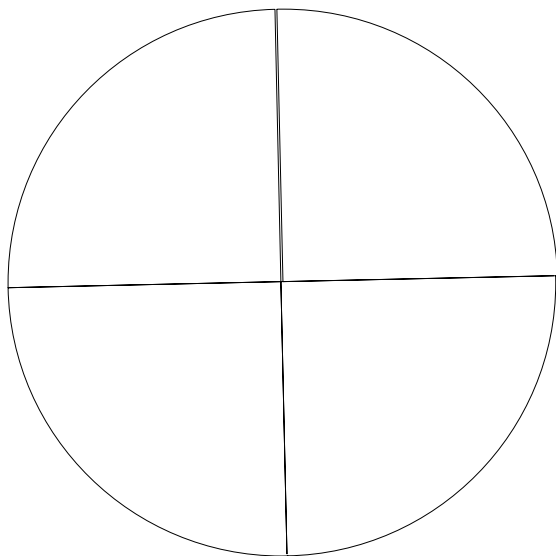
But note other situations Like:



A restaurant is selling Pizza slices for \$1.79 each and four slices make a whole Pizza that is each slice is $\frac{1}{4}$ of a Pizza.

If you a whole Pizza, they charge \$5.99

In case you need 15 slices, a good idea will be to get three whole Pizzas and 3 Slices



That is $3 + \frac{3}{4}$ which is generally written as a mixed fraction of the form $3\frac{3}{4}$

Note that we can express

$$\begin{aligned} 3\frac{3}{4} & \text{ back as} \\ & = 3 + \frac{3}{4} \\ & = \frac{12}{4} + \frac{3}{4} \end{aligned}$$

$$= \frac{15}{4}$$

.....

Arithmetic with mixed fractions:

Example 8:

To find

$$3\frac{1}{6} - 1\frac{11}{12}$$

Let us use two methods:

Method I:

$$\text{Write } 3\frac{1}{6} = \frac{3 \times 6 + 1}{6} = \frac{19}{6}$$

$$\text{and } 1\frac{11}{12} = \frac{1 \times 12 + 11}{12} = \frac{23}{12}$$

Therefore

$$\begin{aligned} & 3\frac{1}{6} - 1\frac{11}{12} \\ &= \frac{19}{6} - \frac{23}{12} \quad \text{note that the lowest common denominator is } 12 \\ &= \frac{19 \times 2}{6 \times 2} - \frac{23}{12} \\ &= \frac{38}{12} - \frac{23}{12} \\ &= \frac{15}{12} \\ &= \frac{5}{4} \\ &= 1\frac{1}{4} \end{aligned}$$

METHOD 2:

you may write

$$3\frac{1}{6} - 1\frac{11}{12}$$

$$= 3\frac{2}{12} - 1\frac{11}{12}$$

Note that $\frac{11}{12}$ is more than $\frac{2}{12}$

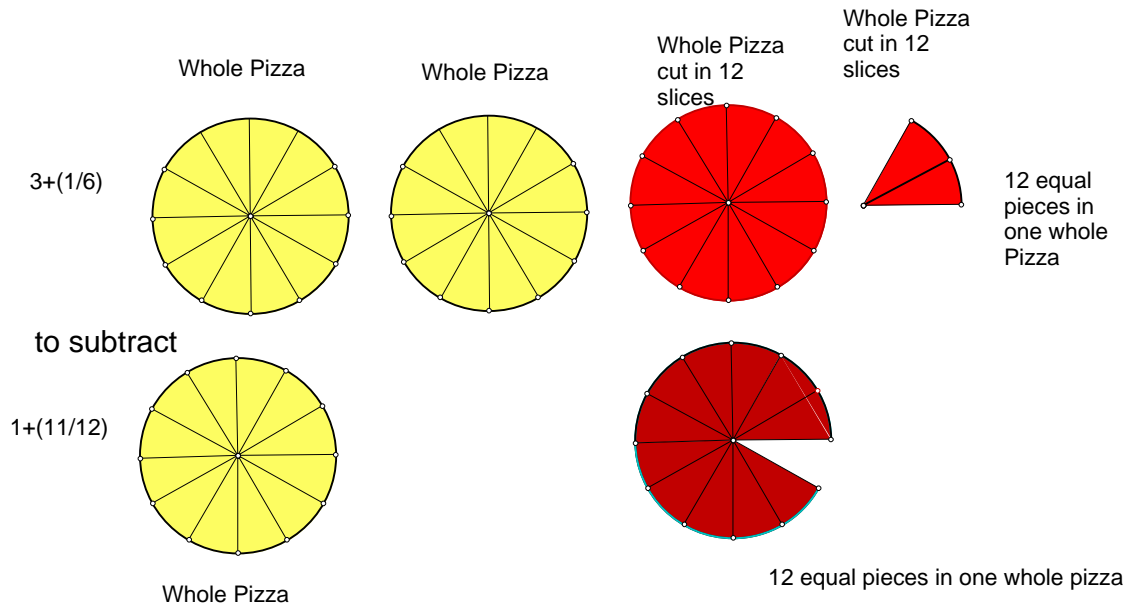
therefore in $3\frac{2}{12}$, we have to transfer, 1 from 3 to make the fractional part larger to be able to subtract

