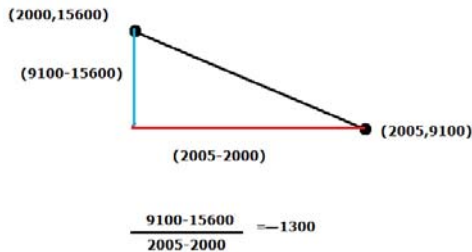


Review MA097 Sheet for the Final (Detailed Answers)

1. The value of a certain type of automobile depreciates linearly. In 2000 the value was \$15,600 and in 2005 the value was \$9100. Use the slope formula to determine the average rate of change of the value of the car. Write your result in a complete sentence with correct units.



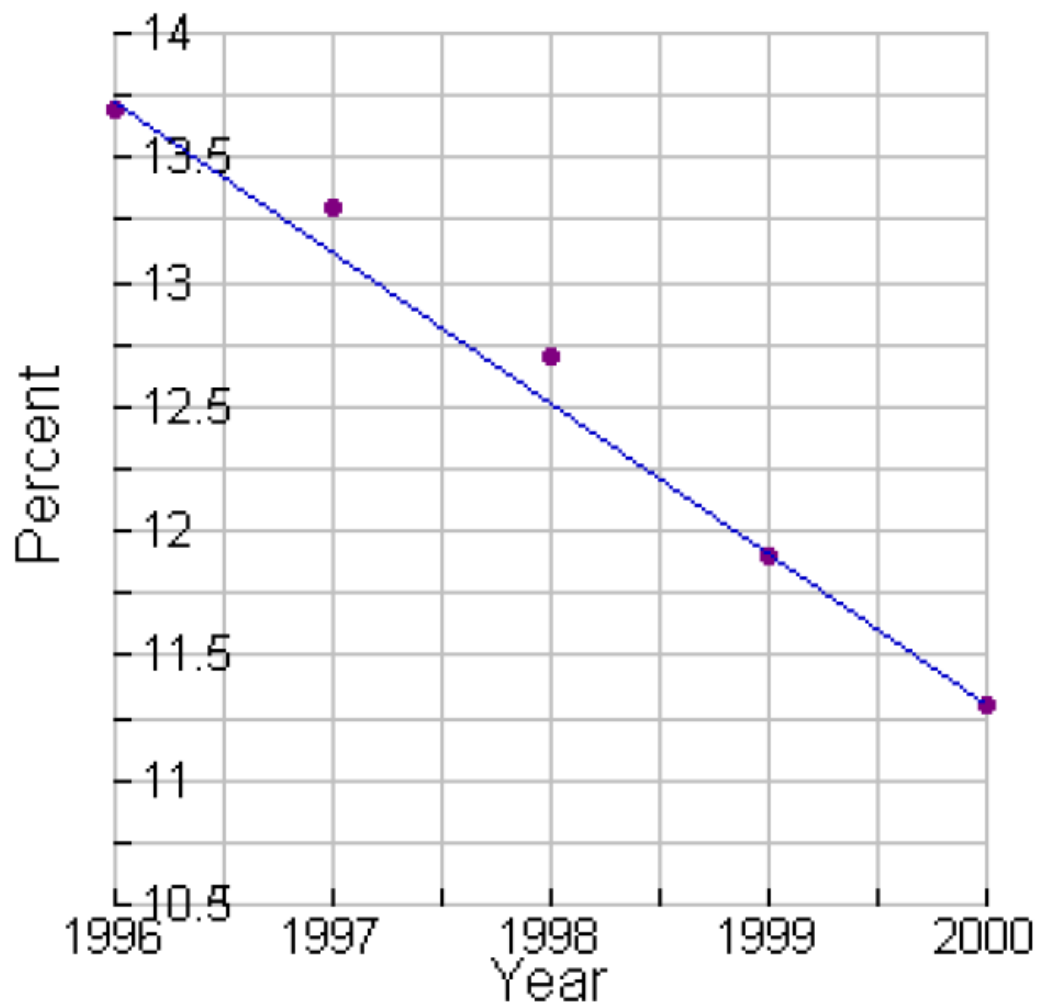
$m = -1300$ dollars/year; The value of the car is decreasing by \$1300 per year.

2. The percentages of Americans living below the poverty level are shown in the table.
- a) Create a scattergram of the given data on the given axes above AND draw a line passing through the starred (*) data points. b) Use the slope formula and the two starred (*) points above to determine the slope of the line. Give your answer as a decimal. Write the meaning of your result from part b in a complete sentence with correct units.

Percentages of Americans Living Below the Poverty Level	
Year	Percent
*1996	13.7
1997	13.3
1998	12.7
1999	11.9
*2000	11.3

answer:

% of Americans Living Below the Poverty Level



a) Refer to graph at the end of the solutions; b) $m = -0.6 \%/year$; From the year 1996 to the year 2000 the percentage of Americans living below the poverty level decreased by 0.6 per year.

