In this unit we are going to see the examples of the interval estimate for the population mean or population proportion based on sample information.

The following are some examples in which such calculations are useful.

1. To estimate the mean time that employees take to finish a certain task.

2. To estimate the mean of the expediture that the tourists had at vacation resort.

3. To estimate the percentage of people who are planning to vote for the incumbent president in the next election.

4. To estimate the average number of mistakes per document at a document processing firm.

First let us review some examples regarding the interval estimation of the population mean.

Take a simple random sample of size n.

The sample mean $\overline{x}$ **is the point estimate**

the margin of error for the estimate is m$= z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$ **where**

$\frac{\alpha}{2}$ **depends on the confidence level, and** σ **is the population standard deviation.**

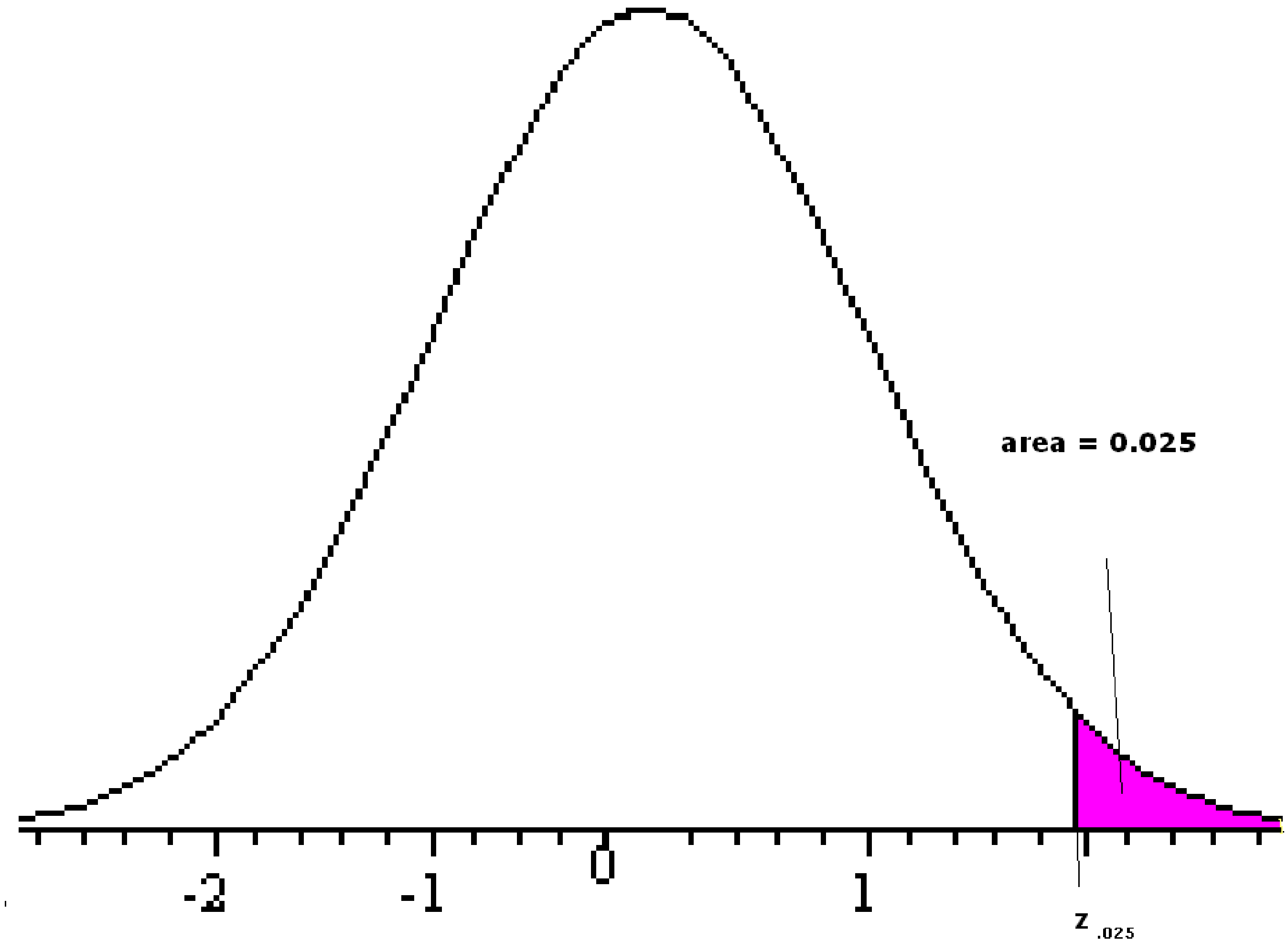
for example for 95% confidence $\frac{\alpha}{2} = \frac{1-.95}{2} = 0.025$

