#9

$f(x) = x^3 - 2x^2$ on $[-1, 1]$

$f'(x) = 3x^2 - 4x$

$f'(x) = 0$

$x = 0, x = \frac{4}{3}$ (OUT)

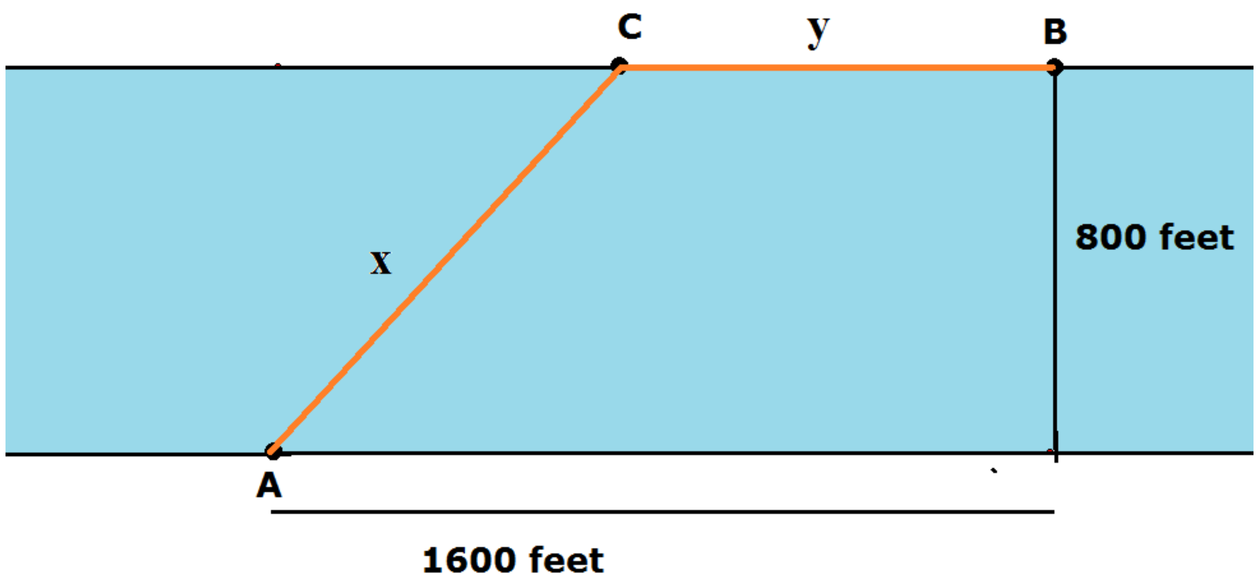
$f(-1) = (-1)^3 - 2(-1)^2 = -3$ MIN

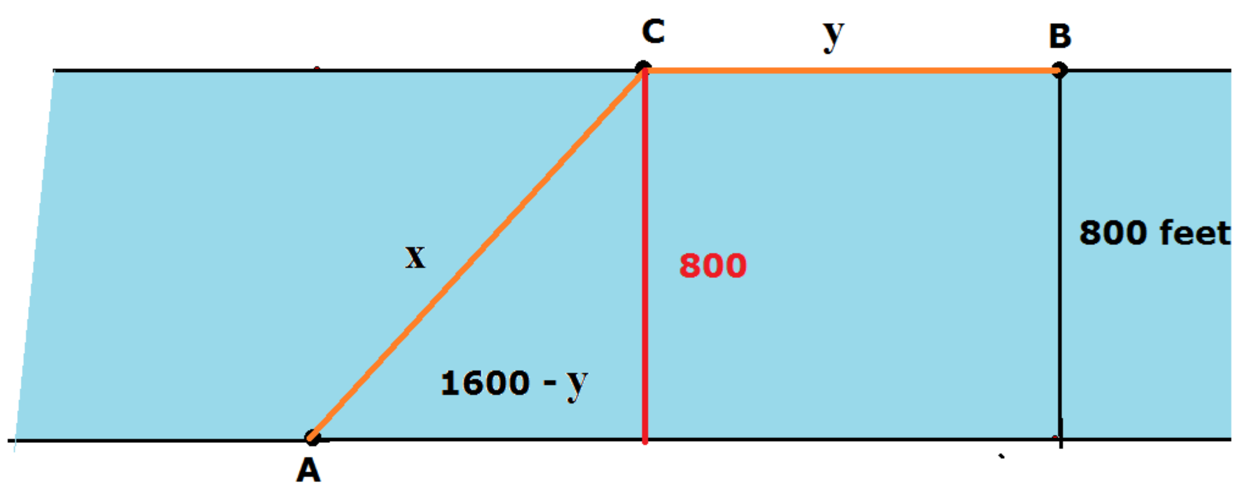
$f(0) = 0$ MAX

$f(1) = 1^3 - 2(1)^2 = -1$

10. A power company needs to lay a cable from point A on one bank of an 800-foot wide straight river to point B on the opposite bank 1600 feet downstream. The power company will choose a point C on the same side of the river as B, and will lay a cable underwater from A to C and on land from C to B. It costs $3 a foot to lay the cable on land and $5 a foot to lay it underwater.

Where the point C should to be chosen to minimize the total cost and what is the minimum cost?





$$x^2 = 800^2 + (1600 - y)^2$$

To minimize

$$5x + 3y = 5\sqrt{800^2 + (1600 - y)^2} + 3y$$

or to find the minimum value of

$$C(y) = 5\sqrt{800^2 + (1600 - y)^2} + 3y \text{ **on** } [0, 1600]$$

$$C'(y) = -\frac{5(1600 - y)}{\sqrt{800^2 + (1600 - y)^2}} + 3$$

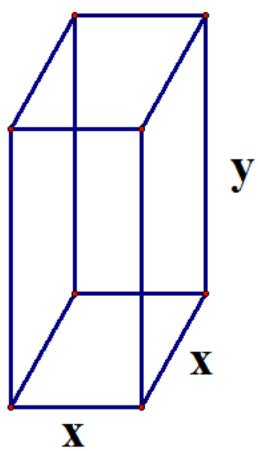