3. Fast-food sales for years since 2000 in the US are shown in the table.
- a) Determine the linear model equation for this data by hand (not the calculator) using the two starred (*) points above. Use the variables t and s for your answer. b) Use the model equation to predict food sales for 2010, and write the meaning of your result in a complete sentence with correct units. c) Use the model equation to predict the year when food sales will be \$200 billion, and write the meaning of your result in a complete sentence with correct units.

Year	Sales (Billions of Dollars)
*2001	88.8
2002	92.5
2003	97.5
*2004	101.4
2005	105.5

If we take t to be the # of years since 2000 (even if do not make this change, model will still yield the same results)

The coordinates of the starred points transform to (1,88.8) and (4,101.4)

$$\text{Slope} = \frac{101.4 - 88.8}{4 - 1} = 4.2$$

Equation is

(1,88.8) is on the line

$$88.8 = 4.2 * 1 + b \quad s = 4.2 t + 84.6$$

$$88.8 = 4.2 + b$$

$$b = 88.8 - 4.2 = 84.6$$

a) $s = 4.2t + 84.6$ b) $s(10) = 126.6$; In 2010, fast-food sales in the US will be \$126.6 billion. c) $t \approx 27$, In 2027, fast-food sales in the US will be \$200 billion.

4. The linear model equation $n = -0.60t + 17.54$ represents the number of world refugees (in millions) at t years since 1990. a) Give the slope of the model and write its meaning in a complete sentence with correct units. b) Determine the n-intercept of the model. Write the ordered pair. Write the meaning of your result in a complete sentence with correct units. c) Determine the t-intercept of the model. Write the ordered pair. Write the meaning of your result in a complete sentence with correct units.

a) $m = -0.60$ million refugees/year; The number of world refugees is decreasing by 0.60 million per year. b) (0, 17.54); In 1990, there were 17.54 million world refugees. c) (29,0); In 2019, there will be no world refugees. (Most likely a model breakdown.)

5. The number of amusement park injuries from roller coasters was 4300 injuries in 2000 and has decreased by about 450 injuries per year. Let t be the years since 2000 and n be the number of injuries from roller coasters.
- Write the linear equation which models this information. Use the variables n and t for your answer.
 - Determine n when $t = -5$. Write the meaning of your result in a complete sentence with correct units.

decrease of 450 injuries per year refers to the slope, therefore the slope is -450

The year 2000 corresponds to $t=0$, the y-intercept is (0,4300) the equation is

$$n = -450t + 4300$$

- b) $n(-5) = 6550$; In 1995, there were 6550 injuries from roller coasters.

6. Average tuitions at four-year colleges are listed below for various years since 1980.
- Use the regression feature of your calculator to determine the linear model equation for public tuition at t years since 1980.
 - Write the meaning of slope for the public linear model in a complete sentence with correct units.
 - Use the calculator to determine the linear model for private tuition at t years since 1980.
 - Write the meaning of slope for the private linear model in a complete sentence with correct units.
 - Compare the two slopes of the two models and write the meaning of the comparison in this situation.

Average Tuitions at Four-Year Colleges		
Year	Public Tuition (dollars)	Private Tuition (dollars)
1984	2074	9202
1989	2395	12146
1994	3188	13844
1999	3632	16454
2004	4694	19710

L1	L2
4	2074
9	2395
14	3188
19	3632
24	4694
-----	-----
L3()=	

**Choose STAT
press CALC
Choose LinReg**

EDIT	TESTS
1: 1-Var Stats	
2: 2-Var Stats	
3: Med-Med	
4: LinReg(ax+b)	
5: QuadReg	
6: CubicReg	
7: QuartReg	

**a) In TI84
Use STAT followed by EDIT
to enter the data
after subtracting 1980 from
the year**

```

LinReg
y=ax+b
a=129.54 — slope
b=1383.04 \ y-intercept

```

Equation is

$y = 129.4x + 1383.04$

a) $y = 129.54x + 1383.04$ b) Public tuition is increasing by \$129.54 per year. c) $y = 506.48x + 7180.48$ d) Private tuition is increasing by \$506.48 per year. d) Private tuition is increasing at a faster rate than public tuition.