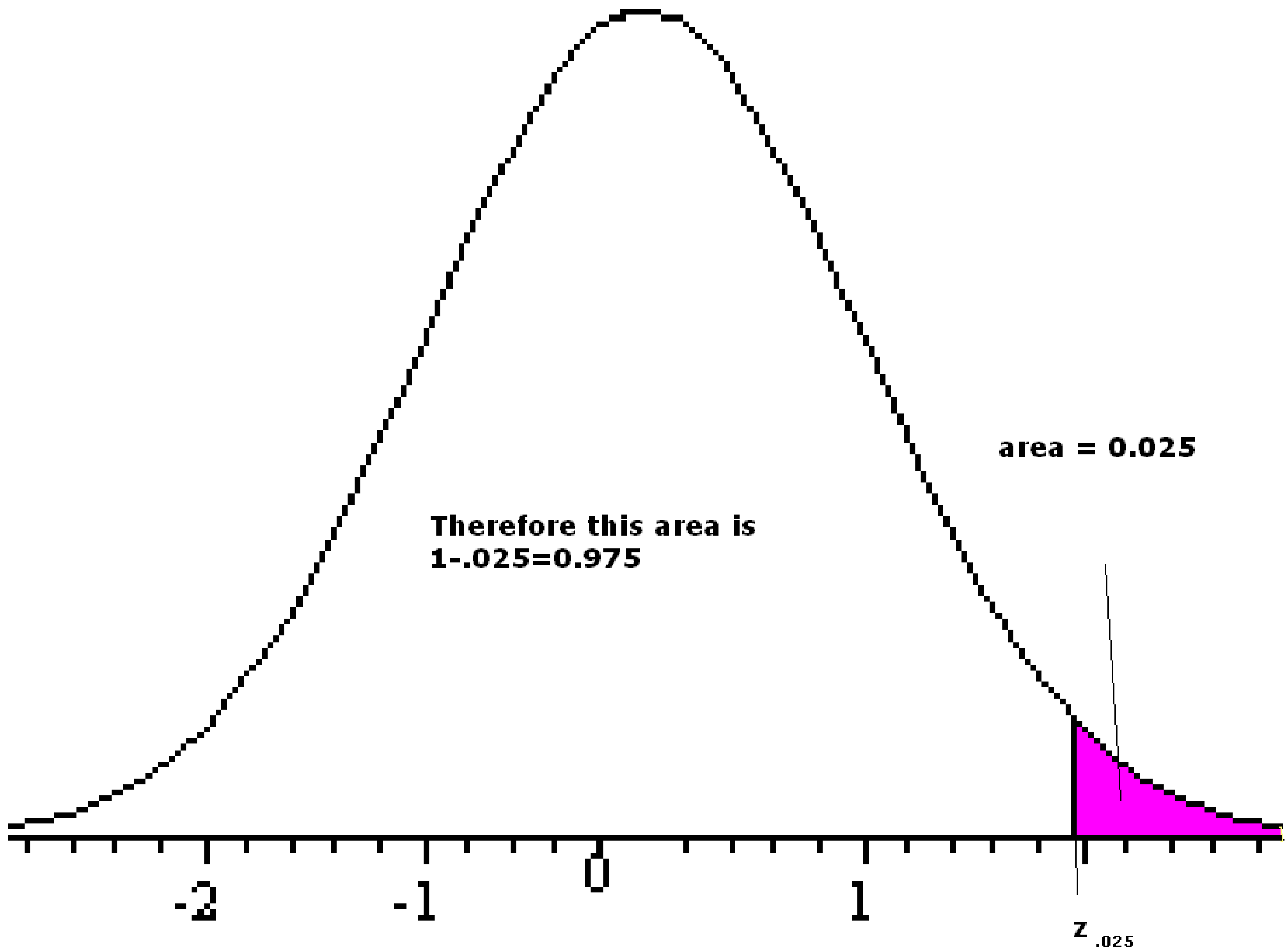
$z_{\alpha/2} = z_{0.025}$ **equals 1.96*. I am recalling the calculations below.**

Remember that $z_{0.025}$ is the value of z such that the area to the right of this value is 0.025.

$$s(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

$s(z)$


area = 0.025
-2
-1
0
1
z .025



Look into the z-table for the value 0.975, you shall find that

$$
\begin{array}{ccc}
 & & .06 \\
 & & \uparrow \\
1.9 & \leftarrow & .475
\end{array}
$$

therefore

$z_{.025} = 1.96$

Note that 95% of the values of z are between -1.96 and 1.96
The population has to be normal unless the sample size is large.

A confidence interval is $(\overline{x} - m, \overline{x} + m)$

Example1:

To estimate the mean driving time (μ) between two branches of a company a simple random sample sample of 50 employees shows a mean of $\overline{x} = 54.36$ minutes. Assume that the population standard deviation $\sigma = 5.9$ minutes is known.

We shall use $\overline{x} = 54.36$ as an estimate for μ.

The margin of error depends on the confidence level.

For 95% confidence level

$$\frac{1-.95}{2} = 0.025$$

m=$z_{.025}\frac{\sigma}{\sqrt{n}}$
σ**: The population standard deviation**
n: sample size

In this case, we have $\sigma = 5.9$ **minutes**
m=$1.96\frac{5.9}{\sqrt{50}} \cong 1.64$

a 95% confidence interval is
$(54.36 - 1.64, 54.36 + 1.64) = (52.72, 56.0)$

that is we are 95% confident that the true value of μ **is in this interval.**

Let me show you an illustration of how this works.