$$\rightarrow 5(1600 - y) = 3\sqrt{800^2 + (1600 - y)^2}$$
$$\rightarrow 25(1600 - y)^2 = 9(800^2 + (1600 - y)^2)$$
$$\rightarrow 25(1600 - y)^2 = 2400^2 + 9(1600 - y)^2$$
$$\rightarrow 16(1600 - y)^2 = 2400^2$$
$$\rightarrow (1600 - y)^2 = \left(\frac{2400}{4}\right)^2$$
$$\rightarrow (1600 - y)^2 = (600)^2$$
$$\rightarrow 1600 - y = \pm 600$$
$$\rightarrow y = 1000 \text{ or } 2200$$

2200 is out of $[0, 1600]$

$$C(0) = 5\sqrt{800^2 + (1600 - 0)^2} + 3(0) = 4000\sqrt{5}$$
$$C(1000) = 5\sqrt{800^2 + (1600 - 1000)^2} + 3(1000) = 8000 \quad \textbf{MIN}$$
$$C(1000) = 5\sqrt{800^2 + (1600 - 1600)^2} + 3(1600) = 8800$$

Minimum cost of $\$8000$ diagonally across the river 600′ then aling the bank 1000′

11. A closed box with a square base has to be built to house an ant colony. The bottom of the box and all four sides will be made of material costing $1 square foot, and the top is to be constructed of glass costing $5 per square foot. What are the dimensions of the box of greatest volume that can be constructed for $72.00?