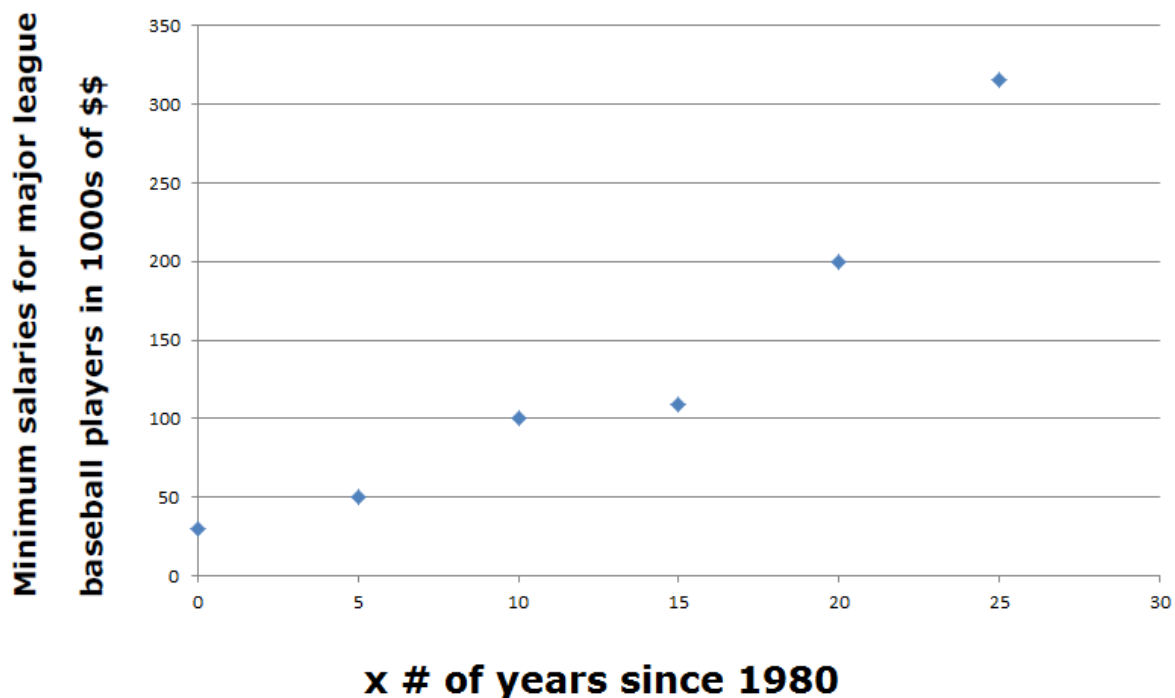
7. Minimum salaries for major league baseball players for various years since 1980 are shown in the table below.

Minimum Salaries for Major League Baseball Players	
Year	Thousands of Dollars
*1980	30
1985	50
1990	100
1995	109
*2000	200
2005	316

a) Create a scattergram of the given data. b) Use the **two starred points** above to determine the exponential model, $y = ab^x$, for this data; round b to four decimal places. Use s and t for your variables. c) Use your model equation to predict the minimum salary for major league baseball players in the year 2011; round to three decimal places. Write your answer in a complete sentence with correct units. d) Use your model to predict the year in which the minimum salary will be \$1 million. (Note: \$1 million = \$1000 thousand.) Write your answer in a complete sentence with correct units.

Answer:

a)



$y = a * b^x$ $x = 0, y = 30$ $30 = a * b^0$ $30 = a * 1$ $a = 30$	$y = 30 * b^x$ $x = 20 \quad y = 200$ $200 = 30 * b^{20}$ $\text{or } 30 * b^{20} = 200$ $b^{20} = \frac{200}{30}$ $b^{20} = (20/3)$ $b = (20/3)^{1/20} \cong 1.0995$ <div style="border: 1px solid black; padding: 2px; width: fit-content; margin: 5px auto;"> $(20/3)^{(1/20)}$ 1.099500515 </div> $y = 30(1.0995)^x$	<p>To find x such that $y = 1000$ i.e.</p> $30(1.0995)^x = 1000$ $(1.0995)^x = \frac{1000}{30}$ $1.0995^x = (100/3)$ $\log 1.0995^x = \log(100/3)$ $x \log 1.0995 = \log(100/3)$ $x = \frac{\log(100/3)}{\log 1.0995}$ <div style="border: 1px solid black; padding: 2px; width: fit-content; margin: 5px auto;"> $\frac{\log(100/3) / \log(1.0995)}{}$ 36.96735298 </div>
--------------------------------------------------------------------	-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------

a) Refer to the graph at the end of the solutions; b) $s(4) = 30(1.0995)^4$ c) $s(31) = 567.771$ In 2011, minimum salaries for Major League Baseball Players will be \$567.771 thousand, that is, \$567,771. d) $t = 37$, In 2017, minimum salaries for Major League Baseball Players will be \$1 million.

8. The exponential model equation $n(t) = 1.20(1.0162)^t$ represents the world population in billions at t years since 1900.
- a) Use the model to predict the world population in 2011. Round to two decimal places. Write your answer in a complete sentence with correct units. b) In what year will the population be 15 billion? Write your answer in a complete sentence with correct units. c) What is the n-intercept? Write the meaning of your answer in a complete sentence with correct units.

a) $n(111) = 1.20(1.0162)^{111}$

$$\boxed{1.2 * 1.0162^{111} = 7.142847858}$$

approximately 7.14 bln

b)

To solve

$$1.2(1.0162)^t = 15$$

for t

$$1.2(1.0162)^t = 15$$

→

$$(1.0162)^t = \frac{15}{1.2}$$

→

$$\log(1.0162)^t = \log(15/1.2)$$

→

$$t \log(1.0162) = \log(15/1.2)$$

→

$$t = \frac{\log(15/1.2)}{\log(1.0162)}$$

$$\frac{\log(15/1.2)}{\log(1.0162)}$$

- 157.1686575

b) $t \approx 157$, In 2057, the world population will be 15 billion.

c) $n = 1.20$, In 1900, the world population was 1.2 billion.