In simple terms, what we mean is that the procedure used to create this interval captures the true population mean 95% of the times. Let us look at 100 confidence intervals that I am creating from a population that has mean of 8 units and standard deviation of 4 units.

I am using sample size of 30. (I shall give reference to a video that you can access to see the method that I used to create these intervals)

The assumed sigma = 4

Variable	N	Mean	StDev	SE Mean	95.0% CI
C1	30	8.843	4.200	0.730	(7.412, 10.275)
C2	30	8.421	4.181	0.730	(6.990, 9.853)
C3	30	7.673	4.558	0.730	(6.242, 9.105)
C4	30	7.228	3.910	0.730	(5.797, 8.659)
C5	30	7.767	4.092	0.730	(6.336, 9.199)
C6	30	9.426	4.573	0.730	(7.994, 10.857)
C7	30	8.602	4.071	0.730	(7.171, 10.033)
C8	30	8.124	3.333	0.730	(6.692, 9.555)
C9	30	8.227	3.710	0.730	(6.796, 9.659)
C10	30	7.640	4.467	0.730	(6.208, 9.071)
C11	30	9.263	4.444	0.730	(7.832, 10.694)
C12	30	8.554	4.949	0.730	(7.122, 9.985)
C13	30	6.964	3.925	0.730	(5.533, 8.395)
C14	30	7.595	4.358	0.730	(6.163, 9.026)
C15	30	7.738	4.269	0.730	(6.307, 9.169)
C16	30	8.094	4.405	0.730	(6.663, 9.526)
C17	30	7.767	3.636	0.730	(6.336, 9.199)
C18	30	8.051	3.763	0.730	(6.620, 9.483)
C19	30	8.771	4.162	0.730	(7.340, 10.203)
C20	30	9.109	4.718	0.730	(7.678, 10.540)
C21	30	8.270	4.391	0.730	(6.839, 9.701)
C22	30	8.636	4.394	0.730	(7.205, 10.067)
C23	30	8.898	4.268	0.730	(7.466, 10.329)
C24	30	7.400	3.229	0.730	(5.969, 8.832)
C25	30	8.739	3.694	0.730	(7.308, 10.170)
C26	30	8.043	3.688	0.730	(6.611, 9.474)
C27	30	8.099	4.200	0.730	(6.668, 9.530)
C28	30	7.211	3.623	0.730	(5.779, 8.642)
C29	30	7.855	3.722	0.730	(6.424, 9.287)
C30	30	7.521	2.976	0.730	(6.090, 8.952)

C61	30	6.800	2.595	0.730	(5.369,	8.231)
C62	30	9.151	4.376	0.730	(7.720,	10.582)
C63	30	8.452	3.907	0.730	(7.021,	9.884)
C64	30	8.717	5.592	0.730	(7.285,	10.148)
C65	30	8.069	4.541	0.730	(6.638,	9.501)
C66	30	7.970	5.404	0.730	(6.539,	9.401)
C67	30	8.857	4.568	0.730	(7.426,	10.289)
C68	30	7.556	3.275	0.730	(6.125,	8.988)
C69	30	8.418	3.698	0.730	(6.987,	9.849)
C70	30	7.391	3.012	0.730	(5.960,	8.822)
C71	30	7.365	4.171	0.730	(5.934,	8.796)
C72	30	7.254	3.722	0.730	(5.823,	8.686)
C73	30	6.561	3.274	0.730	(5.130,	7.993)
C74	30	7.073	3.173	0.730	(5.641,	8.504)
C75	30	7.541	4.518	0.730	(6.109,	8.972)
C76	30	7.545	3.258	0.730	(6.113,	8.976)
C77	30	9.350	4.608	0.730	(7.918,	10.781)
C78	30	6.685	3.188	0.730	(5.254,	8.116)
C79	30	8.792	3.730	0.730	(7.361,	10.224)
C80	30	7.841	4.321	0.730	(6.410,	9.273)
C81	30	9.064	4.347	0.730	(7.632,	10.495)
C82	30	8.229	4.786	0.730	(6.798,	9.660)
C83	30	7.438	3.973	0.730	(6.007,	8.869)
C84	30	8.480	2.981	0.730	(7.048,	9.911)
C85	30	8.986	4.090	0.730	(7.555,	10.418)
C86	30	7.215	3.259	0.730	(5.783,	8.646)
C87	30	7.698	2.861	0.730	(6.267,	9.130)
C88	30	6.901	2.664	0.730	(5.469,	8.332)
C89	30	8.200	4.452	0.730	(6.768,	9.631)
C90	30	7.171	3.016	0.730	(5.740,	8.603)
C91	30	7.915	4.943	0.730	(6.483,	9.346)
C92	30	8.063	3.698	0.730	(6.632,	9.495)
C93	30	7.810	3.628	0.730	(6.379,	9.242)
C94	30	7.509	2.820	0.730	(6.078,	8.940)
C95	30	8.336	4.271	0.730	(6.904,	9.767)
C96	30	8.630	4.002	0.730	(7.199,	10.061)

Note that only 5 (red ones) of the 100 intervals failed to capture the population mean that is 8

95 out of 100 intervals have captured the the population mean 8.