To maximize

$$x^2y = x^2\left(\frac{72-6x^2}{4x}\right) = \frac{72x-6x^3}{4}$$

$$V(x) = \frac{72x-6x^3}{4}$$

$$V'(x) = \frac{72-18x^2}{4}$$

$$\frac{72-18x^2}{4} = 0, \text{ Solution : } -2, 2$$

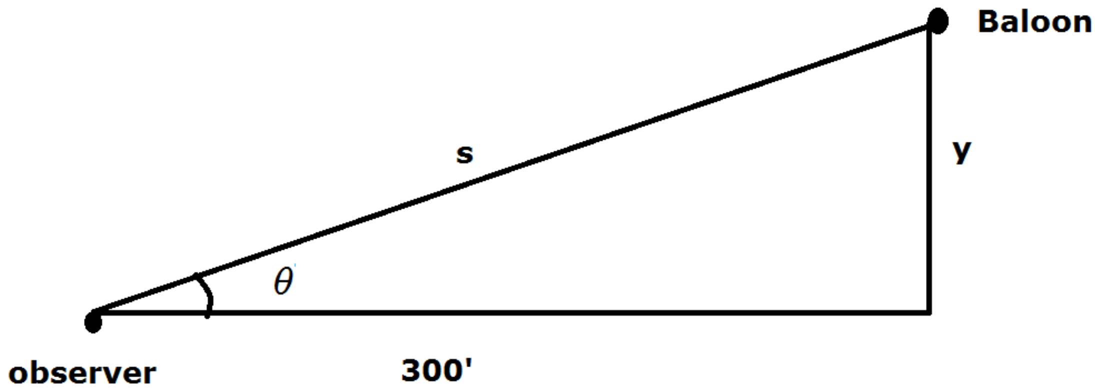
only $x = 2$ **is in the domain**

$$V''(x) = \frac{-38x^2}{4} < 0 \qquad x > 0$$

max at $x = 2$

$2'$ **by** $2'$ **by** $\dfrac{72-6(2)^2}{4\times 2} = 6'$

12. A weather balloon is rising vertically at the rate of 10 ft/s. An observer is standing on the ground 300 ft horizontally from the point where the balloon was released.
(a) At what rate is the distance between the observer and the balloon changing when the balloon is 400 ft high?
(b) At what rate is the angle of elevation of the observer's line of sight to the balloon changing when the balloon is 400 feet high?



$$s^2 = y^2 + 300^2$$

differentiate wrt t

$$2s\frac{ds}{dt} = 2y\frac{dy}{dt}$$

$$s\frac{ds}{dt} = y\frac{dy}{dt}$$

$$y = 400 \rightarrow s = \sqrt{400^2 + 300^2} = 500'$$

$$500\left(\left.\frac{ds}{dt}\right|_{y=400}\right) = 400(10)$$

$$\left.\frac{ds}{dt}\right|_{y=400} = \frac{4000}{500} = 8 \text{ f/s}$$

If the angle of elevation is θ

$$\tan\theta = \frac{y}{300}$$

differentiate wrt t

$$\sec^2\theta\left(\frac{d\theta}{dt}\right) = \frac{1}{300}\frac{dy}{dt}$$

$$\rightarrow \cos^2\theta\sec^2\theta\left(\frac{d\theta}{dt}\right) = \frac{\cos^2\theta}{300}\frac{dy}{dt}$$

$$\rightarrow \frac{d\theta}{dt} = \frac{\cos^2\theta}{300}(10)$$

when $y = 400$

$$\cos\theta = \frac{300}{500}$$

$$\left.\frac{d\theta}{dt}\right|_{y=400} = \frac{(300/500)^2}{300}(10) = 0.012 \text{ rad/sec}$$

Equation of the line through (1,5) and slope about 1.5
y-5=(1.5)(x-1)

17. The graph of the derivative of a certain function f appears below.

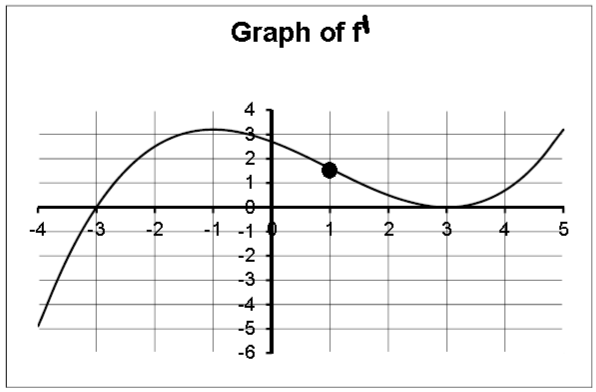
(a) Suppose $f(1) = 5$. Find an equation of the line tangent to the graph of f at (1,5).