9. A person invests an amount of \$5000 in an account at 4% interest compounded annually.

a) Write an exponential model, $y = ab^x$, for this investment.

Use A and t for your variables. b) Use your model to predict the amount of the investment in 5 years. Write your answer in a complete sentence with correct units.

c) Use your model to predict when the original amount of the investment will double. Write your answer in a complete sentence with correct units.

a) $A(t) = 5000(1.04)^t$

b) $A(5) = 6083.26$; In 5 years, the investment will be worth \$6083.26.

c) **To solve**

$$5000(1.04)^t = 10000$$

$$(1.04)^t = 2$$

$$\log(1.04)^t = \log 2$$

$$t \log 1.04 = \log 2$$

$$t = \frac{\log 2}{\log 1.04}$$

$\log(2)/\log(1.04)$ 17.67298769

In about 18 years, the investment will double.

10. The number of lawsuits filed against tobacco companies is shown in the table below for various years since 1990.
- Use the regression feature on your calculator to determine the exponential model equation for this data. Round to three decimal places. Use n and t for your variables.
 - Use your model equation to predict when the number of lawsuits will be 5000. Write your answer in a complete sentence with correct units.
 - Use your model to predict the number of lawsuits that will be filed in 2011. Write your answer in a complete sentence with correct units.

Number of Lawsuits Filed Against Tobacco Companies	
Year	Number of Lawsuits
1993	49
1994	73
1995	200
1996	352
1997	733

Press **STAT** → **EDIT**

L1	L2	L3	3
4	2074		
9	2395		
14	3188		
19	3632		
24	4694		
ERASE THE EXISTING VALUES			
L1=			

Use the arrow key to select the list name

L1	L2	L3	1
4	2074		
9	2395		
14	3188		
19	3632		
24	4694		
L1={4, 9, 14, 19, 2...			

then press clear

L1	L2	L3	1
4	2074		
9	2395		
14	3188		
19	3632		
24	4694		
L1 =			

then enter

L1	L2	L3	1
	2074		
	2395		
	3188		
	3632		
	4694		
L1() =			

L1 is empty

similarly erase L2

L1	L2
-----	████████

enter the data

L1	L2
0	48
1	73
2	200
3	352
4	533

Press STAT
then CALC
then choose
ExpReg

```
EDIT [2nd][DEL] TESTS
4:LinReg(ax+b)
5:QuadReg
6:CubicReg
7:QuartReg
8:LinReg(a+bx)
9:LnReg
0:ExpReg
```

```
ExpReg
y=a*b^x
a=5.454077672
b=2.010497601
```

$$n(t) = 5.454(2.01)^t$$

b)

To solve

$$5.454(2.01)^t = 5000$$

$$(2.01)^t = \frac{5000}{5.454}$$

$$\log(2.01)^t = \log(5000/5.454)$$

$$t \log(2.01) = \log(5000/5.454)$$

$$t = \frac{\log(5000/5.454)}{\log(2.01)}$$

$$\frac{\log(5000/5.454)}{\log(2.01)} = 9.770096933$$

In 2000, the number of lawsuits filed against tobacco companies was 5000.

c)

$n(21) = 12,700,837$; Model breakdown: 12 million lawsuits is unrealistic.

11.

The BlackBerry is a wireless handheld device. The numbers of BlackBerry Subscribers are shown on the table for various years. Let $f(t)$ be the number of BlackBerry subscribers (in millions) at t years since

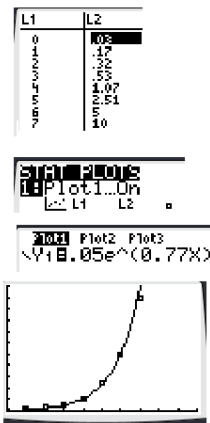
2000. A possible equation of f is: $f(t) = 0.05e^{0.77t}$

a) Verify that f models the situation well. b) According to the model, how many BlackBerry subscribers were there in 2008? c) Predict when there will be 500 million subscribers.