Example 2:

To assess the the average damage after a snow storm, a county official calls a random sample of 100 residents and finds that
$\overline{x} = \$897$ **and s=$768. Use** $\sigma = 768$ **and obtain**

a) a 95% confidence interval for μ**.**
b) a 99% confidence interval for μ**.**

a) 95% confidence interval

$\frac{1-.95}{2} = \mathbf{0.025}$

$z_{.025} = \mathbf{1.96}$

m$= 1.96\frac{768}{\sqrt{100}} = 150.53$

$(897 - 150.53, 897 + 150.53) = (746.47, 1047.53)$

b)
99% confidence interval

$\frac{1-.99}{2} = \mathbf{0.005}$

to obtain $z_{.005}$**, look for 0.495 in the table.**

		.07	.08
		↑	↑
2.5	←	.4949	.4951

$\mathbf{z}_{.005}=\frac{2.57+2.58}{2}$

$\mathbf{z}_{.005}=$ **2.575**

$\mathbf{m}=2.575\frac{768}{\sqrt{100}}=197.76$

$(897-197.76, 897+197.76)=(699.24, 1094.76)$

As the confidence level increases, the interval becomes wider because the error increases.

Here, we used the value of s for σ because the sample size is large, for small sample sizes, we shall use the following

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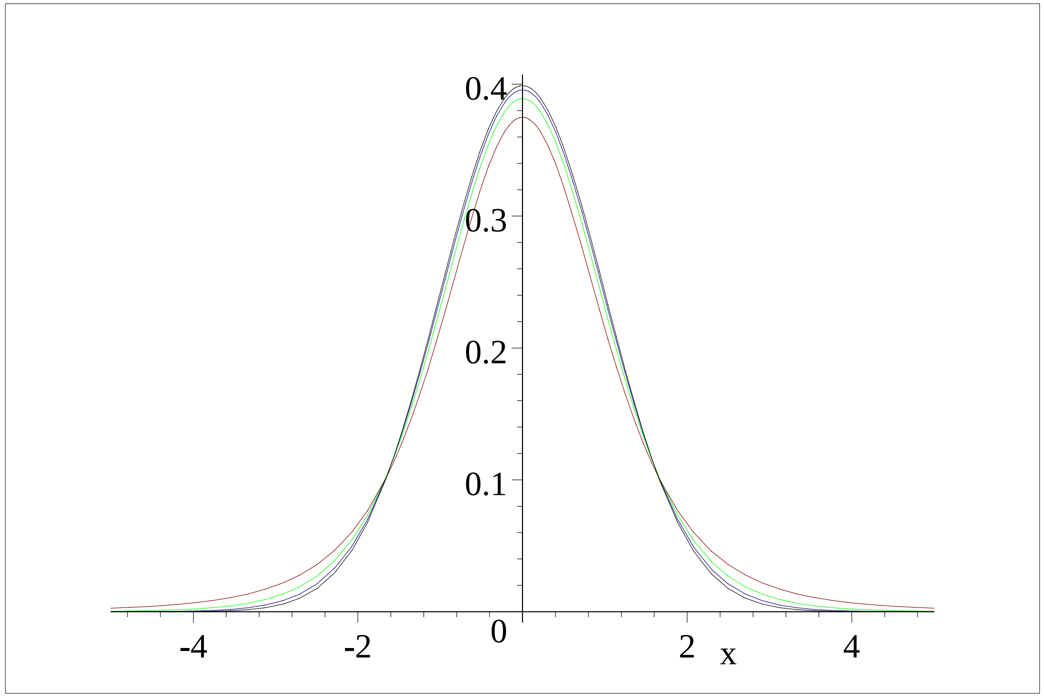
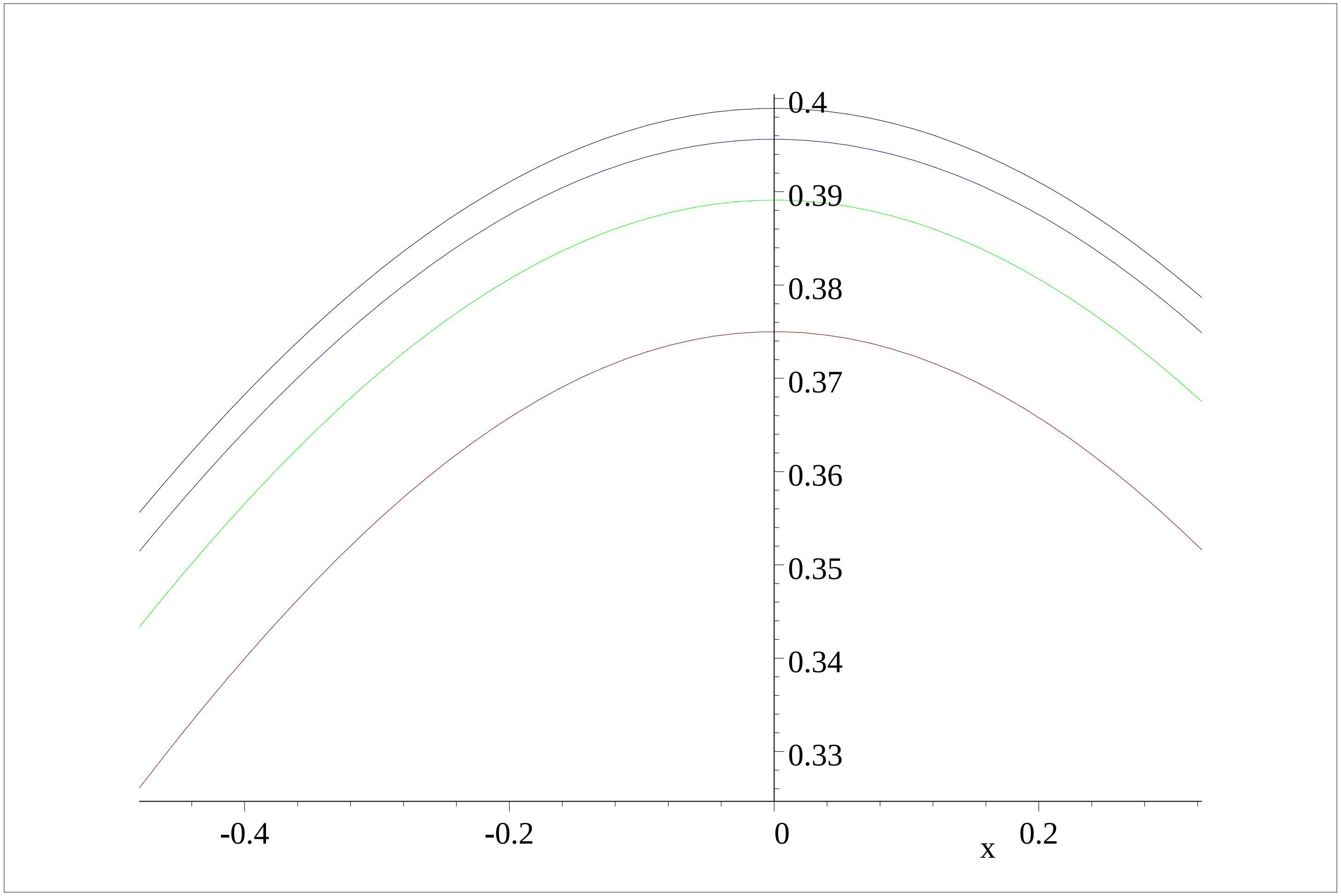
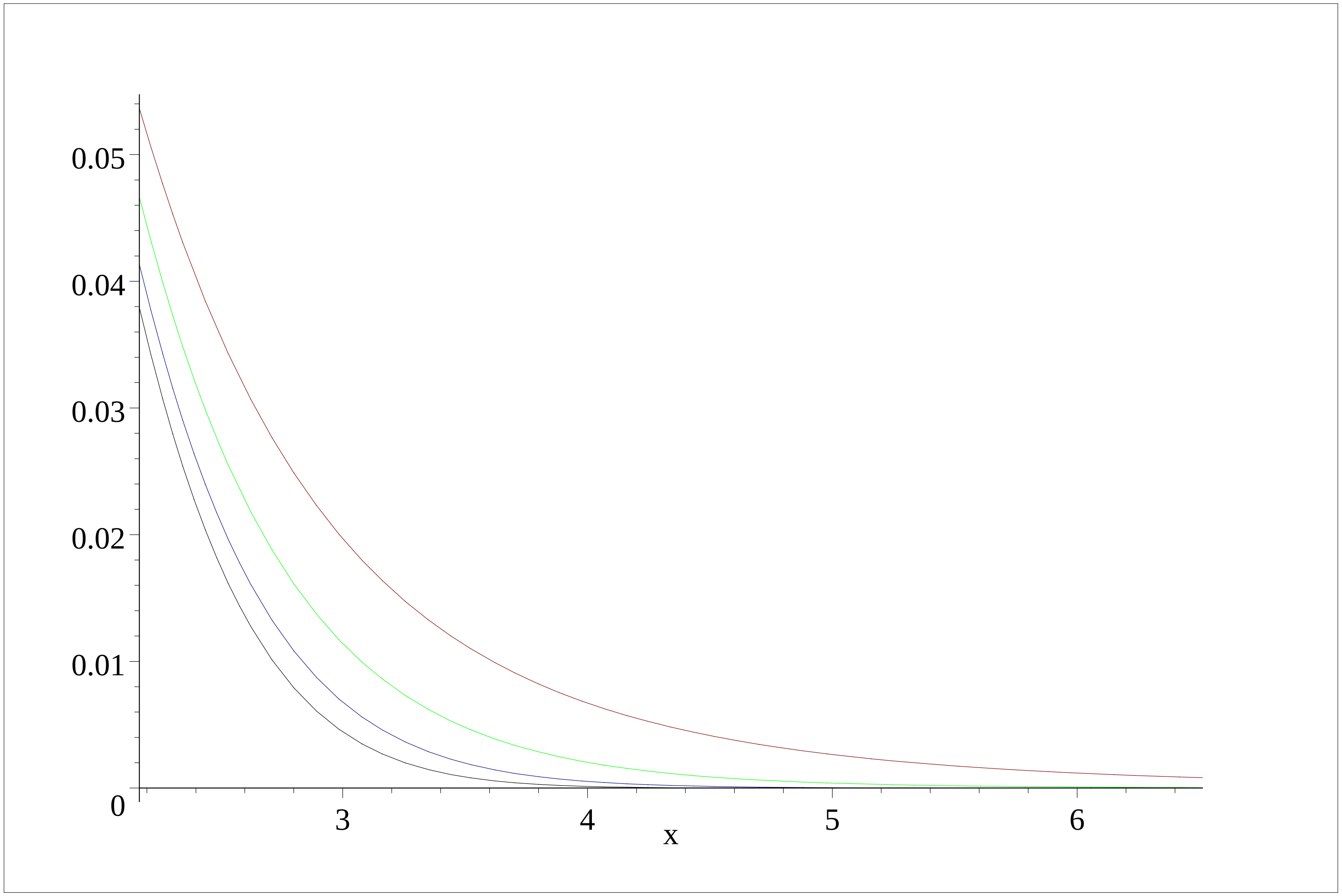
Estimation for the cases when σ is not known and a large sample is not possible
In such cases we use t-distribution.