That is, look at $3\frac{2}{12}$ as $3 + \frac{2}{12}$ and then as $2 + 1 + \frac{2}{12} = 2 + \frac{12}{12} + \frac{2}{12} = 2 + \frac{14}{12} = 2\frac{14}{12}$

Therefore

$$\begin{aligned} & 3\frac{2}{12} - 1\frac{11}{12} \\ &= 2\frac{14}{12} - 1\frac{11}{12} \\ &= 1\frac{3}{12} \\ &= 1\frac{1}{4} \end{aligned}$$

as illustrated by the following picture

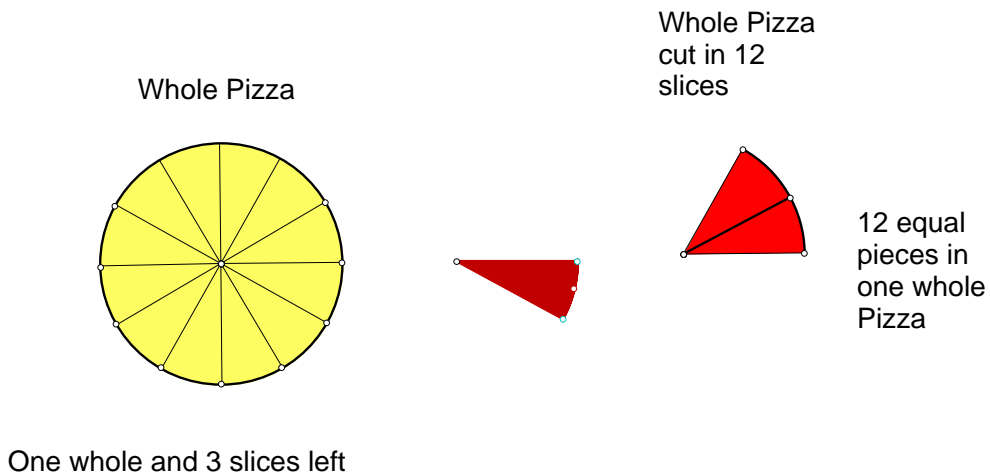


Note that during the subtraction part
 One whole Pizza on the top will be required to subtract the
 whole Pizza on the bottom

and

11 slices on the top will be required to cancel the 11 slices on
 the bottom.

WE shall be left with 1 whole Pizza and 3 slices



Method 3:



Example 9:

A division problem

$$3\frac{4}{7} \div 2\frac{8}{21}$$
$$= \frac{25}{7} \div \frac{50}{21}$$

$$3\frac{4}{7} = \frac{3 \times 7 + 4}{7} = \frac{25}{7} \quad \text{and} \quad 2\frac{8}{21} = \frac{2 \times 21 + 8}{21} = \frac{50}{21}$$

$$= \frac{25}{7} \cdot \frac{21}{50}$$

flipping the divisor

changed from division to multiplication by

$$= \frac{3}{2}$$

$$= 1\frac{1}{2}$$

Example 10:

A multiplication problem

$$\begin{aligned} & 5\frac{3}{5} \cdot 2\frac{1}{7} \\ &= \frac{28}{5} \cdot \frac{15}{7} \\ &= 4 \times 3 \\ &= 12 \end{aligned}$$