Year	Number of Subscribers (millions)
2000	0.03
2001	0.17
2002	0.32
2003	0.53
2004	1.07
2005	2.51
2006	5.0
2007	10.0

Solution:

Year	t	# of subscribers in millions
2000	0	0.03
2001	1	0.17
2002	2	0.32
2003	3	0.53
2004	4	1.07
2005	5	2.51
2006	6	5
2007	7	10



scatter plot along with a graph of $f(t) = 0.05e^{0.77t}$

$$f(t) = 0.05e^{0.77t}$$

to find $f(t)$ when $t = 2008 - 2000 = 8$

$$.05 * e^{.77 * 8} = 23.67140374$$

$$f(t) = 0.05e^{0.77t}$$

To find the year when, the number of subscribers will be 500 millions or

$$f(t) = 500$$

First, we have to find t such that

$$0.05e^{0.77t} = 500$$

$$e^{0.77t} = \frac{500}{.05}$$

$$e^{0.77t} = 10000$$

$$\ln e^{0.77t} = \ln 10000$$

$$0.77t \ln e = \ln 10000$$

$$\rightarrow 0.77t = \ln 10000$$

$$\ln e = 1$$

$$t = \frac{\ln 10000}{0.77} \cong 11.961481$$

The revenue will be 500 millions in 2012 if the same trend continues

12.

A company started business in 1995. Assume that a company has a profit per year shown in the table below. (Negative profits are losses.) Let the model $P(t) = -0.19t^2 + 2.56t - 2.38$ represent the amount of profit in millions of dollars, where t is the number of years since 1995.

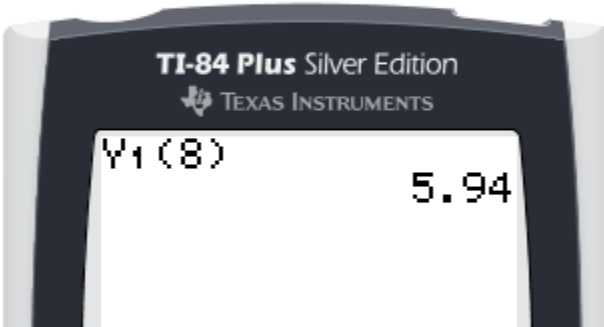
- a) Find $P(8)$. Write the meaning of your result in a complete sentence using correct units. b) Find t when $P(t) = 4.23$. Write the meaning of your result in a complete sentence using correct units. c) Find t when $P(t) = 0$. Write the meaning of your result in a complete sentence using correct units. d) In which year will the company have a maximum profit? e) Use the regression feature on your calculator to determine the quadratic model equation for this data and compare the calculator model to the given model.

Year	Profit millions of dollars
1995	-2.4
1996	0
1997	2.0
1998	3.6
1999	4.8
2000	5.7
2001	6.2

Solution:

Year	t	Profit in millions of \$\$\$
1995	0	-2.4
1996	1	0
1997	2	2
1998	3	3.6
1999	4	4.8
2000	5	5.7
2001	6	6.2

Plot1 Plot2 Plot3
 $Y_1 = -.19X^2 + 2.56X - 2.38$



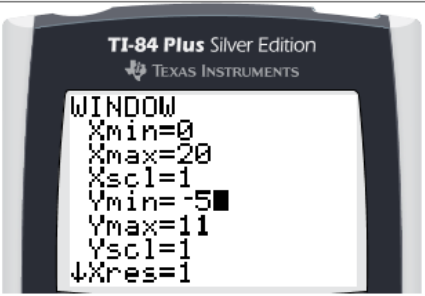
TI-84 Plus Silver Edition
 TEXAS INSTRUMENTS
 Y1(8) 5.94

Key Press History Large Screen

DISTR VARS ENTRY SOLVE ENTER
 ENTRY SOLVE ENTER (K V P 8
) L ENTRY SOLVE ENTER

a) In 2003, the profit was \$5.94 million

Plot1 Plot2 Plot3
 $Y_1 = -.19X^2 + 2.56X - 2.38$
 $Y_2 = 4.3$

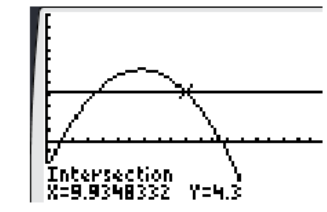


TI-84 Plus Silver Edition
 TEXAS INSTRUMENTS
 WINDOW
 Xmin=0
 Xmax=20
 Xscl=1
 Ymin=-5
 Ymax=11
 Yscl=1
 Xres=1

Key Press History Large Screen

TBLSET F2 WINDOW 2
 CATALOG 0 2 (-)
 L5 5 U

change the window



Intersection
 $X = 9.9348332$ $Y = 4.3$

b) In 1998 and 2005, the profit is \$4.23 million.