$g(x)$ **(please ignore this function, it is here only for sketching the graphs.)**



red: t with 4 degrees of freedom
green: t with 10 degrees of freedom
blue: t with 30 degrees of freedom


0.4
0.39
0.38
0.37
0.36
0.35
0.34
0.33
-0.4
-0.2
0
0.2
x



A tidistribution is also bell shaped with center at 0 but is flatter than the z-distribution.
The shape of t-distribution depends on degrees of freedom.
In case sample size is small, we use t-distribution.
An assumption for the t-distribution is that the population is normal.

If the population is normal
and we take simple random samples of size n,
then

$$\frac{\overline{x} - \mu}{\left(\frac{s}{\sqrt{n}}\right)}$$ **shows a t-distribution with $n - 1$ degrees of freedom.**

For the margin of error, we use the rule

m $= \frac{t_{\alpha/2}\ s}{\sqrt{n}}$

where $t_{\alpha/2}$ depends on the confidence level and the degrees of freedom that is $n - 1$ in one-sample procedures.

Some critical values of t are given in the table that is in the back of the book.

Example 3:

To estimate the overall mean time that customers spend these days on a long distance call, a surveyer takes a simple random sample of 32 customers and finds that $\overline{x} = 19.7$ minutes and s=17.2 minutes. Construct a 95% confidence interval for μ the overall mean time that the customers spend on a call.

Recall that the margin of error $m = \frac{t_{\alpha/2} s}{\sqrt{n}}$

$\frac{\alpha}{2} = \frac{1 - .95}{2} =$ **0.025**

The degrees of freedom is $n-1$
that is
32 − 1 = 31

Look at the table E.3 in back of the book and note that

$$\begin{array}{ccc} & & .025 \\ & & \downarrow \\ 31 & \rightarrow & 2.0395 \end{array}$$

therefore

$t_{.025} = 2.0395$

the margin of error for this confidence interval

$$m = 2.0395\frac{17.2}{\sqrt{32}} = 6.2012204049888438069$$

therefore a 95% confidence interval is

$(19.7 - 6.2, 19.7 + 6.2) = (13.5, 25.9)$ **minutes**

Example 4 (To demonstrate TI83plus)

For a simple random sample of 23 mangoes, the mean weight is 12.4 **oz and the standard deviation** $s = 1.15$ **oz. Use this data to obtain a 99% confidence interval for the mean weight**

of all mangoes of this type.

Assuming a normal distribution for the populations of the weights of all the mangoes,