(b) Suppose $f(-1) = -2$. Could $f(3) = -6$? Why or why not?

(c) Estimate $f''(-3)$.

(d) At which values of x does $f(x)$ have points of inflection? **x=-1,x=3**

(e) At which value of x in the interval $[-4,5]$ does $f(x)$ achieve its largest value? Its smallest value?



f is an increasing function on [-1,3]
therefore f(3) can not be smaller than f(-1)

(-4,-3) the function decreases
(-3,3) the function increases, MIN at x=-3
(3,5) function keeps increasing, MAX at x=5

#24

$$\int_{2}^{4} x \ln x dx$$

Left Reimann Sum with $n = 50$

width of the intervals $\frac{1}{25}$

$$\left(\left(2 + \frac{i}{25}\right) \ln\left(2 + \frac{i}{25}\right)\right)\left(\frac{1}{25}\right)$$

Left sum

$$\sum_{i=0}^{49} \left(\left(2 + \frac{i}{25}\right) \ln\left(2 + \frac{i}{25}\right)\right)\left(\frac{1}{25}\right) = 6.620975285129496496$$

Right sum

$$\sum_{i=1}^{50} \left(\left(2 + \frac{i}{25}\right) \ln\left(2 + \frac{i}{25}\right)\right)\left(\frac{1}{25}\right) = 6.7873306084638833707$$

average

$$\frac{6.6209752851294964965 + 6.7873306084638833707}{2} = 6.704$$

One of the following is $\int x\ln x dx$

$$F(x) = x\ln x - x \qquad G(x) = \frac{x^2(2\ln x - 1)}{4} \qquad H(x) = \frac{x^2\ln x}{2}$$

$$\frac{DF}{dx} = \ln x + x\left(\frac{1}{x}\right) - 1 = \ln x$$

$$\frac{dG}{dx} = \frac{2x(2\ln x - 1) + x^2\left(\frac{2}{x}\right)}{4} = \frac{4x\ln x - 2x + 2x}{4} = x\ln x$$

$$\int x\ln x dx = \frac{x^2(2\ln x - 1)}{4}$$

$$\int_2^4 x\ln x dx = \frac{4^2(2\ln 4 - 1)}{4} - \frac{2^2(2\ln 2 - 1)}{4} = 6.7040605278392343318$$

27. You are designing right circular cylindrical cans with volumes of 1000 cubic centimeters. The manufacturer of these cans will take waste into account. There is no waste in cutting the aluminum for the sides, but the top and the bottom of radius $\mathbf{r}$ will be cut from squares that measure $\mathbf{2r}$ on a side. The total amount of the aluminum used up by each can will therefore be

$$\mathbf{A = 8r^2 + 2\pi rh}$$

. Find the values of $\mathbf{r}$ and $\mathbf{h}$ which will minimize the amount of aluminum used.

$$A = 8r^2 + 2\pi rh$$

$$\pi r^2 h = 1000$$
$$h = \frac{1000}{\pi r^2}$$

To minimize

$$A(r) = 8r^2 + 2\pi r\left(\frac{1000}{\pi r^2}\right) = 8r^2 + \frac{2000}{r}$$

$$A'(r) = 16r - \frac{2000}{r^2}$$

$$16r - \frac{2000}{r^2} = 0 \to r = 5 \text{ cm}$$

$$A''(r) = 16 + \frac{4000}{r^3} > 0 \text{ for } r > 0$$

min at $r = 5$ **cm.**

$$h = \frac{1000}{\pi(5)^2} = \frac{40}{\pi} \text{ cm.}$$

30. The velocity of an object moving in a horizontal line is given by $v(t) = 8 - 2t$ (in ft/sec.)

Find

a) The displacement during the time interval $0 \le t \le 6$
b) The total distance traveled during the time interval $0 \le t \le 6$

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a)

$$\int_0^6 v(t)dt = \int_0^6 (8 - 2t)dt = 12 \text{ feet}$$

b)

$$\int_0^6 |v(t)|dt = \int_0^6 |(8 - 2t)|dt = 20 \text{ feet}$$