Plot1 Plot2 Plot3
 $Y1 = -.19X^2 + 2.56X - 2.38$

TI-84 Plus Silver Edition
 TEXAS INSTRUMENTS

Key Press History Large Screen

2ND CALC F4 L2 Z
 TRACE 2
 ENTER ENTER

24th
 $X = 1.004589$ $Y = 0$

TI-84 Plus Silver Edition
 TEXAS INSTRUMENTS

Key Press History Large Screen

2ND CALC F4 L2 Z
 TRACE 2
 ENTER ENTER
 ENTRY SOLVE ENTER

24th
 $X = 12.469095$ $Y = 1E-12$

c) In 1996 and 2007, the company had 0 profit

Plot1 Plot2 Plot3
 $Y1 = -.19X^2 + 2.56X - 2.38$

TI-84 Plus Silver Edition
 TEXAS INSTRUMENTS

Key Press History Large Screen

2ND CALC F4 L4 T
 TRACE 4
 ENTRY SOLVE ENTER ENTRY SOLVE ENTER
 ENTRY SOLVE ENTER

Maximum
 $X = 6.7368437$ $Y = 6.2431579$

d) vertex: ; A maximum profit of 6.24 million was in the year 2002 (between 2001 and 2002).

b)

$$-.19t^2 + 2.58t - 2.4 = 4.23$$

**You may use the quadratic formula
or use the intersect feature**

```

Plot1 Plot2 Plot3
Y1=-.19X^2+2.58X
-2.4
Y2=4.23

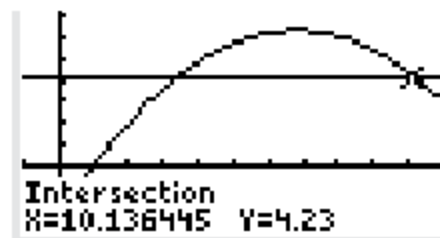
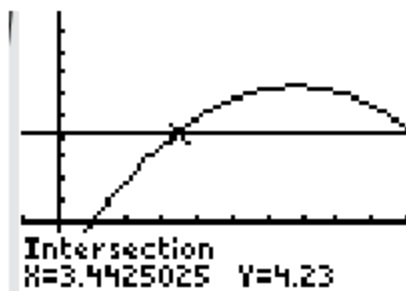
```

**You may press 2nd then calc
to see and use the intersect feature**

```

2nd [2] [2] [2] [2] [2] [2] [2] [2]
1:value
2:zero
3:minimum
4:maximum
5:intersect

```



$t \approx 3, 10$; In 1998 and 2005, the profit is \$4.23 million.

c)

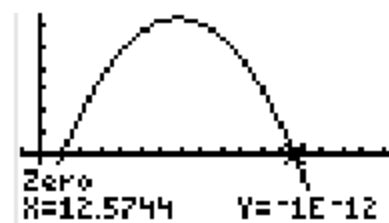
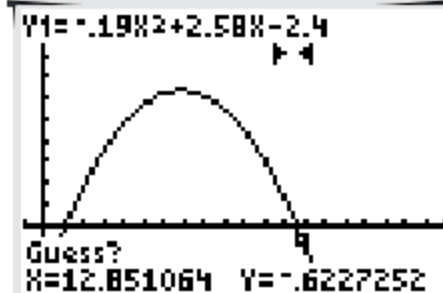
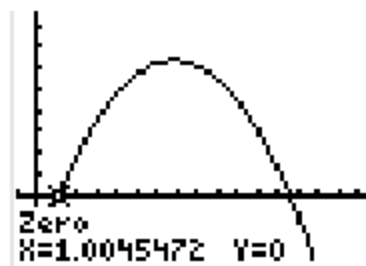
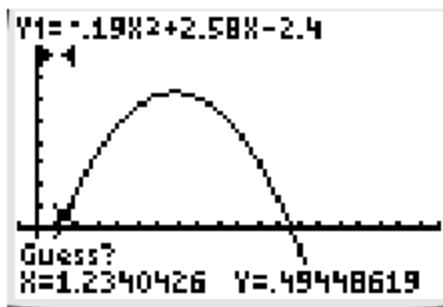
$$-.19t^2 + 2.58t - 2.4 = 0$$

**You may use the quadratic formula
or use the zero feature**

```
Plot1 Plot2 Plot3
Y1 = -.19X^2+2.58X
-2.4
```

**You may press 2nd then calc
to see and use the zero feature**

```
2nd > CALC >
1: value
2: zero
3: minimum
4: maximum
5: intersect
6: dy/dx
7: ∫f(x)dx
```

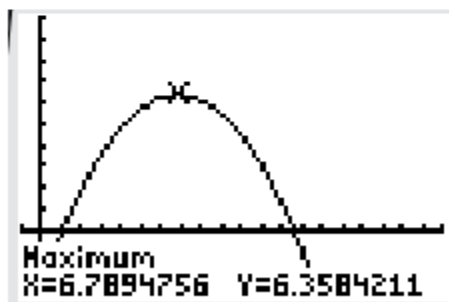


$t = 1, 12$; In 1996 and 2007, the company had 0 profit.

d)