IN TI83plus

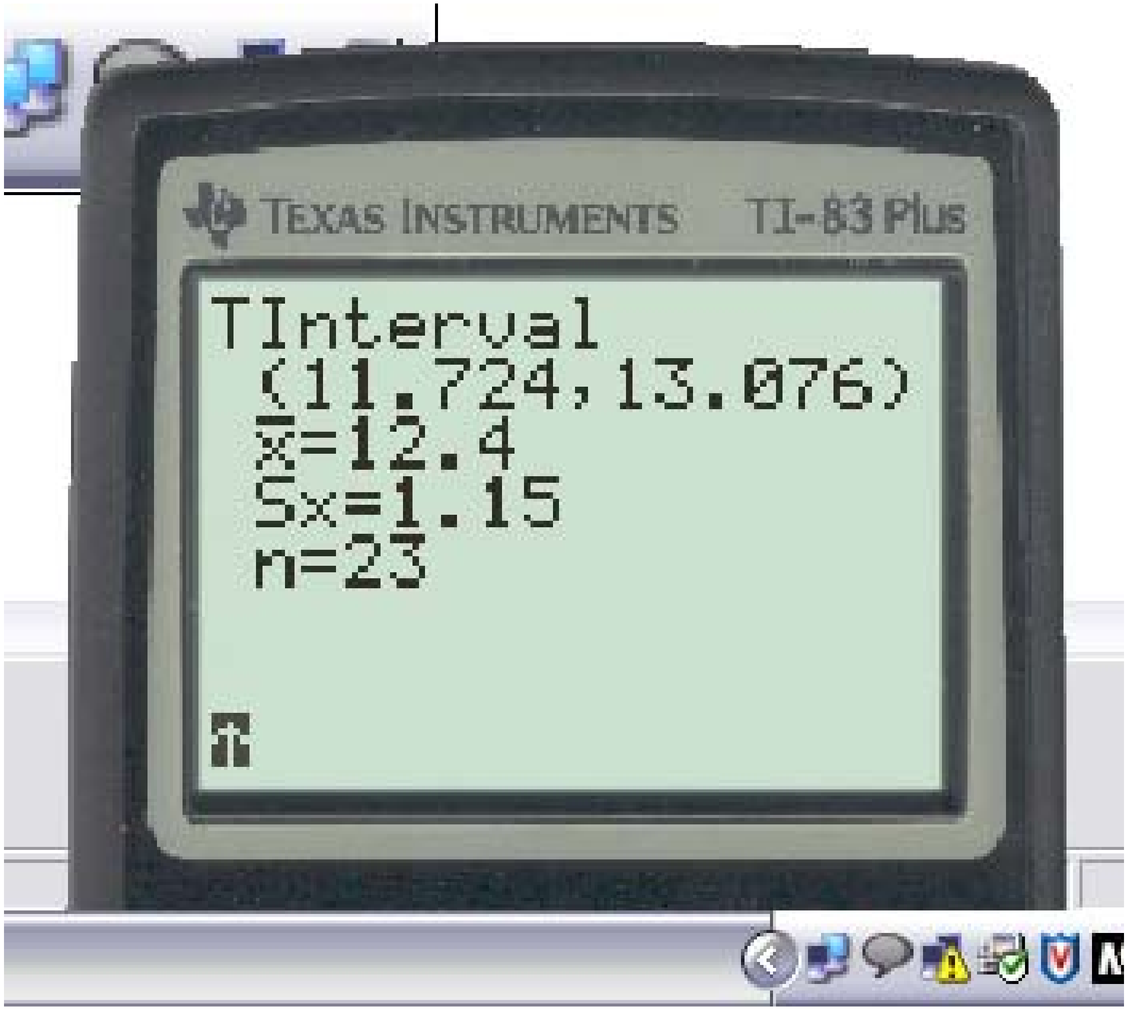
click on

STAT
TESTS
then chose t-interval


TEXAS INSTRUMENTS
TI-83 Plus
EDIT CALC TESTS
3↑2-SampZTest…
4:2-SampTTest…
5:1-PropZTest…
6:2-PropZTest…
7:ZInterval…
8:TInterval…
9↓2-SampZInt…
TI-83+


TEXAS INSTRUMENTS
TI-83 Plus
TInterval
Inpt:Data Stats
x̄:12.4
Sx:1.15
n:23
C-Level:.99

ask it to calculate with the above values



a 99% confidence interval is $(11.724, 13.076)$ **oz**

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Determination of Sample size for a desired margin of error at a given confidence interval

If E is the margin of error, recall that

m $= \frac{z_{\alpha/2} \times \sigma}{\sqrt{n}}$

solve for n:

n $= \left(\frac{z_{\alpha/2}\sigma}{m} \right)^2$

Example 5.

Given that the population standard deviation of the heights of the adult males is 2.9 inches. You would like to construct a confidence interval for the mean height of all the adult males in a certain region with a margin of error of no more than 0.75 inches.

Find the required sample size if the confidence level has to be

a) 90%
b) 95%
c) 99%

For
a) note that

$\frac{\alpha}{2} = \frac{1-.9}{2} =$ **0.05**

To find $z_{.05}$ **we have to look for the value of z in the z-table such that the area to the left of this value is 0.95**

In the z-table, when we look into the area side, we do not see 0.95 exactly, but see two values .9495 and .9505 that are equally close to this value,

one approach is to look at the average of these two values,

$$\begin{array}{cccc} & & .04 & .05 \\ & & \uparrow & \uparrow \\ 1.6 & \leftarrow & .9495 & .9505 \end{array}$$

$z_{.05} = \frac{1.64 + 1.65}{2} =$ **1.645**

therefore

$n = \left(\frac{1.645 \times 2.9}{.75}\right)^2 = 40.458080444444444445$ **we need a simple random sample of 41 adult males to get an interval with 90% confidence**

b)

For 95% confidence interval, we shall use

$\frac{1-.95}{2} = \mathbf{0.025}$

z$_{.025}$= **1.95**

therefore

$n = \left(\frac{1.96 \times 2.9}{.75}\right)^2 = 57.436188444444444445$**, we need a simple random sample of at least 58 adults**

c)

For a 99% confidence interval, we shall use

$\frac{1-.99}{2} = \mathbf{0.005}$

z$_{.005}$= **2.575**

therefore we need

$n = \left(\frac{2.576 \times 2.9}{.75}\right)^2 = 99.212224284444444444$**, we need a simple random sample of at least 100 adults**

NOTE THAT

We need a larger sample size for a larger confidence level

Example 6.