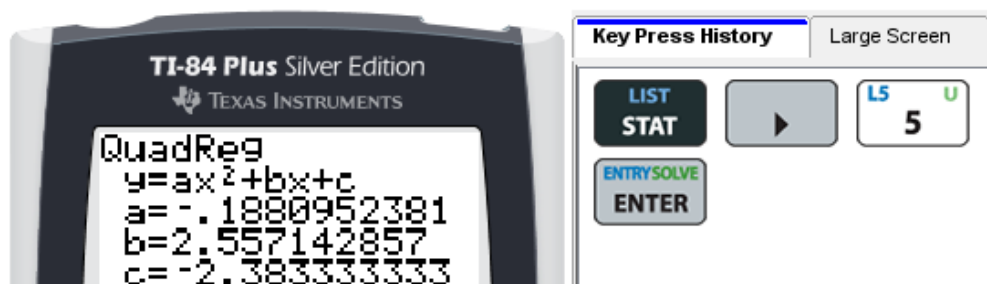
Use the max feature from 2nd calc to get the vertex of

```
Plot1 Plot2 Plot3
Y1= .19X^2+2.58X
-2.4
```



e)

L1	L2
0	-2.4
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0



close to

$$P(t) = -0.19t^2 + 2.56t - 2.38$$

12. A person standing on top of a building throws a stone into the air so that it hits the ground at the base of the building. The stone's height in feet after t seconds is given by the equation: $h = -16t^2 + 30t + 200$.

- a) Find the t -intercepts. Write your answer in a complete sentence with correct units. b) Find the h -intercept. Write your answer in a complete sentence with correct units. c) Find the vertex. Write your answer in a complete sentence with correct units. d) Sketch the graph indicating the intercepts and vertex.

a)

$$h(t) = -16t^2 + 30t + 200$$

for the t -intercepts, we shall solve

$$-16t^2 + 30t + 200 = 0$$

$$8t^2 - 15t - 100 = 0 \quad \text{divided by } -2$$

$$8t^2 - 15t - 100 = 0$$

May use the quadratic formula or the zero feature

Solutions are

$$\frac{5}{16}\sqrt{137} + \frac{15}{16} = 4.5952187220998828466$$

$$\frac{15}{16} - \frac{5}{16}\sqrt{137} = -2.7202187220998828466$$

(does not apply here)

The ball will hit the ground in 4.6 seconds.

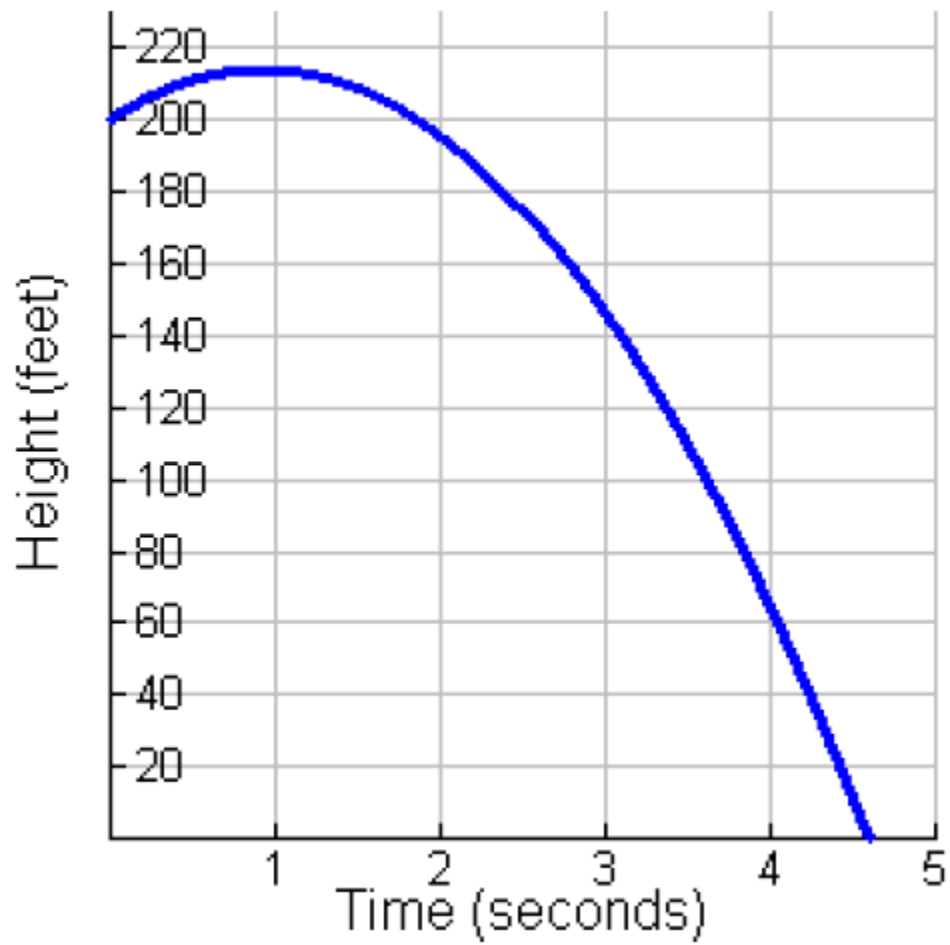
b)

$h = 200$; The ball is at 200 feet when it is thrown.

c)

Use the vertex formula or the max feature to find the answer

c) $t = 0.9375$ and $h(0.9375) = 214.0625$, vertex: $(0.9375, 214.0625)$, The ball reaches a MAXIMUM height of 214.0625 feet after 0.9375 seconds.