Given that the population standard deviation of the heights of the adult males is 2.9 inches. You would like to construct a confidence interval for the mean height of all the adult males in a certain region with a confidence level of 95%. How large a simple random sample must be taken to construct an interval with

a) margin of error of no more than 1.00 inches?
b) margin of error of no more than 0.75 inches?
c) margin of error of no more than 0.50 inches?

Here for all the three cases, we have $z_{.025} = 1.96$

For a)

$n = \left(\frac{1.96 \times 2.9}{1.00}\right)^2 = 32.307856$ **, at least 33 adult males**

b)

$n = \left(\frac{1.96 \times 2.9}{0.75}\right)^2 = 57.436188444444444445$**, at least 58 adult males (as done above)**

c)

$n = \left(\frac{1.96 \times 2.9}{0.50}\right)^2 = 129.231424$**, at least 130 adult males**

NOTE THAT

We need a larger sample size for a smaller margin of error

Confidence Interval for Population Proportion is a very widely present example, specially if you watch news.(http://www.pollingreport.com)

Example 7:
Jim would like to build a new shopping mall in an area with movie theatures, because at present the malls in that area do not have theatures. To eastimate the poroportion of people who want a new mall, an agency calls a siomple random sample of 400 residents, 279 say "YES."

Sample proportion $\tilde{p} = \dfrac{279}{400} = 0.6975$

69.75%

Need margin of error

E=$z_{\alpha/2}\sqrt{\dfrac{\tilde{p}(1-\tilde{p})}{n}}$ **,**
n is the sample size,
$\tilde{p}$ **is the sample proportion:**

For 99% confidence interval
$\dfrac{1-.99}{2} = \mathbf{0.005}$
$z_{.005} = 2.575$ **from the table**
area between 0 and $z_{.005}$ **is** $.5 - .005 = 0.495$

		.07	**.08**
		$\uparrow$	$\uparrow$
2.5	$\leftarrow$	**.4949**	**.4951**

use $z_{.005} = 2.575$

m$=2.575\sqrt{\dfrac{.6975(1-.6975)}{400}} = 5.9140097091435139105 \times 10^{-2} \cong 0.0591$

a 99% confidence interval is

$(.6975 - .0591, .6975 + .0591) = (0.6384, 0.7566)$

An approximate statement:

Based on our sample, we are 99% confidence that % of people wanting a new mall is somewhere in 63.83% to 75.67%.

For 95% interval

$z_{.025} = 1.96$

m$=1.96\sqrt{\dfrac{.6975(1-.6975)}{400}} = 4.501537488 \times 10^{-2}$

..

Example 8:

To estimate the proportion of people who are in favor of construction of a new highway in the area, a simple random sample of 348 residents is taken. 219 of them are in favor of a new highway. Construct a 95% confidence interval for the true overall proportion of people who would like to see a new highway in the area.

$\overline{p} = \frac{219}{348} \cong \mathbf{0.63}$

95% confidence,
the margin of error

$\mathbf{1.96}\sqrt{\frac{0.63(1-0.63)}{348}} = \mathbf{5.072681396 \times 10^{-2}} \cong \mathbf{0.051}$

a 95% confidence interval is

$(.63 - .051, .63 + .051) = (0.579, 0.681)$

How large a sample must be taken to estimate this proportion with a margin of error of no more than $\pm .03$ **at 99% confidence level?**

n$= \frac{(z_{\alpha/2})^2 p(1-p)}{m^2}$

E=0.03

For 99% confidence, $z_{.005} = 2.575$

p: an educated guess for the population proportion

in the absence of any such guess, take p=0.5

Here one guess is p=0.63 based on the previous result

n=$\frac{2.575^2 \cdot .63(1-.63)}{.03^2} = 1717.331876$

a sample of at least 1718 is needed

Sometimes people use the value in a previous interval that is closer to 0.5, for example they will use from $(0.579, 0.681)$
the value .579 for the guess.

$$g(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

$$\mathbf{f}(\mathbf{x},\mathbf{v}) = \frac{\Gamma(\frac{v+1}{2})}{\Gamma(\frac{v}{2})\sqrt{\pi v}}\left(1+\frac{1}{v}x^2\right)^{-\frac{v+1}{2}}$$

$$\mathbf{f}(\mathbf{x},\mathbf{5})$$