13. An insurance company offers a \$25,000 life insurance policy for men who are smokers. The monthly rates are listed for various ages. Let $r(t)$ represent the monthly rate in dollars for a man t years of age who smokes.
- Use your calculator to draw a scattergram of the data. What kind of function (linear, quadratic, or exponential) best fits the data?
 - Use the regression feature of your calculator to find the appropriate model for $r(t)$. Round to four decimal places.
 - Find $r(25)$ and write your result in a complete sentence with correct units.

Age	Monthly Rate in Dollars
30	50.85
35	61.74
40	92.27
45	134.97
50	193.25
55	290.18
60	444.54
65	674.06

L1	L2
30	50.85
35	61.74
40	92.27
45	134.97
50	193.25
55	290.18
60	444.54

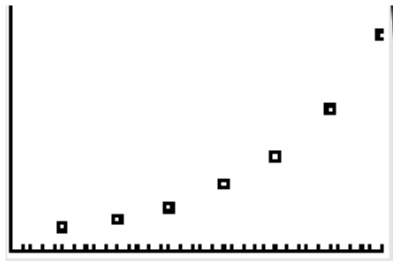
turn the plot on
make sure to adjust the
window for the table values

```

WINDOW
Xmin=25
Xmax=60
Xscl=1
Ymin=0
Ymax=500
Yscl=1
Xres=1

```

press graph



Let us rule the linear out
and fit quadratic and exp

```
QuadReg  
y=ax2+bx+c  
a=.6648095238  
b=-46.66052381  
c=871.8746429
```

$$r(t) = 0.665t^2 - 46.661t + 871.87$$

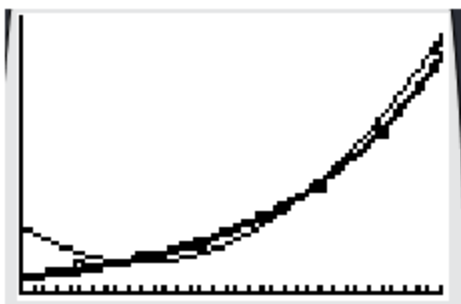
```
ExpReg  
y=a*bx  
a=4.648930658  
b=1.078546207
```

$$r(t) = 4.6489(1.0785)^t$$

enter the above two
in the calculator

darken one
of the
expressions
to
distinguish

```
Y1=.665X2-46.66  
1X+871.87  
Y2=4.6489*1.078  
50X
```

Exponential model is closer

$$4.6489 * 1.0785^{25}$$

$$= 30.75062452$$

Rate of a 25 year old smoker is \$30.75/month

15. The table lists women's and men's total enrollments at all institutions of higher learning in Country X for various years. The enrollments (in millions) of women and men, respectively, where t is the number of years since 1980, can be modeled by the following system.

$$W(t) = 0.14t + 5.69$$

$$M(t) = 0.07t + 5.62$$

- Find $(W + M)(t)$. What does it mean in the situation?
- Find $(W + M)(12)$. What does it mean in the situation?
- When was the total enrollment of men and women the same? What was that enrollment?

Year	Women	Men
1988	6.3	5.9
1990	7.4	6.5
1994	8.1	6.8
2002	8.9	7.1
2006	9.2	7.4

a) $(W + M)(t) = 0.21t + 11.31$ The function represents the combined enrollment of women and men, in millions, at t years since 1980.

b) $(W + M)(12) = 13.83$ million. In 1992, there were 13.83 million students enrolled at all institutions of higher learning in Country X

c)

To solve

$$.14t + 5.69 = .07t + 5.62$$

$$.14t - .07t = 5.62 - 5.69$$

$$0.07t = -.07$$

$$t = -\frac{.07}{.07}$$

$$t = -1$$

$$.14(-1) + 5.69 = 5.55$$

Solution: (-1.00, 5.55); This means that in 1979, both women's and men's enrollment was the same at about 5.6 million students.

15. Given $2x + 3y = 18$, determine a) the x -intercept
b) y -intercept and c) slope. d) Sketch the graph.

a) set $y = 0$ $2x + 3(0) = 18 \rightarrow 2x = 18 \rightarrow x = 9$ $(9, 0)$

b) set $x = 0$ $2(0) + 3y = 18 \rightarrow 3y = 18 \rightarrow y = 6$ $(0, 6)$

c) One way $\frac{0-6}{9-0} = -\frac{2}{3}$

another solve $2x + 3y = 18$ for y

to get $3y = -2x + 18 \rightarrow y = -\frac{2}{3}x + 6$

slope is $-\frac{2}{3}$

